Landau-Ginzburg Vacua of String, M- and F-Theory at c=12 (hep–th/9812195)

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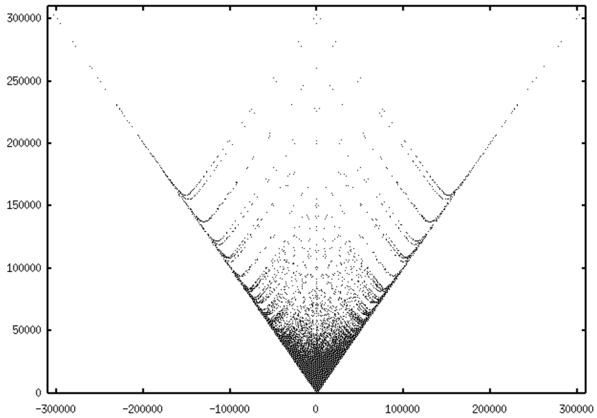
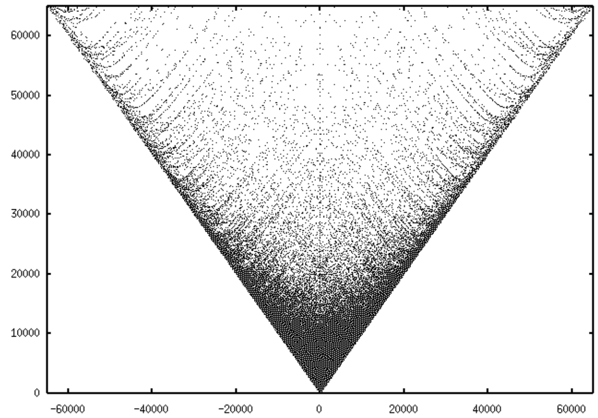
Theories in more than ten dimensions play an important role in understanding nonperturbative aspects of string theory. Four dimensional compactifications of such theories can be constructed via Calabi–Yau (CY) fourfolds. These models will be analyzed particularly efficiently in the Landau–Ginzburg (LG) phase of the linear $\sigma-$model, when available. We focus on those $\sigma-$models which have both a LG phase and a geometric CY phase described by a hypersurface in weighted projective five–space. Some of the pertinent properties of these 1,100,055 models, such as cohomology, mirror symmetry and fibration structure, are presented.

Using an N=2 superspace formulation we can view the LG phase of a linear $U(1)$ $\sigma-$model to be described by the action $\mathcal{A}=\int d^2zd^4\theta K(\Phi_i,\overline{\Phi}_i)+\int d^2zd^2\theta^- W(\Phi_i)+h.c.$ K is the Kähler potential and the superpotential W is a holomorphic function of the chiral superfields Φ_i. The superpotentials of interest are quasihomogeneous polynomials with an isolated, completely degenerate singularity. Let Φ_i have the weight $q_i = k_i/d$. Then, a catastrophe is associated to W to be the zero locus of polynomials of degree d in variables of weight k_i. This defines an affine variety. In the manifold context we are now interested in complex four dimensional Kähler manifolds with c_1=0. In order to compactify the variety we embed the hypersurface defined by the zero locus into a projective space with weights k_i. For a theory with six fields this construction is completely sufficient to pass from the LG theory to a string vacuum, provided $d=\sum_{i=1}^{6} k_i$, i.e. c_1=0.

If we require the existence of a transverse polynomial, Bertini's theorem will put constraints on the weights k_i. Anomaly cancellation forces the LG theory to have central charge c=12. These conditions lead to a programmable algorithm for the construction of all possible weight sets [1]. In the fourfold case running the program takes several decades of CPU time, whereas for threefolds [1] the run finished after ten minutes.

We have constructed 1,100,055 LG theories at c=12, where 360,346 configurations exhibit a trivial quadratic factor Φ_6^2 in W and the remaining 739,709 correspond to more general theories. The configuration with the maximal degree is $\mathbb{P}_{(1806,151662,931638,2173882,3260733)}[6521466]$ with Hodge numbers $h^{(2,2)}$=1,213,644, $h^{(1,1)}$=303,148, $h^{(2,1)}$=0, $h^{(3,1)}$=252. This class of models leads to 667,954 different spectra (Hodge triplets). They determine the complete Hodge diamond ($h^{(1,1)}$, $h^{(2,1)}$, $h^{(3,1)}$, $h^{(2,2)}$) of the geometric CY phase because the four Hodge numbers depend on each other via the linear relation $44+4h^{(3,1)}+4h^{(1,1)}=2h^{(2,1)}+h^{(2,2)}$. Thus, the Euler number $\chi=h^{(2,2)}+2h^{(3,1)}+2h^{(1,1)}-4h^{(2,1)}+4$ will be divisible by six: $\chi=6(8+h^{1,1}-h^{2,1}+h^{3,1})$. Only 204 of all the models have a negative χ with only 24 distinct values $-252\leq\chi\leq-6$. Our data completely contain the brute force results gained by checking the partitions of $d\leq4000$ into k_i [2], a pretty good test of the code. There are 583,824 distinct pairs of combinations ($h^{(3,1)}+h^{(1,1)}$, $h^{(3,1)}-h^{(1,1)}$), shown in Figs. 1 & 2.

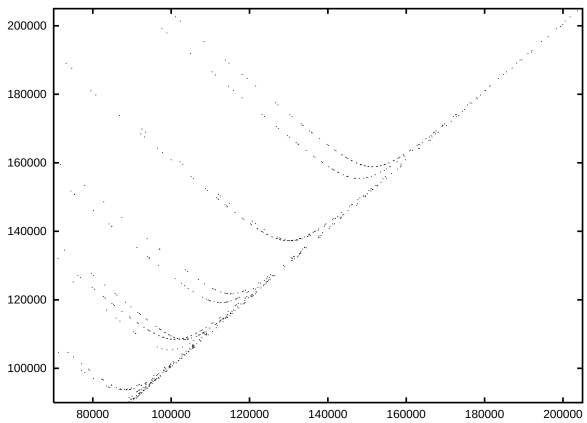




Figs. 1 & 2: *plot of* $h^{(3,1)}+h^{(1,1)}$ *vs.* $h^{(3,1)}-h^{(1,1)}$ *for all* CY_4 *hypersurfaces in weighted* $\mathbb{P}_5$ *& a zoom thereof*

Apparently, the space of CY fourfold hypersurfaces exhibits a high degree of mirror symmetry. The degree is roughly 70% and 1205 of the 202,492 mirror pairs are Hodge theoretically self–mirror, i.e. $h^{(3,1)}=h^{(1,1)}$.

In the fourfold mirror plot of Fig. 1 a remarkable parabolic structure appears in the upper regions of the plot as it also does in the threefold case. In the lower region of Fig. 1 this feature is obscured by the high density of points. The close–up view in Fig. 2 shows that this feature persists even for manifolds with smaller cohomology groups. In Fig. 3 we present a close–up view of such curves which suggests that they do indeed describe parabolas (regression coefficient $r\approx0.999$). A further zoom in Fig. 4 shows that the parabolic structure is not perfect. However, the two points not fitting the curve may belong to a different parabola.



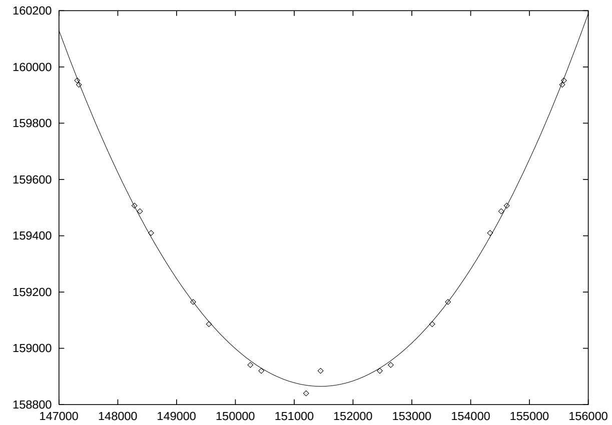
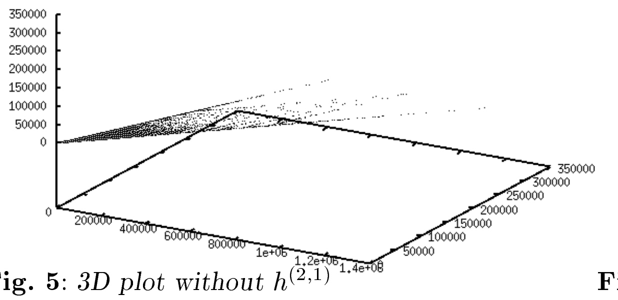


Fig. 3: *close–up view of the parabola–like curves* **Fig. 4**: *the points belonging to a single parabola*

Since three Hogde numbers are independent, we also consider 3D plots omitting $h^{(2,1)}$ and $h^{(3,1)}$, respectively.



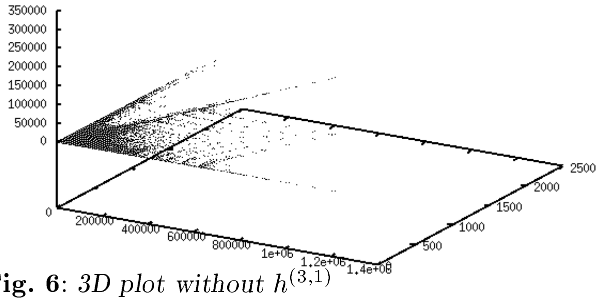
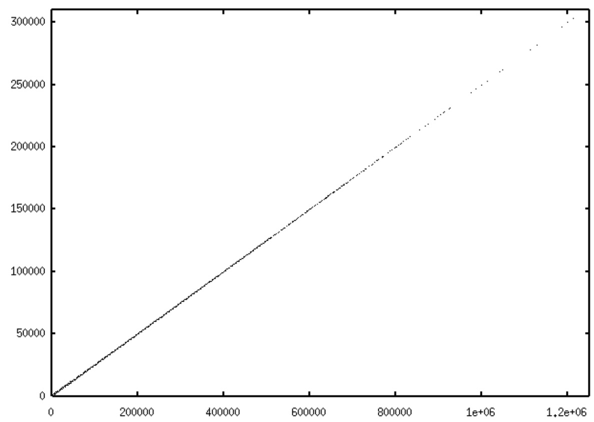


Fig. 5: *3D plot without* $h^{(2,1)}$ **Fig. 6**: *3D plot without* $h^{(3,1)}$

The points in Figs. 5 and 6 seem to form a plane, so we should look for an empirical linear relation among the Hodge numbers. We do indeed find that $h^{(2,2)} \approx 4.00037\,(h^{(3,1)}+h^{(1,1)})-7.6$ is satisfied with surprisingly good accuracy ($r \approx 1-10^{-5}$). This is illustrated by plotting $h^{(2,2)}$ vs. $h^{(3,1)}+h^{(1,1)}$ in Fig. 7. However, looking at $4.00037\,(h^{(3,1)}+h^{(1,1)})-h^{(2,2)}-7.6$ vs. $\log h^{(2,2)}$ in Fig. 8 demonstrates that this relation does not hold exactly.



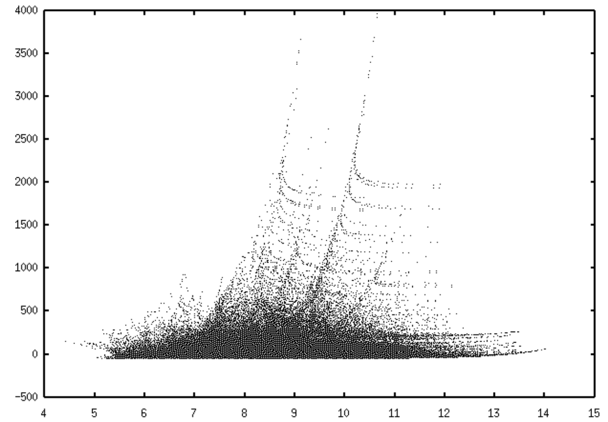


Fig. 7: *plot of* $h^{(2,2)}$ *vs.* $h^{(3,1)}+h^{(1,1)}$ **Fig. 8**: *deviation from linearity in Fig. 7*

In the context of dualities involving F–theory fibered manifolds have become of particular interest. There are some simple sufficient criteria for the existence of fibrations, given in terms of the weights only [3]. These criteria are not necessary and hence will not yield an exhaustive list but they provide a wealth of examples.

A search for fibrations whose generic smooth fiber is a CY threefold results in 35,540 examples. Restricting further leads to 8,270 K3 and finally 4,305 elliptic fibrations. Because of their nested fibration structure these are the simplest type examples to analyze problems in F/heterotic duality: $\mathrm{T}^2 \to \mathrm{K3} \underset{\searrow \mathbf{P}_1}{\longrightarrow} \mathrm{CY}_3 \underset{\searrow \mathbf{P}_1}{\longrightarrow} \mathrm{CY}_4 {}_{\searrow \mathbf{P}_1}$

For F–theory we only need to consider more general fourfolds which are either K3 fibered or elliptic. We obtain 49,751 elliptic and 27,099 K3 fibrations. 13,285 of the latter ones pass the test of being also elliptic.

Refs.[1] A. Klemm, R. Schimmrigk, *Landau–Ginzburg String Vacua*, Nucl. Phys. **B411** (1994) 559–583

[2] M. Kreuzer, H. Skarke, *Calabi–Yau Fourfolds and Toric Fibrations*, J.Geom.Phys. **26** (1998) 272–290

[3] S. Hosono, B.H. Liam, S.T. Yau, *Calabi–Yau Varities and Pencils of* K3 *Surfaces*, alg–geom/9603020