Physikalisches Institut
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Theoretische Physik

Exercise 1
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Crash Course in Theoretical Particle Physics

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–Homework Exercises–

1.3 Properties of Gamma Matrices

(a) Take one of the gamma matrix representations given on the in-class exercise sheet and show explicitly that the anticommutation relation indeed holds!

(b) The following exercises are to be solved **without using** a particular representation but only by using the anticommutation relations and the normalisation property $(\gamma^0)^\dagger = \gamma^0, (\gamma^i)^\dagger = -\gamma^i$. For convenience we introduce the notation,

$$\gamma^5 = i\,\gamma^0\gamma^1\gamma^2\gamma^3\,. \tag{1}$$

Show that:

$$\left(\gamma^5\right)^\dagger = \gamma^5\,, \qquad \left(\gamma^5\right)^2 = \mathbb{1}\,, \qquad \left\{\gamma^5, \gamma^\mu\right\} = 0\,. \tag{2}$$

(c) Prove the following trace theorems.

$$\mathrm{tr}\left(\gamma^\mu\gamma^\nu\right) = 4g^{\mu\nu} \tag{3a}$$
$$\mathrm{tr}\left(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right) = 4\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right) \tag{3b}$$
$$\mathrm{tr}\left(\gamma^{\mu_1}\ldots\gamma^{\mu_n}\right) = 0\,, \qquad \text{for } n \text{ odd} \tag{3c}$$
$$\mathrm{tr}\,\gamma^5 = 0 \tag{3d}$$
$$\mathrm{tr}\left(\gamma^\mu\gamma^\nu\gamma^5\right) = 0 \tag{3e}$$
$$\mathrm{tr}\left(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5\right) = -4i\epsilon^{\mu\nu\rho\sigma} \qquad \text{with } \epsilon^{0123} = 1 \tag{3f}$$

Hint: Traces are cyclic, that is $\mathrm{tr}(ABC..XY) = tr(YABC..X)$

(d) Show the following contraction identities:

$$\gamma^\mu\gamma_\mu = 4\cdot\mathbb{1} \tag{4a}$$
$$\gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu \tag{4b}$$
$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho}\,\mathbb{1} \tag{4c}$$
$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu \tag{4d}$$

1.4 The Dirac Equation (continued)

(a) With the given normalization constant $N = \sqrt{E+m}$, verify that for the free spinor solutions we found in class

$$\bar{u}(p)u(p) = -\bar{v}(p)v(p) = 2m$$

(b) The charge conjugation operator (C) takes a Dirac spinor ψ into the "charge-conjugate" spinor ψ_c, given by

$$\psi_c = C\gamma^0\psi^*$$

In the representation used in this exercise, $C = i\gamma^2\gamma^0$. Find the explicit charge-conjugates of $u^{(1)}$ and $u^{(2)}$, and compare them with $v^{(1)}$ and $v^{(2)}$. Remember $u^{(i)} = u(p, \phi = \phi_i)$ and $v^{(i)} = v(p, \chi = \chi_i)$. *Hint: There is an identity for sigma matrices which might be useful:* $(\sigma^i)^* = -\sigma^2(\sigma^i)\sigma^2$