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## Crash Course in Theoretical Particle Physics

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–CLASS EXERCISES–

### 4.1 Drawings of quantum interactions

In the following exercise, draw all Feynman diagrams with no internal loops that correspond to the given process with the given interaction Lagrangian.

- (a)  $-\mathcal{L}_{\text{int}} = \frac{\lambda}{4!}\phi^4$ ,  $\phi(p_1) + \phi(p_2) \rightarrow \phi(k_1) + \phi(k_2)$
- (b)  $-\mathcal{L}_{\text{int}} = \frac{\kappa}{3!}\phi^3$ ,  $\phi(p_1) + \phi(p_2) \rightarrow \phi(k_1) + \phi(k_2)$
- (c)  $-\mathcal{L}_{\text{int}} = \frac{\kappa}{2}\Phi\phi^2$ ,  $\Phi(p) \rightarrow \phi(k) + \phi(q)$
- (d)  $-\mathcal{L}_{\text{int}} = \frac{\lambda}{4}\Phi^2\phi^2$ ,  $\Phi(p) \rightarrow \phi(k) + \phi(q)$
- (e)  $-\mathcal{L}_{\text{int}} = \frac{\kappa}{2}\phi^2\Phi + \frac{\lambda}{4}\Phi^2\phi^2$ ,  $\phi(p_1) + \phi(p_2) \rightarrow \Phi(k_1) + \Phi(k_2)$

### 4.2 Feynman graph for the matrix element in ABC theory

Consider a theory containing three *different* real scalar fields  $A, B, C$ , described by  $\phi_A, \phi_B$  and  $\phi_C$  and the Lagrangian

$$\mathcal{L} = \sum_{i=A,B,C} \frac{1}{2} (\partial^\mu \phi_i \partial_\mu \phi_i - m_i^2 \phi_i^2) - A\phi_A\phi_B\phi_C. \quad (1)$$

- (a) Determine the lowest-order amplitude for  $\phi_A(p) + \phi_B(k) \rightarrow \phi_A(p') + \phi_B(k')$ . [*Hint*: There are two diagrams.]
- (b) Specify  $p, k, p', k'$  in the center of mass frame given that  $A$  and  $B$  have both mass  $m$ ,  $\phi_A$  has energy  $E$  and is scattered by an angle  $\theta$ .
- (c) Find  $d\sigma/d\Omega$  for this process in the CM frame, assuming  $m_A = m_B = m, m_C = 0$ . Express your answer in terms of the incident energy,  $E$ , and the scattering angle  $\theta$ .
- (d) Find  $d\sigma/d\Omega$  for this process in the lab frame, assuming  $B$  is much heavier than  $A$ , and remains stationary.  $A$  is incident with energy  $E$ . [*Hint*: Use results from the previous exercise sheet!]
- (e) For the CM frame result, find the total cross section,  $\sigma$ .