# Learning from WIMPs 

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- Cosmic Microwave Background anisotropies (WMAP) imply $\Omega_{\mathrm{DM}} h^{2}=0.105_{-0.013}^{+0.007}$


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- Can also (trivially) write down "tailor-made" WIMP models
- In standard cosmology, roughly weak cross section automatically gives roughly right relic density for thermal WIMPs! (On logarithmic scale)
- Roughly weak interactions may allow both direct and indirect detection of WIMPs


## WIMP production

Let $\chi$ be a generic DM particle, $n_{\chi}$ its number density (unit: $\mathrm{GeV}^{3}$ ). Assume $\chi=\bar{\chi}$, i.e. $\chi \chi \leftrightarrow$ SM particles is possible, but single production of $\chi$ is forbidden by some symmetry.

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Evolution of $n_{\chi}$ determined by Boltzmann equation:

$$
\frac{d n_{\chi}}{d t}+3 H n_{\chi}=-\left\langle\sigma_{\mathrm{ann}} v\right\rangle\left(n_{\chi}^{2}-n_{\chi, \mathrm{eq}}^{2}\right)
$$

$H=\dot{R} / R$ : Hubble parameter
$\langle\ldots\rangle:$ Thermal averaging
$\sigma_{\text {ann }}=\sigma(\chi \chi \rightarrow$ SM particles $)$
$v$ : relative velocity between $\chi$ 's in their cms
$n_{\chi, \text { eq }}: \chi$ density in full equilibrium

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Gives

$$
\Omega_{\chi} h^{2} \propto \frac{1}{\left\langle v \sigma_{\mathrm{ann}}\right\rangle} \sim 0.1 \text { for } \sigma_{\mathrm{ann}} \sim \mathrm{pb}
$$

## Thermal WIMPs: Assumptions

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- Universe must have been sufficiently hot:
$T_{R}>T_{F} \simeq m_{\chi} / 20$
Can we test these assumptions, if $\Omega_{\chi}$ and "all" particle physics properties of $\chi$ are known?


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Use non-relativistic expansion of cross section:

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\sigma_{\mathrm{ann}}=a+b v^{2}+\mathcal{O}\left(v^{4}\right) \Longrightarrow\left\langle\sigma_{\mathrm{ann}} v\right\rangle=a+6 b / x
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## Low temperature scenario (cont.'d)

Using explicit form of $H, Y_{\chi, \text { eq }}$, Boltzmann eq. becomes

$$
\begin{gathered}
\frac{d Y_{\chi}}{d x}=-f\left(a+\frac{6 b}{x}\right) x^{-2}\left(Y_{\chi}^{2}-c x^{3} \mathrm{e}^{-2 x}\right) . \\
f=1.32 m_{\chi} M_{\mathrm{Pl}} \sqrt{g_{*}}, c=0.0210 g_{\chi}^{2} / g_{*}^{2}
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For $T_{0} \ll T_{F}$ : Annihilation term $\propto Y_{\chi}^{2}$ negligible: defines 0 -th order solution $Y_{0}(x)$, with

$$
Y_{0}(x \rightarrow \infty)=f c\left[\frac{a}{2} x_{R} \mathrm{e}^{-2 x_{R}}+\left(\frac{a}{4}+3 b\right) \mathrm{e}^{-2 x_{R}}\right] .
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For intermediate temperatures, $T_{0} \lesssim T_{F}$ : Define 1 st-order solution

$$
Y_{1}=Y_{0}+\delta
$$

$\delta<0$ describes pure annihilation:

$$
\frac{d \delta}{d x}=-f\left(a+\frac{6 b}{x}\right) \frac{Y_{0}(x)^{2}}{x^{2}} .
$$

$\delta(x)$ can be calculated analytically: $\delta \propto \sigma_{\text {ann }}^{3}$

## Low temperature scenario (cont.'d)

Get good results for $\Omega_{\chi} h^{2}$ for all $T_{0} \leq T_{F}$ through "resummation":

$$
Y_{1}=Y_{0}\left(1+\frac{\delta}{Y_{0}}\right) \simeq \frac{Y_{0}}{1-\delta / Y_{0}} \equiv Y_{1, r}
$$

$Y_{1, r} \propto 1 / \sigma_{\text {ann }}$ for $|\delta| \gg Y_{0}$ MD, Imminniyaz, Kakizaki, hep-ph/0603165

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$a=10^{-8} \mathrm{GeV}^{-2}$


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Can extend validity of new solution to all $T$, including $T \gg T_{0}$, by using $\Omega_{\chi}\left(T_{\max }\right)$ if $T_{0}>T_{\max } \simeq T_{F}$

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Note: $\Omega_{\chi}\left(T_{0}\right) \leq \Omega_{\chi}\left(T_{0} \gg T_{F}\right)$

## Application: lower bound on $T_{0}$ for thermal WIMP

MD, Imminniyaz, Kakizaki, arXiv:0704.1590 [hep-ph]
If $n_{\chi}\left(T_{0}\right)=0$, demanding $\Omega_{\chi} h^{2} \simeq 0.1$ imposes lower bound on $T_{0}$ :

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$\Longrightarrow T_{0} \geq \frac{m_{x}}{23} \quad$ Holds independent of $\sigma_{\text {ann }}!$ If $T_{0} \simeq m_{\chi} / 22$ : Get right $\Omega_{\chi} h^{2}$ for wide range of cross sections!

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- Around decoupling: $z \ll 1 \Longrightarrow$ use Taylor expansion

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A(z)=A\left(z_{F, \mathrm{st}}\right)+\left(z-z_{F, \mathrm{st}}\right) A^{\prime}\left(z_{F, \mathrm{st}}\right)+\left(z-z_{F, \mathrm{st}}\right)^{2} A^{\prime \prime}\left(z_{F, \mathrm{st}}\right) / 2
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- Successful $\mathrm{BBN} \Longrightarrow k \equiv A(z \rightarrow 0)=1.0 \pm 0.2$


## Constraining $H(T)$ (cont.d)

Assume $T_{0} \gg T_{F} \Longrightarrow \Omega_{\chi} h^{2} \propto \frac{1}{\int_{0}^{z^{F} A(z)(a+6 b z) d z}}$

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Relative constraint on $A\left(z_{F, \mathrm{st}}\right)$ weaker than that on $\Omega_{\chi} h^{2}$.

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- Detection needs ultrapure materials in deep-underground location; way to distinguish recoils from $\beta, \gamma$ events; neutron screening;. .
- Is being pursued vigorously around the world!


## Direct WIMP detection: theory

Counting rate given by

$$
\frac{d R}{d Q}=A F^{2}(Q) \int_{v_{\min }}^{v_{\text {se }}} \frac{f_{1}(v)}{v} d v
$$

$Q$ : recoil energy
$A=\rho \sigma_{0} /\left(2 m_{\chi} m_{r}\right)=$ const.: encodes particle physics
$F(Q)$ : nuclear form factor
$v$ : WIMP velocity in lab frame
$v_{\text {min }}^{2}=m_{N} Q /\left(2 m_{r}^{2}\right)$
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$f_{1}(v)$ : normalized one-dimensional WIMP velocity distribution

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In principle, can invert this relation to measure $f_{1}(v)$ !

## Direct reconstruction of $f_{1}$

MD \& C.L. Shan, astro-ph/0703651

$$
f_{1}(v)=\mathcal{N}\left\{-2 Q \frac{d}{d Q}\left[\frac{1}{F^{2}(Q)} \frac{d R}{d Q}\right]\right\}_{Q=2 m_{r}^{2} v^{2} / m_{N}}
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Need to know slope of recoil spectrum!
$d R / d Q$ is approximately exponential: better work with logarithmic slope

## Determining the logarithmic slope of $d R / d Q$

- Good local observable: Average energy transfer $\langle Q\rangle_{i}$ in $i$-th bin


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- To maximize information: use overlapping bins ("windows")


## Recoil spectrum: prediction and simulated measurement



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## Statistical exclusion of constant $f_{1}$



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Need several hundred events to begin direct reconstruction!

## Determining moments of $f_{1}$

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Can incorporate finite energy (hence velocity) threshold Moments are strongly correlated!

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\left\langle v^{n}\right\rangle & \equiv \int_{0}^{\infty} v^{n} f_{1}(v) d v \\
& \propto \int_{0}^{\infty} Q^{(n-1) / 2} \frac{1}{F^{2}(Q)} \frac{d R}{d Q} d Q \\
& \rightarrow \sum_{\text {events } a} \frac{Q_{a}^{(n-1) / 2}}{F^{2}\left(Q_{a}\right)}
\end{aligned}
$$

Can incorporate finite energy (hence velocity) threshold Moments are strongly correlated!

High moments, and their errors, are underestimated in
"typical" experiment: get large contribution from large $Q$

## Determination of first 10 moments



## Constraining a "late infall" component



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## Determining the WIMP mass

MD \& C.L. Shan, in progress
Can determine $m_{\chi}$ from requirement that different targets yield same moments of $f_{1}$



## Range of WIMP mass from simulation Preliminary!



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- Learning about WIMPs: Can determine $m_{\chi}$ from moments of $f_{1}$ measured with two different targets.

