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Physikalisches Institut
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Condensed Matter Theory I — WS09/10

Exercise 5

(Please return your solutions before Fr., 18.12., 12:00h)

5.1 Sound waves in a Fermi liquid (part 2) (15 points)

We consider the collision dominated regime (1st sound): $\omega_{s1} \ll 1/\tau$.
Make the ansatz of a time-dependent, local equilibrium distribution

$$n_{\mathbf{k}\sigma}(\mathbf{r},t) = n^0_{\mathbf{k}\sigma}(E_{\mathbf{k}\sigma} - \epsilon_F(\mathbf{r},t))$$
$$\epsilon_F(\mathbf{r},t) = \epsilon^0_F + \Delta\epsilon^0_F\, e^{\mathrm{i}(\mathbf{k}_{s1}\mathbf{r}-\omega_{s1}t)}$$

with a complex sound frequency ω_{s1} (damping) to solve the Boltzmann equation:

$$\frac{dn_{\mathbf{k}\sigma}}{dt} = \frac{\partial n_{\mathbf{k}\sigma}}{\partial t} + \mathbf{v}_{\mathbf{k}} \cdot \frac{\partial n_{\mathbf{k}\sigma}}{\partial \mathbf{r}} - \frac{\partial E_{\mathbf{k}\sigma}}{\partial \mathbf{r}}\frac{\partial n^0_{\mathbf{k}\sigma}}{\partial \mathbf{k}} = I\{n_{\mathbf{k}\sigma}\}\,.$$

Why does the collision integral $I\{n_{\mathbf{k}\sigma}\}$ vanish in the collision dominated regime?
Determine the 1st sound dispersion $\mathrm{Re}(\omega_{s1}(\mathbf{k}))$ and the damping rate $\mathrm{Im}(\omega_{s1})$.

5.2 Landau quantization (15 points)

We consider a 2-dimensional system of non-interacting electrons under the influence of a homogeneous magnetic field of strength B in z-direction.

(a) The electromagnetic vector potential in Landau gauge is given by

$$\mathbf{A}_L = \begin{pmatrix} 0 \\ Bx \\ 0 \end{pmatrix}.$$

Use minimal coupling to show that the Hamiltonian describes a harmonic oscillator in x-direction and free motion in y- and z-direction. Write down the energy eigenfunctions. What is the degeneracy of the energy states (Landau levels)?

(b) Because of gauge freedom, the Landau gauge is not the only valid choice for the vector potential. Consider the isotropic gauge

$$\mathbf{A}_I = \frac{1}{2}\begin{pmatrix} -By \\ Bx \\ 0 \end{pmatrix}.$$

Calculate $\Delta\mathbf{A} = \mathbf{A}_I - \mathbf{A}_L$. Show that $\Delta\mathbf{A}$ can be written as a gradient field $\Delta\mathbf{A} = \nabla\Theta$ and conclude that this gauge describes the same physics as the Landau gauge.

(c) The eigenfunction ψ for the isotropic case can be constructed out of the function for the Landau gauge by multiplying with a phase factor $\exp(\mathrm{i}\Theta(x,y))$. How do the eigenfunctions look like?

(d) What do you expect for the electron energy distribution in an infinitely extended 2-dimensional system?