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Theoretische Physik

Exercise 3
7th October 2015
WS 15/16

Crash Course in Theoretical Particle Physics

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–CLASS EXERCISES–

3.1 Mandelstam variables

In a two-body scattering event, $A + B \longrightarrow C + D$, it is convenient to introduce *Mandelstam variables*

$$s = (p_A + p_B)^2, \tag{1a}$$
$$t = (p_A - p_C)^2, \tag{1b}$$
$$u = (p_A - p_D)^2, \tag{1c}$$

where the p's are the energy-momentum four-vectors and the squares are taken in a Lorentz invariant manner, namely $p^2 = g_{\mu\nu} p^\mu p^\nu$. The virtue of the Mandelstram variables is that they are Lorentz invariants, with the same value in any inertial system. Experimentally though, the more accessible parameters are energies and scattering angles.

(a) Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.

(b) Find the centre-of-mass (CM) energy of A, in terms of s, t, u and the masses. Note that the CM frame is defined by $\vec{\boldsymbol{p}}_A + \vec{\boldsymbol{p}}_B = \vec{\boldsymbol{0}}$.

(c) Find the energy of A in the rest frame of particle B in terms of s, t, u and the masses.

(d) Find the total CM energy ($E_{\text{total}} = E_A + E_B = E_C + E_D$).

3.2 Cross section for two-particle scattering

Let us consider the general two body process $1 + 2 \to 3 + 4$ scattering, where now the numbers label the momenta of the particles. The differential cross section is given by

$$d\sigma = \frac{1}{4E_1 E_2 |\vec{v}|} (2\pi)^4 \delta^{(4)}(p_3 + p_4 - p_1 - p_2) \frac{d^3 p_3}{2E_3 (2\pi)^3} \frac{d^3 p_4}{2E_4 (2\pi)^3} |F|^2, \tag{2}$$

where F is the reduced (Feynman) amplitude.

(a) In both the CM and laboratory frames, verify that the incident flux factor satisfies the equation

$$4E_1 E_2 |\vec{v}| = 4[(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{1/2}. \tag{3}$$

(b) Consider the case of elastic scattering $A + B \to A + B$ in the rest frame of particle B, assuming the target is so heavy ($m_B \gg E_A$) that its recoil is negligible. Use the RHS of Eq. (3) to determine the differential cross section $d\sigma/d\Omega$, where $d^3p = p^2 dp d\Omega$.

(c) Using the Mandelstam variable s from Problem 1, show that

$$2[(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{1/2} = \{[s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]\}^{1/2} \tag{4}$$

where RHS $\equiv \lambda^{1/2}(s, m_1^2, m_2^2)$.

(d) Perform suffiently many phase space integrals in Eq. (2) to get the differential cross section per unit solid angle in the CM frame:

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s \lambda^{1/2}(s, m_1^2, m_2^2)} \lambda^{1/2}(s, m_3^2, m_4^2)|F|^2. \tag{5}$$

(e) Now express the same cross section in terms of the Mandelstam variable t, when there is cylindrical symmetry about the beam axis:

$$\frac{d\sigma}{dt} = \frac{|F|^2}{16\pi\lambda(s, m_1^2, m_2^2)}. \tag{6}$$