Reconstructing the Velocity Distribution of WIMPs from Direct Dark Matter Detection Data

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Deriving $f_1(v)$ from a scattering spectrum

Reconstructing $f_1(v)$ from experimental data

Considering an annual modulated event rate

Summary

Deriving $f_1(v)$ from a scattering spectrum

- Differential rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\rm min}}^{\infty} \left[\frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\rm min} = \alpha \sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the energy Q in the detector.

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{\rm r}^2} \qquad \alpha \equiv \sqrt{\frac{m_{\rm N}}{2 m_{\rm r}^2}} \qquad m_{\rm r} = \frac{m_\chi m_{\rm N}}{m_\chi + m_{\rm N}}$$

ρ_0: WIMP density near the Earth
σ_0: total cross section ignoring the form factor suppression
$F(Q)$: elastic nuclear form factor

Deriving $f_1(v)$ from a scattering spectrum

- Normalized one-dimensional velocity distribution function

$$f_1(v) = \mathcal{N} \left\{ -2Q \cdot \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2}$$

$$\mathcal{N} = \frac{2}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$

- Moments of the velocity distribution function

$$\langle v^n \rangle = \mathcal{N}(Q_{\rm thre}) \left(\frac{\alpha^{n+1}}{2} \right) \left[\frac{2\, Q_{\rm thre}^{(n+1)/2}}{F^2(Q_{\rm thre})} \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}} + (n+1) I_n(Q_{\rm thre}) \right]$$

$$\mathcal{N}(Q_{\rm thre}) = \frac{2}{\alpha} \left[\frac{2\, Q_{\rm thre}^{1/2}}{F^2(Q_{\rm thre})} \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}} + I_0(Q_{\rm thre}) \right]^{-1}$$

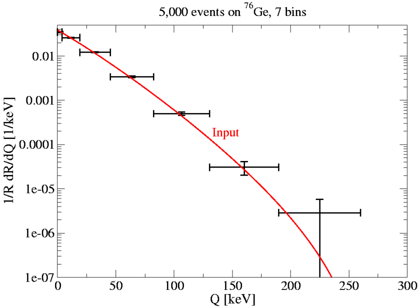
$$I_n(Q_{\rm thre}) = \int_{Q_{\rm thre}}^\infty Q^{(n-1)/2} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ$$

Reconstructing $f_1(v)$ from experimental data

- Experimental data

$$Q_n - \frac{b_n}{2} \le Q_{n,i} \le Q_n + \frac{b_n}{2} \qquad i = 1,\ 2,\ \cdots,\ N_n, \quad n = 1,\ 2,\ \cdots,\ B$$

- Theoretically predicted scattering spectrum



Reconstructing $f_1(v)$ from experimental data

- Ansatz: in the nth Q-bin

$$\left(\frac{dR}{dQ}\right)_n \equiv \left(\frac{dR}{dQ}\right)_{Q \simeq Q_n} = \widetilde{r}_n e^{k_n (Q - Q_n)} \equiv r_n e^{k_n (Q - Q_{s,n})}$$

$$\widetilde{r}_n \equiv \left(\frac{dR}{dQ}\right)_{Q = Q_n} \qquad r_n \equiv \frac{N_n}{b_n}$$

- Recoil spectrum at $Q = Q_n$

$$\widetilde{r}_n = \frac{N_n}{b_n} \left(\frac{\widetilde{k}_n}{\sinh \widetilde{k}_n}\right) \qquad \widetilde{k}_n \equiv \left(\frac{b_n}{2}\right) k_n$$

- Logarithmic slope and shifted point in the nth Q-bin

$$\overline{Q}_n - Q_n = \frac{b_n}{2} \left(\coth \widetilde{k}_n - \frac{1}{\widetilde{k}_n}\right) \qquad \overline{Q}_n = \frac{1}{N_n} \sum_{i=1}^{N_n} Q_{n,i}$$

$$Q_{s,n} = Q_n + \frac{1}{k_n} \ln \left(\frac{\sinh \widetilde{k}_n}{\widetilde{k}_n}\right)$$

Reconstructing $f_1(v)$ from experimental data

- Reconstructed one-dimensional velocity distribution function

$$f_{1,r}(v_{s,\mu}) = \mathcal{N}\left[\frac{2Q_{s,\mu}r_\mu}{F^2(Q_{s,\mu})}\right]\left[\frac{d}{dQ}\ln F^2(Q)\bigg|_{Q=Q_{s,\mu}} - k_\mu\right]$$

$$v_{s,\mu} = \alpha\sqrt{Q_{s,\mu}}$$

$$\mathcal{N} = \frac{2}{\alpha}\left[\sum_a \frac{1}{\sqrt{Q_a}\,F^2(Q_a)}\right]^{-1}$$

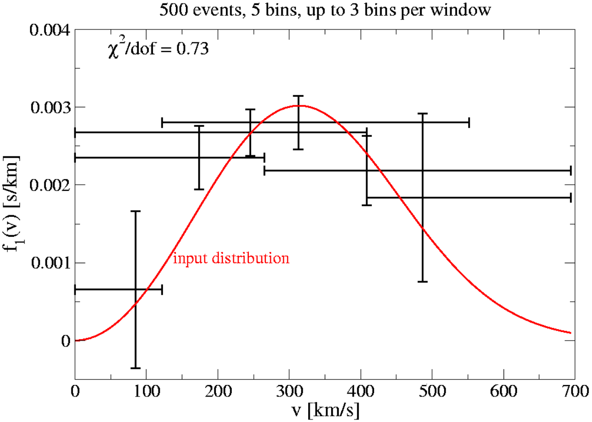
- Determining moments of the velocity distribution function

$$\langle v^n \rangle = \mathcal{N}\left(\frac{\alpha^{n+1}}{2}\right)\cdot(n+1)I_n$$

$$I_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}$$

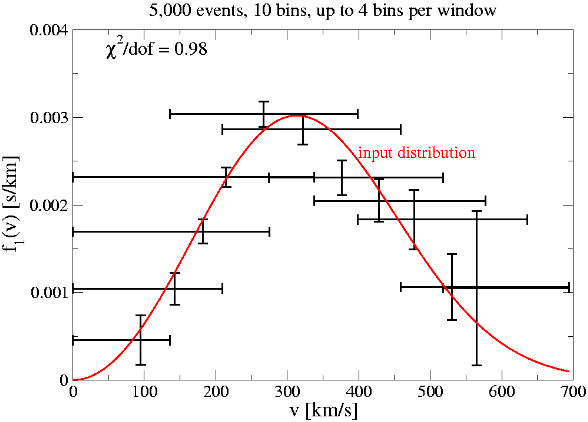
Reconstructing $f_1(v)$ from experimental data

- Reconstruction with simulated experimental data (500 events, 5 bins, up to 3 bins per window)



Reconstructing $f_1(v)$ from experimental data

- Reconstruction with simulated experimental data
  (5,000 events, 10 bins, up to 4 bins per window)



Considering an annual modulated event rate

- Fourier cosine series ($\omega \equiv 2\pi/365$)

$$\left(\frac{dR}{dQ}\right)_t = \left(\frac{dR}{dQ}\right)_{(0)} + \left(\frac{dR}{dQ}\right)_{(1)} \cos(\omega t) + \left(\frac{dR}{dQ}\right)_{(2)} \cos(2\omega t) + \cdots$$

$$f_1(v,t) = f_{1,(0)}(v) + f_{1,(1)}(v)\cos(\omega t) + f_{1,(2)}(v)\cos(2\omega t) + \cdots$$

$$\left(\frac{dR}{dQ}\right)_{(m)} = \mathcal{A}F^2(Q)\int_{v_{\rm min}}^{\infty}\left[\frac{f_{1,(m)}(v)}{v}\right]dv \qquad m = 0,\ 1,\ 2,\ \cdots$$

- Determining N_n and $\overline{(Q-Q_n)^\lambda}|_n$

$$N_n = \int_{Q_n-b_n/2}^{Q_n+b_n/2}\left(\frac{dR}{dQ}\right)_{(0)} dQ = \frac{1}{365}\int_0^{365}\int_{Q_n-b_n/2}^{Q_n+b_n/2}\left(\frac{dR}{dQ}\right)_t dQ\, dt = \frac{N_{n,1\ {\rm yr}}}{365}$$

$$\overline{(Q-Q_n)^\lambda}|_n = \frac{1}{N_{n,1\ {\rm yr}}}\sum_{i=1}^{N_{n,1\ {\rm yr}}}(Q_{n,i}-Q_n)^\lambda$$

Summary

- From a recoil spectrum dR/dQ of the elastic WIMP-nucleus scattering we can derive $f_1(v)$ and $\langle v^n \rangle$ of WIMPs.

- From experimental data of direct WIMP detection measured over some whole years we can reconstruct $f_1(v)$ and $\langle v^n \rangle$ directly.

- Our reconstructions of $f_1(v)$ and $\langle v^n \rangle$ are independent of each other and of some as yet unknown quantities, e.g. WIMP density near the Earth. The only information needed is the WIMP mass.