Physikalisches Institut
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Theoretische Physik

Exercise 2
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Crash Course in Theoretical Particle Physics

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–Homework Exercises–

2.4 Fun with equations of motion (continued)

(a) Calculate the equation of motion for a vector field A_μ with

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu \tag{1}$$

with $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. This equation of motion is called 'Proca' equation. *Hint: μ, ν are summed within the Lagrangian and within the Euler-Lagrange-Eqautions you also sum over an internal index μ within the term $\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu X)}$. This can easily lead to the wrong answer if you are not careful with your index naming! Hence, you better use different indices in your Euler-Lagrange-Equation!*

(b) Derive Maxwell's equations without sources from the previous exercise. Remember that light does not have a mass and that one identifies the classical electric and magnetic fields as $E^i = -F^{0i}$ and $\epsilon^{ijk}B^k = -F^{ij}$.

2.5 Current conservation in QED Coupling two fields ψ_i with charge q_i to an electromagnetic field A_μ leads to the following (QED) Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_j \bar{\psi}_j(i\not{\partial} - m_j - q_j \not{A})\psi_j. \tag{2}$$

with $\not{X}$ being defined as $\gamma^\mu X_\mu$ for any four-vector X_μ.

1. Show that it is invariant under transformations of the type

$$\psi_i \to e^{i\alpha q_i}\psi_i.$$

   for constant α.

2. Show that the corresponding Noether current is

$$-J_N^\mu \equiv J_Q^\mu = \sum_i q_i \bar{\psi}_i \gamma^\mu \psi_i. \tag{3}$$

3. Write down the equations of motion for fermion fields ψ_i and their conjugates $\bar{\psi}_i$ using the Lagrangian (2).
4. Using these equations of motion, show that the current J_Q^μ of eq. (3) is conserved, $\partial_\mu J_Q^\mu = 0$.
5. Is the sum over i in eq. (3) necessary for the current to be conserved? What does this mean?