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Advanced Theoretical Condensed Matter Physics — SS11

Exercise 8

(Please have your solutions ready by Mon. 4.06.2011)

In the lecture you have heard about the Keldysh formalism and how it can be used to calculate nonequilibrium quantites. You have also heard how one can derive kinetic equations from the Keldysh nonequlibrium formalism, among them the famous Boltzmann equation. In this exercise sheet we will start from the Boltzmann equation as derived in the lecture, obtain from it the *diffusive* Boltzmann equation, and finally calculate its *stationary* solution for the specific case of a 1D wire spanning two leads with a voltage drop applied between them.

8.1. Diffusive Motion of Noninteracting Electrons in a One-Dimensional Wire

We look at *noninteracting* electrons. The physical system we want to describe is illustrated in Fig. 1. The wire is *disordered*; therefore multiple scattering between the electrons and the impurities is dominant, and hence we can look at the diffusive approximation to the Boltzmann equation. We apply a potential difference V across the wire in order that a current could be driven across it from left to right. We want to obtain an expression for the stationary distribution function $\bar{f}(x,\omega)$ of electrons at an arbitrary point x, where $|x| \leq L$. For convenience we write down the classical form of Boltzmann equation as derived in the lecture

$$\frac{\partial}{\partial t} f(t,\mathbf{r},\mathbf{p},E) + \mathbf{v_p} \cdot \frac{\partial}{\partial \mathbf{r}} f(t,\mathbf{r},\mathbf{p},E) + \mathbf{F} \cdot \frac{\partial}{\partial \mathbf{p}} f(t,\mathbf{r},\mathbf{p},E) = 0 \tag{1}$$

for noninteracting electrons. We are interested in the *stationary* distribution in the wire at point x, summed over $\mathbf{p}$:

$$\bar{f}(x) = \sum_{\mathbf{p}} f(t = \text{const.}, x, \mathbf{p}, E = E_F) \tag{2}$$

We also define 2 important quantites for the solution of the problem: we define the *charge density*

$$\rho(t,\mathbf{r}) = \sum_{\mathbf{p}} f(t,\mathbf{r},\mathbf{p}) \tag{3}$$

and the *current density*

$$\mathbf{j}(t,\mathbf{r}) = \sum_{\mathbf{p}} \mathbf{v_p} f(t,\mathbf{r},\mathbf{p}) \tag{4}$$

(3) and (4) is connected by Fick's law:

$$\mathbf{j} = -D\nabla\rho \tag{5}$$

a) Using the definition (2) and the quantities in (3) and (4), show that

$$\frac{\partial f}{\partial t} - D\nabla^2 f = 0 \tag{6}$$

i.e., that the distribution function obeys a diffusion equation.

b) We want to obtain a stationary, i.e., time-independent solution to (6). We note that, due to the potential applied across the wire, the chemical potential of the left and right reservior will be shifted relative to each other by the applied voltage. In other words, the distribution functions defining the left and right reserviors, $f(x = 0)$ and $f(x = L)$ respectively, are specified by their respective chemical potentials:

$$\bar{f}(x = 0) = f_L(E) = \frac{1}{e^{\beta(E+\mu_L)} + 1}$$
$$\bar{f}(x = L) = f_R(E) = \frac{1}{e^{\beta(E+\mu_R)} + 1}$$

where

$$\mu_{L/R} \equiv \mu \pm \frac{eV}{2}$$

Using the boundary conditions as described above, solve (6) for the stationary case $\bar{f}(x, E)$.

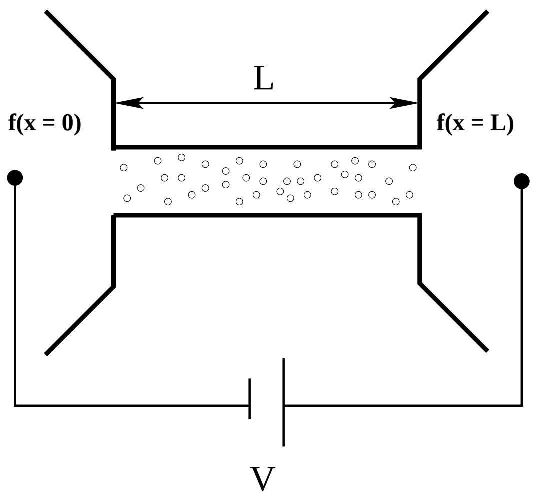

Figure 1: Ths system with the relevant parameters labeled.

c) Sketch the resulting stationary distribution function $\bar{f}(x, E)$.