Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 5
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Crash Course in Theoretical Particle Physics

Prof. Manuel Drees
Daniel Schmeier

–Class Exercises–

5 Scattering in Yukawa theory

Let us consider the Yukawa theory described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}},$$

where

$$\mathcal{L}_{\text{free}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_\psi)\psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{m_\phi^2}{2}\phi^2, \tag{1}$$

$$\mathcal{L}_{\text{int}} = -\kappa \phi \bar{\psi} \psi. \tag{2}$$

Consider the $2 \to 2$ scattering of a fermion-antifermion pair

$$\psi(\vec{k_1}, s_1) + \bar{\psi}(\vec{k_2}, s_2) \to \psi(\vec{p_1}, s_1') + \bar{\psi}(\vec{p_2}, s_2').$$

We work in the lowest non-trivial order in perturbation theory. Let us do the calculation for the differential cross section in all detail.

1. Draw the two Feynman diagrams.
2. Write down the matrix element F using the Feynman rules given in class.
3. Calculate $F^\dagger$.
4. Remember that we do spin sums if we do not observe the spin of the initial or final state particles.

$$|\bar{F}|^2 = \frac{1}{4} \sum_{s_1 s_2 s_1' s_2'} |F|^2.$$

All terms $F_1^\dagger F_1, F_2^\dagger F_2$ and $2\text{Re}(F_1^\dagger F_2)$ can be calculated independently and the procedure is always as follows:

- Explain why we can rearrange spinors (that is: vectors!) as in $(\bar{u}_1 v_2)(\bar{u}_3 v_4) = (\bar{u}_3 v_4)(\bar{u}_1 v_2)$ and rearrange the spinors in $|F|^2$ such that you bring as many $u_i \bar{u}_i$ pairs next to each other.
- Explain why we can (and should) artificially add traces to the calculation, as in $\bar{u}_1 v_2 \bar{v}_2 u_1 = \text{Tr}(\bar{u}_1 v_2 \bar{v}_2 u_1)$. Do that for all terms in $|F|^2$
- Perform the spin sums: $\sum_s u(p,s)\bar{u}(p,s) = \not{p} + m, \sum_s v(p,s)\bar{v}(p,s) = \not{p} - m$.(Make sure to not mix up m_ψ and m_ϕ!)

- Remember the formulae for gamma matrices, which straight-forwardly apply to 'slashed' vectors:

$$\mathrm{Tr}(\not{a}) = 0 \tag{3}$$

$$\mathrm{Tr}(\not{a}\not{b}) = 4a \cdot b \tag{4}$$

$$\mathrm{Tr}(\not{a}\not{b}\not{c}) = 0 \tag{5}$$

$$\mathrm{Tr}(\not{a}\not{b}\not{c}\not{d}) = 4\Big((a \cdot b)(c \cdot d) + (a \cdot d)(b \cdot c) - (a \cdot c)(b \cdot d)\Big) \tag{6}$$

5. Now we go into the center of mass frame: Specify the momenta and calculate the scalar products within $|F|^2$.

6. Compute the differential cross section $\mathrm{d}\sigma/\mathrm{d}\cos\theta$. Discuss the behavior at high energy, $s \gg m_\phi^2, m_\psi^2$.