

Abundance of Thermal Relics in Non-standard Cosmological Scenarios

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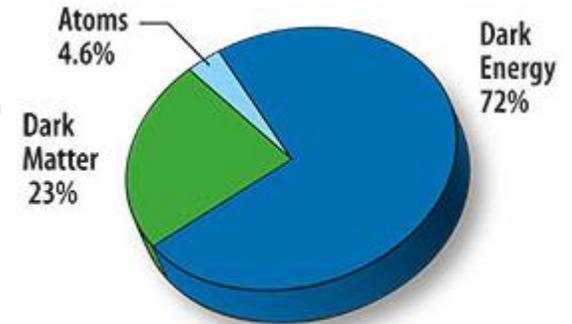
Refs:

- PRD73 123502 (2006)
- PRD76 103524 (2007)
- work in progress

Tohoku University

1. Motivation

- Observations of
 - cosmic microwave background
 - structure of the universe
 - etc.



[<http://wmap.gsfc.nasa.gov>]

→ Non-baryonic dark matter: $\Omega_{\text{DM}} h^2 = 0.1143 \pm 0.0034$

Physics beyond the standard model (SM) of particle physics necessary

- Weakly interacting massive particles (WIMPs) χ are good candidates for cold dark matter (CDM)

The predicted thermal relic abundance naturally explains the observed dark matter abundance: $\Omega_{\chi, \text{standard}} h^2 \sim 0.1$

- Neutralino (LSP); 1st KK mode of the B boson (LKP); etc.

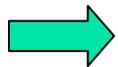
Investigation of early universe using DM abundance

- The abundance of thermal relics (e.g. DM) is determined by the Boltzmann equation:

$$\dot{n}_\chi + 3H n_\chi = -\langle \sigma_{\text{eff}} v \rangle (n_\chi^2 - n_{\chi,\text{eq}}^2)$$

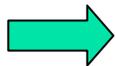
(and the reheat temperature: T_R)

Numerical calculation needed in evaluating the relic density in many cases



Analytic methods should be developed in various scenarios

- The (effective) cross section σ_{eff} can be determined from collider and DM detection experiments



We can test the standard CDM scenario and investigate conditions of very early universe: T_R, H, \dots

$\Omega_{\chi,\text{standard}} h^2 > \Omega_{\chi,\text{DM}}$ Non-thermal production, ...

$\Omega_{\chi,\text{standard}} h^2 < \Omega_{\chi,\text{DM}}$ Low T_R , modified H , Late production of entropy, ...

Outline

This work

- Analytic treatment applicable to low-reheat-temperature scenarios
- Dark matter = thermal WIMPs
 - constraints on the reheating temperature and
 - on modifications of the Hubble parameter
- Analytic treatment that connects the hot and cold relic solutions and late entropy production by semi-relativistic relics

1. Motivation
2. Standard calculation of WIMP relic abundance (review)
3. Low-temperature scenario
4. Constraints on the very early universe from WIMP dark matter
5. Abundance of semi-relativistic relics and its application
6. Summary

2. Standard calculation of the WIMP relic abundance

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- Conventional assumptions for WIMPs as DM particle:

- $\chi = \bar{\chi}$, single production of χ is forbidden

- WIMP abundance n_χ is determined by the Boltzmann eq.:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle(n_\chi^2 - n_{\chi,\text{eq}}^2)$$

$H = \dot{R}/R$: Hubble expansion parameter

$\langle\sigma v\rangle$: thermal average of the annihilation cross section

$\sigma(\chi\chi \rightarrow \text{SM particles})$ times relative velocity v

$n_{\chi,\text{eq}}$: equilibrium number density

- Introduce $Y_{\chi(\text{,eq})} = \frac{n_{\chi(\text{,eq})}}{s}$, $x = \frac{m_\chi}{T}$

$$\rightarrow \frac{dY_\chi}{dx} = -\frac{\langle\sigma v\rangle s}{Hx}(Y_\chi^2 - Y_{\chi,\text{eq}}^2)$$

Standard formula

- High temperature ($T \geq T_F$):
 - Thermal equilibrium was maintained:

$$\Gamma = n_\chi \langle \sigma v \rangle > H = R/\dot{R}$$

- χ decoupled when non-relativistic in RD era:

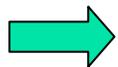
$$H = \frac{\pi T^2}{M_{\text{Pl}}} \sqrt{\frac{90}{g_*}}$$

$$\langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2),$$

$$n_{\chi, \text{eq}} = g_\chi \left(\frac{m_\chi T}{2\pi} \right)^{3/2} e^{-m_\chi/T}$$

- Low temperature ($T \leq T_F$):

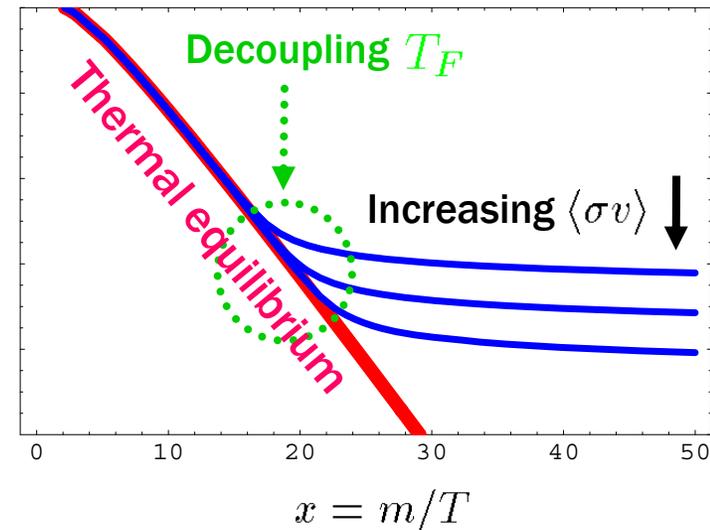
- WIMP production negligible :



$$\Omega_{\chi, \text{standard}} h^2 \simeq 0.1 \times \left(\frac{a + 3b/x_F}{10^{-9} \text{ GeV}^{-2}} \right)^{-1} \left(\frac{x_F}{22} \right) \left(\frac{g_*}{90} \right)^{-1/2} \sim \Omega_{\text{DM}} h^2$$

Co-moving number density

$$Y_\chi = n_\chi/s$$



$$\Omega_\chi h^2 = 2.7 \times 10^8 Y_\chi \left(\frac{m_\chi}{1 \text{ GeV}} \right)$$

3. Low-temperature scenario

[c.f. Giudice,Kolb,Riotto(2001), Gelmini,Gondolo(2006)]

- T_R : Reheat temperature

The initial abundance is assumed to be negligible: $Y_\chi(x_0) = 0$, $x_0 = \frac{m_\chi}{T_R}$

- Zeroth order approximation:

$T_R < T_F$ \rightarrow χ annihilation is negligible:

$$\frac{dY_0}{dx} = 0.028 g_\chi^2 g_*^{-3/2} m_\chi M_{\text{Pl}} e^{-2x} x \left(a + \frac{6b}{x} \right)$$

\rightarrow The solution is proportional to the cross section:

At late times,

$$Y_0(x \gg x_0) \simeq 0.014 g_\chi^2 g_*^{-3/2} m_\chi M_{\text{Pl}} e^{-2x_0} x_0 \left(a + \frac{6b}{x_0} \right)$$

This solution should be smoothly connected to the standard result

First order approximation

- Add a correction term describing annihilation to Y_0 : $Y_1 = Y_0 + \delta$ ($\delta < 0$)
- As long as $|\delta| \ll Y_0$, the evolution equation for δ is

$$\frac{d\delta}{dx} = -1.3 \sqrt{g_*} m_\chi M_{\text{PL}} \left(a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}$$

➡ The solution is proportional to σ^3

At late times,

$$\delta(x \gg x_0) \simeq -2.5 \times 10^{-4} g_\chi^4 g_*^{-5/2} m^3 M_{\text{Pl}}^3 e^{-4x_0} x_0 \left(a + \frac{3b}{x_0} \right) \left(a + \frac{6b}{x_0} \right)^2$$

- $|\delta|$ soon dominates over Y_0 for not very small cross section

➡ Y_1 fails to track the exact solution

Re-summed ansatz

- It is noticed that $Y_0 \propto \sigma > 0$, $\delta \propto \sigma^3 < 0$

For large cross section,

$Y_\chi(x \rightarrow \infty)$ should be $\propto 1/\langle\sigma v\rangle$

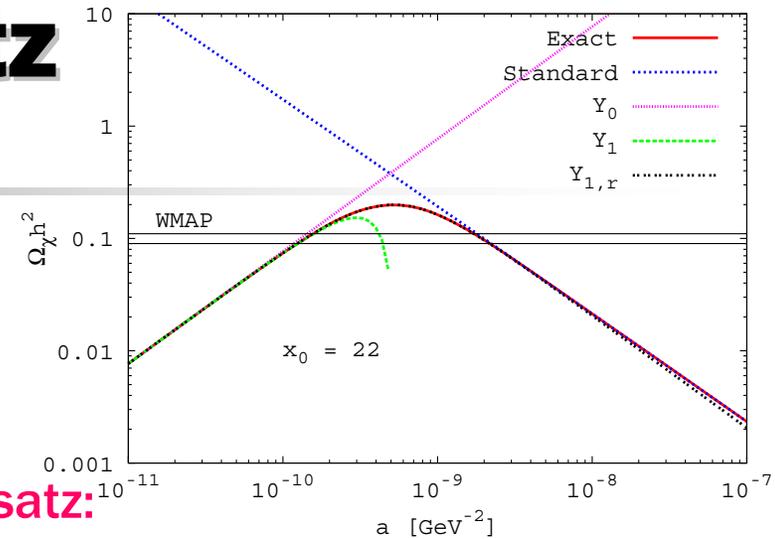
- This observation suggests the re-summed ansatz:

$$Y(x) = Y_0 + \delta = Y_0 \left(1 + \frac{\delta}{Y_0} \right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$

- For $|\delta| \gg Y_0$, $Y_{1,r}(x) \simeq -\frac{Y_0^2}{\delta} \propto \frac{1}{\sigma}$

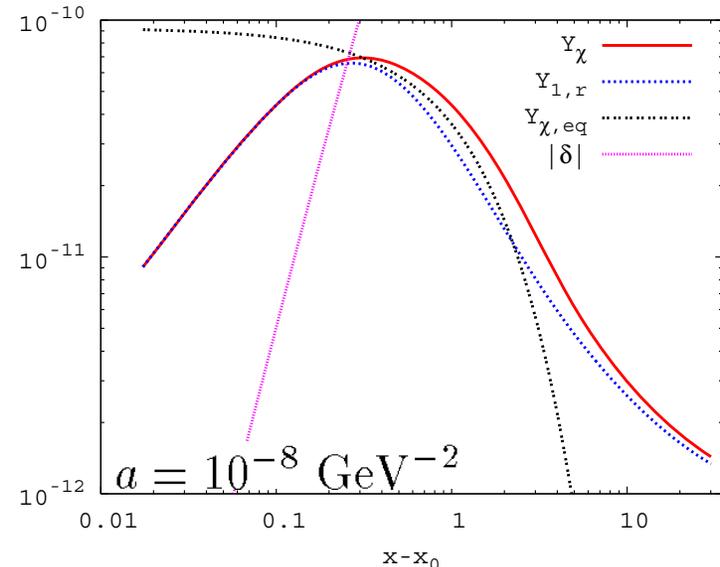
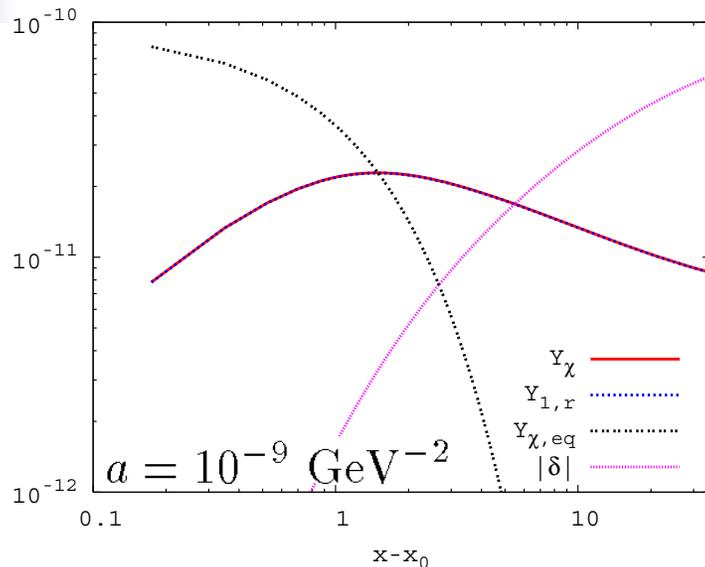
At late times, $Y_{1,r}(x \rightarrow \infty) = \frac{x_0}{1.3 \sqrt{g_*} m_\chi M_{\text{Pl}} (a + 3b/x_0)}$

- In the case where χ production is negligible but the initial abundance is sizable, $Y_{1,r}$ is exact



$x_0 \rightarrow x_F \rightarrow$ Standard formula

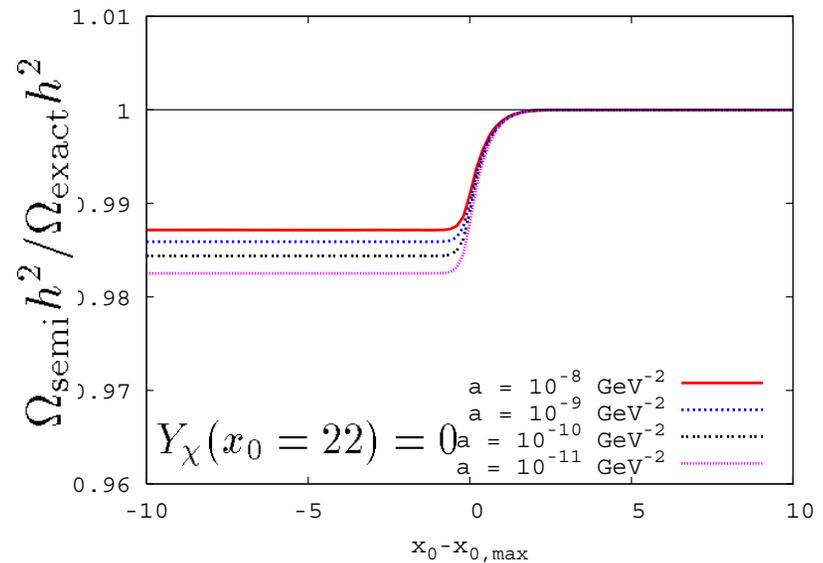
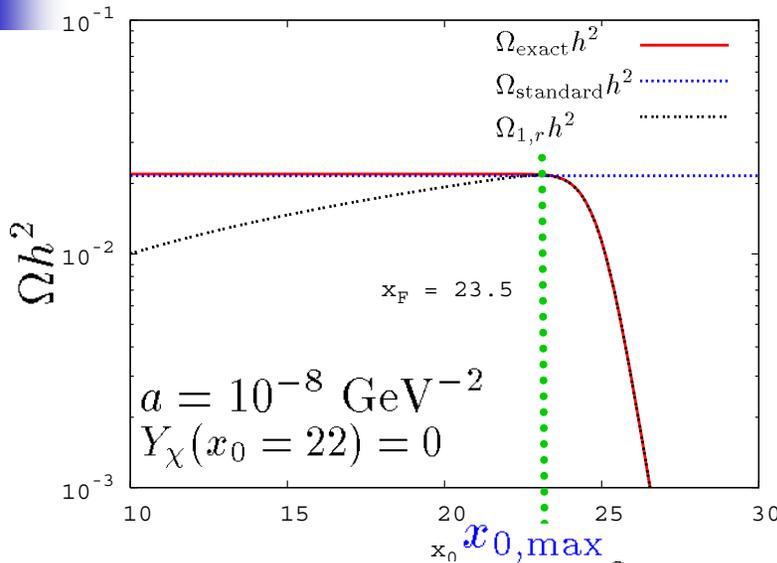
Evolution of solutions



Y_χ : Exact result, $Y_{1,r}$: Re-summed ansatz, $b = 0$, $Y_\chi(x_0 = 22) = 0$

- The re-summed ansatz $Y_{1,r}$ describes the full temperature dependence of the abundance when equilibrium is not reached
- For larger cross section the deviation becomes sizable for $x - x_0 \sim 1$, but the deviation becomes smaller for $x \gg x_0$

Semi-analytic solution



- $Y_{1,r}(x_0, x \rightarrow \infty) (\propto \Omega_{1,r} h^2)$ has a maximum (left)
- New semi-analytic solution can be constructed: $\Omega_{\text{semi}} h^2$ (right)

For $x_0 > x_{0,\text{max}}$, use $Y_{1,r}(x_0)$; for $x_0 < x_{0,\text{max}}$, use $Y_{1,r}(x_{0,\text{max}})$

The semi-analytic solution $\Omega_{\text{semi}} h^2$ reproduces the correct final relic density $\Omega_{\text{exact}} h^2$ to an accuracy of a few percent

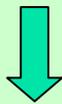
4. Constraints on the very early universe from WIMP DM

- Out-of-equilibrium case: $\sigma \nearrow \rightarrow \Omega h^2 \nearrow$; $T_0 = m_\chi/x_0 \nearrow \rightarrow \Omega h^2 \nearrow$
- Equilibrium case: $\sigma \nearrow \rightarrow \Omega h^2 \searrow$; $\Omega_\chi h^2$ is independent of T_R

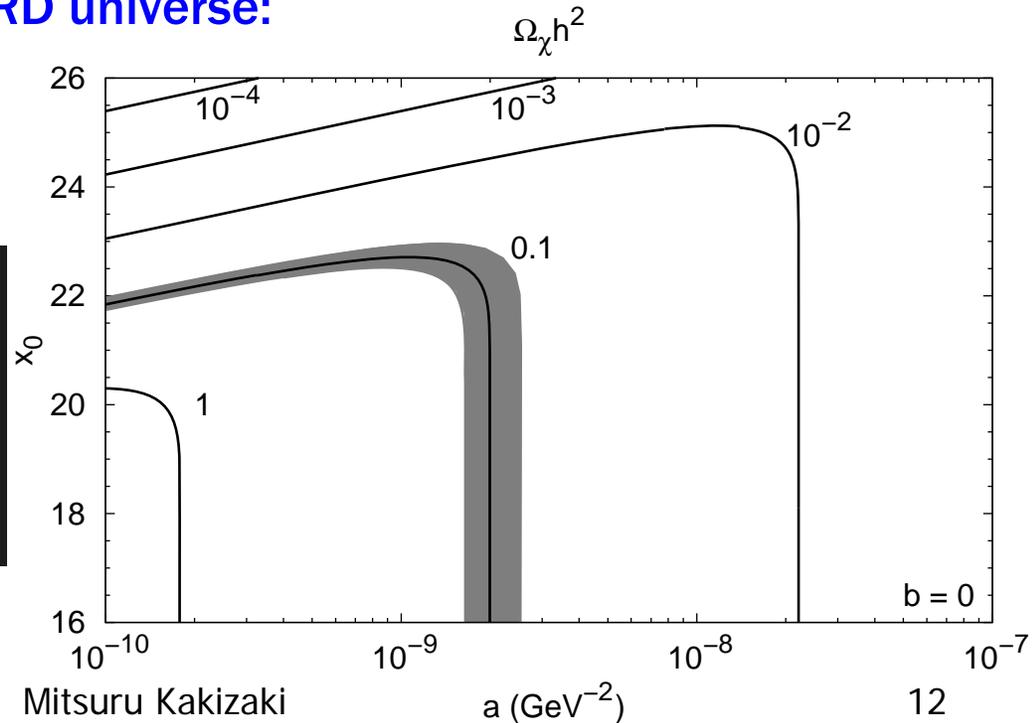
- Thermal relic abundance in the RD universe:

■ $0.8 < \Omega_{\text{DM}} h^2 < 0.12$

Requirement that $\Omega_\chi h^2 \simeq 0.1$



Lower bound on the reheat temperature: $T_R > m_\chi/23$



Modified expansion rate

- Various cosmological models predict a non-standard early expansion
 [e.g. Scherrer et al.,PRD(1985); Salati,PLB(2003);
 Ferrigno et al.,PRD(2003); Chung et al., PRD (1999); ...]
- ➡ Predicted WIMP relic abundances are also changed
- When WIMPs were in full thermal equilibrium, in terms of the modification parameter $A(x) = H_{\text{st}}(x)/H(x)$ the relic abundance is

$$\Omega_\chi h^2 = 0.1 \left(\frac{I(x_F)}{8.5 \times 10^{-10} \text{ GeV}^{-2}} \right)^{-1}$$

$$I(x_F) = \int_{x_F}^{\infty} dx \frac{\sqrt{g_*} \langle \sigma v \rangle A(x)}{x^2}, \quad x_F = \ln \left[\sqrt{\frac{45}{\pi^5}} \xi m_\chi M_{\text{Pl}} g_\chi \frac{\langle \sigma v \rangle A(x)}{\sqrt{x g_*}} \right] \Big|_{x=x_F}$$

If $A(x) = 1$, $x_F = x_{F,\text{st}}$ and we recover the standard formula

This formula is capable of predicting the final relic density correctly

Constraints on modifications of the Hubble parameter

- In terms of $z \equiv T/m_\chi = 1/x$

we need to know $A(z)$ only for $z_{\text{BBN}} = 10^{-5} - 10^{-4} \leq z \leq z_F \sim 1/20 \ll \mathcal{O}(1)$

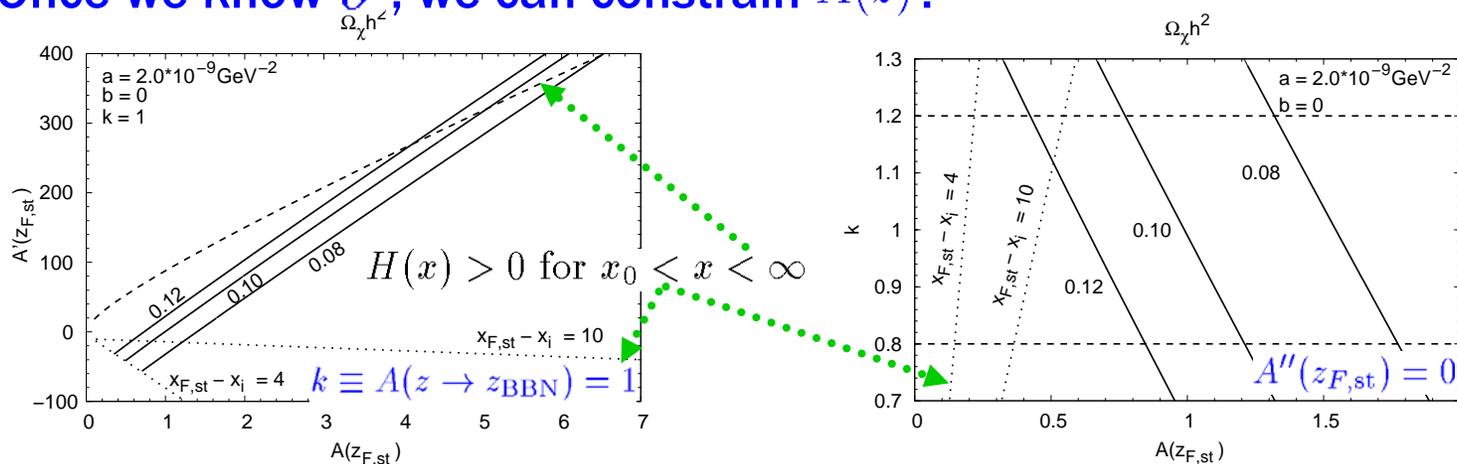
→ This suggests a parametrization of $A(z)$ in powers of $(z - z_{F,\text{st}})$:

$$A(z) = A(z_{F,\text{st}}) + (z - z_{F,\text{st}})A'(z_{F,\text{st}}) + \frac{1}{2}(z - z_{F,\text{st}})^2 A''(z_{F,\text{st}})$$

subject to the BBN limit: $0.8 \leq k \equiv A(z \rightarrow z_{\text{BBN}}) \leq 1.2$

- Once we know σ , we can constrain $A(z)$:

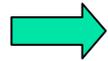
x_i : Maximal temperature where



$\Omega_\chi h^2$ depends on all $H(T)$ → Larger allowed region for $H(T_F)$

5. Abundance of semi-relativistic relics and entropy production

- Precise evaluation of the abundance of particles that freeze out when they are semi-relativistic ($x_F \sim 3$) is complicated



Goal: simple analytic treatment that describes the transition from non-relativistic to relativistic relics

- Assume the Maxwell-Boltzmann distribution:

$$Y_{\chi, \text{eq}} \equiv \frac{n_{\chi, \text{eq}}}{s} = 0.115 \frac{g_{\chi}}{g_{*s}} x^2 K_2(x) \quad (K_n(x): \text{modified Bessel function})$$



Thermal average of cross section σ :

$$\langle \sigma v \rangle = \frac{1}{8m_{\chi}^4 T K_2^2(m_{\chi}/T)} \int_{4m_{\chi}^2}^{\infty} ds \sigma(s - 4m_{\chi}^2) \sqrt{s} K_1(\sqrt{s}/T)$$

Ansatz for approximate cross sections

- Consider neutrinos as stable relic:
- Annihilation cross section:

$$\sigma v^{\text{Dirac } \nu} = \frac{G_F^2 s}{16\pi}$$

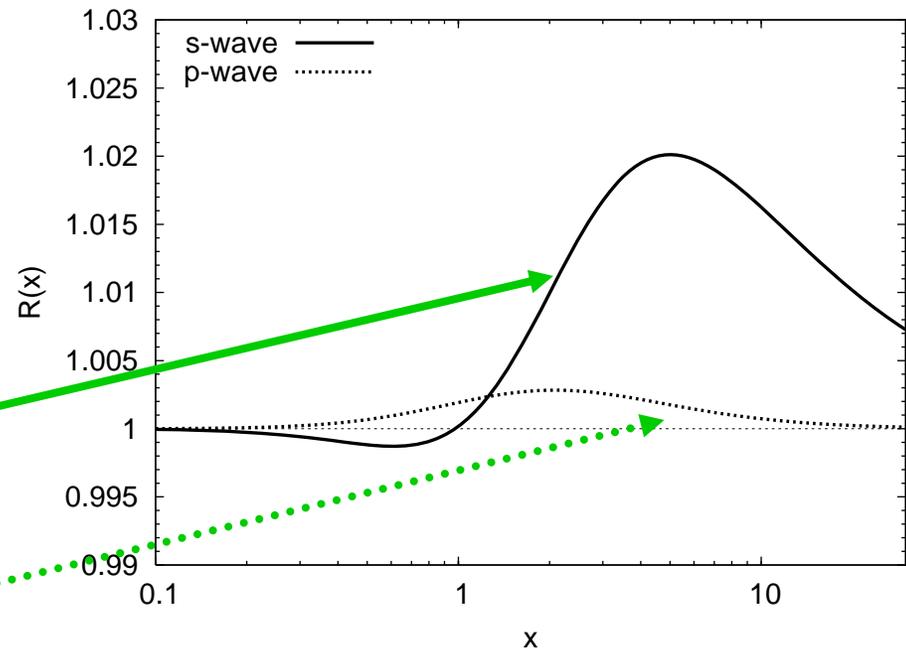
$$\sigma v^{\text{Majorana } \nu} = \frac{G_F^2 s v^2}{16\pi}$$

- Ansatz for the thermally-averaged annihilation cross section:

$$\langle \sigma v \rangle_{\text{app}}^{\text{Dirac}} = \frac{G_F^2 m_\chi^2}{16\pi} \left(\frac{12}{x^2} + \frac{5+4x}{1+x} \right)$$

$$\langle \sigma v \rangle_{\text{app}}^{\text{Majorana}} = \frac{G_F^2 m_\chi^2}{16\pi} \left(\frac{12}{x^2} + \frac{3+6x}{(1+x)^2} \right)$$

- $\langle \sigma v \rangle_{\text{app}} / \langle \sigma v \rangle_{\text{exact}}$ MB :



The approx. cross sections reproduce the exact results with accuracy of a few %

Approximate abundance of semi-relativistic relics

- Define the freeze-out temperature by

$$\Gamma(x_F) = H(x_F)$$

where

$$\Gamma(x_F) = n_{\chi, \text{eq}}(x_F) \langle \sigma v \rangle(x_F)$$

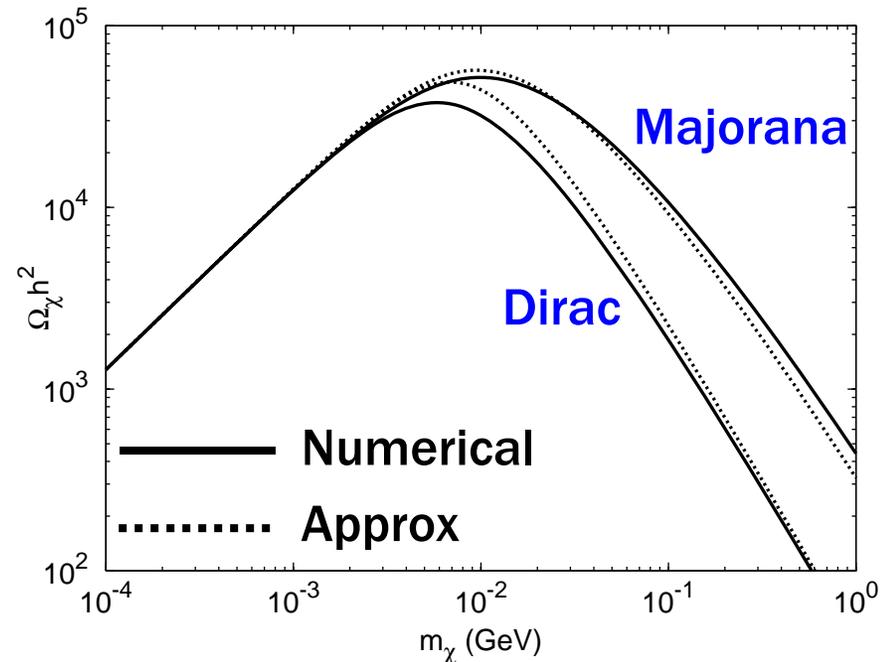
(different from
the standard definition of x_F)

- Assume the relic abundance does not change after decoupling

➡ Final abundance:

$$Y_{\chi, \infty} = Y_{\chi, \text{eq}}(x_F)$$

- Comparison between the numerical and approx solutions



Applications of semi-relativistic relics

- As DM candidates

Hypothetical semi-relativistic relics should decouple before BBN

$$\longrightarrow m_\chi \sim T_F > T_{\text{BBN}} \simeq \text{MeV} \longrightarrow \Omega_\chi h^2 > 10^3$$

The relic abundance is too high!

- As source of large entropy production [Steinhardt, Turner (1983)]

Out-of-equilibrium decay of relic particles produces entropy

$$\text{Ratio of the final to initial entropy: } \frac{S_f}{S_i} = g_*^{1/4} \frac{m_\chi Y_{\chi,i} \tau_\chi^{1/2}}{M_{\text{Pl}}^{1/2}} \propto \Omega_\chi h^2$$

Semi-relativistic relics can produce significant entropy!

Example: sterile neutrino

- Consider a sterile neutrino mixed with an active neutrino (mixing angle: θ)
- Decay rate of the sterile neutrino:

$$\Gamma_\chi = \frac{G_F^2 m_\chi^5}{192\pi^3} \sin^2 \theta, \quad \frac{G_F m_\chi^3}{16\pi} \sin^2 \theta$$

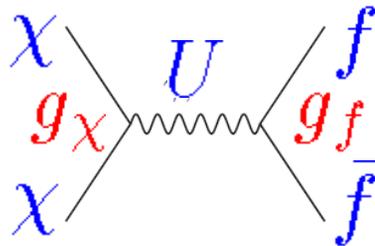
(for small m_χ) (for large m_χ)

should be large enough not to spoil BBN

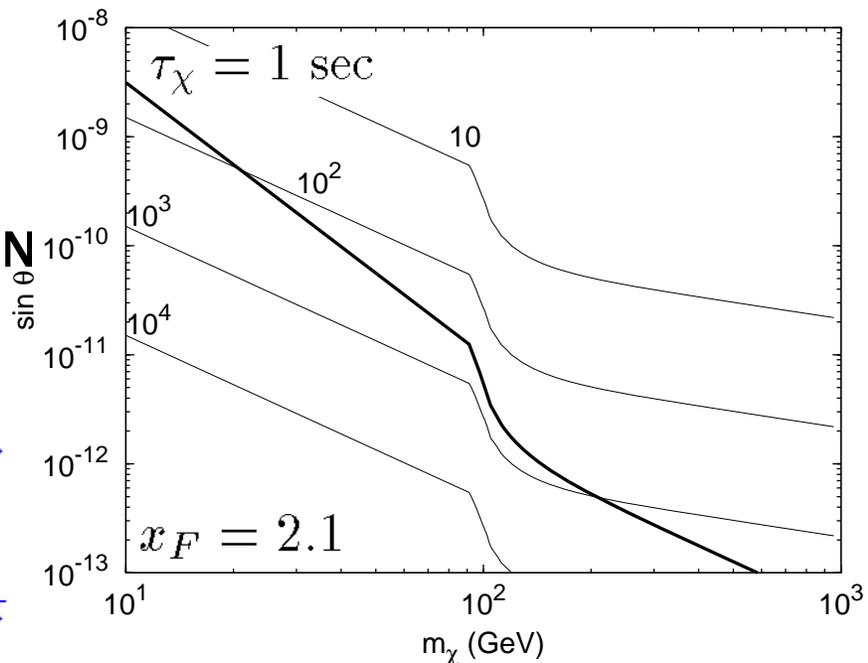
- By introducing a new heavy particle, U, large pair annihilation can be induced:

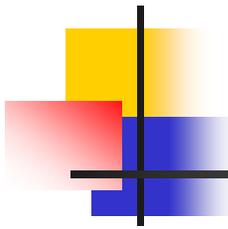
$$\sigma v = \frac{sv^2}{12\pi} \frac{g_\chi^2 g_f^2}{M_U^4}$$

→ $x_F \sim 3$ possible



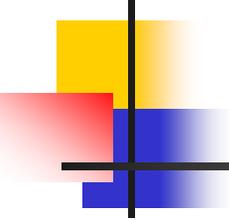
- Entropy production S_f/S_i by the decay of semi-relativistic sterile neutrinos



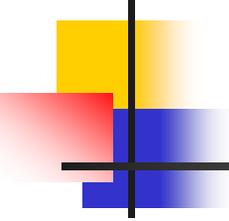


5. Summary

- Using the DM relic density we can probe very early universe at around $T \sim m_\chi/20 \sim \mathcal{O}(10)$ GeV (well before BBN $T_{\text{BBN}} \sim \mathcal{O}(1)$ MeV)
- We find an approximate analytic formula for the WIMP abundance that is valid for all $T_R \leq T_F$
- $\Omega_{\chi,\text{thermal}} h^2 = \Omega_{\text{DM}} h^2$
  Lower bound on the reheat temperature: $T_R > m_\chi/23$
- The sensitivity of $\Omega_{\chi,\text{thermal}} h^2$ on $H(T_F)$ is weak
- We find an approximate analytic formula for the abundance of semi-relativistic relics
- Semi-relativistic relics are useful for producing a large amount of entropy



Backup slides



Hot relics

- Hot relics (decouple for $x_F < 3$):

$Y_{\chi,\text{eq}}(x)$ almost constant

➡ Final abundance is insensitive to the freeze out temperature:

$$Y_{\chi,\infty} = Y_{\chi,\text{eq}}(x_F) = \frac{45}{2\pi^4} \frac{g_\chi}{g_{*s}(x_F)}$$