Single Higgs-boson production through photon fusion at Linear Colliders within the general 2HDM

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Outline

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Two-Higgs doublet model

Canonical extension of the SM Higgs sector with a second $SU(2)_L$ doublet with weak hypercharge $Y = +1$

$$\Phi_1 = \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix} \quad (Y = +1) \qquad \Phi_2 = \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix} \quad (Y = +1)$$

The most general $\mathcal{CP}$-conserving, gauge invariant, renormalizable Higgs potential spontaneously breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$

$$\begin{aligned} V(\Phi_1, \Phi_2) \quad = \quad & \lambda_1 \left(\Phi_1^\dagger \Phi_1 - v_1^2\right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2 - v_2^2\right)^2 \\ & + \lambda_3 \left[\left(\Phi_1^\dagger \Phi_1 - v_1^2\right) + \left(\Phi_2^\dagger \Phi_2 - v_2^2\right)\right]^2 \\ & + \lambda_4 \left[\left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) - \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right)\right] \\ & + \lambda_5 \left[\mathrm{Re}\left(\Phi_1^\dagger \Phi_2\right) - v_1 v_2\right]^2 + \lambda_6 \left[\mathrm{Im}\left(\Phi_1^\dagger \Phi_2\right)\right]^2 \end{aligned}$$

We also impose the discrete symmetry $\Phi_i \to (-1)^i \, \Phi_i$, in order to avoid tree-level Flavor Changing Neutral Currents (FCNC)

- Symmetry softly broken by: $\lambda_5 \, \mathrm{Re}\left(\Phi_1^\dagger \Phi_2\right)^2$

Two-Higgs doublet model

Canonical extension of the SM Higgs sector with a second $SU(2)_L$ doublet with weak hypercharge $Y = +1$

$$\Phi_1 = \begin{pmatrix} \Phi_1^+ \\ \frac{v_1 + \phi_1^0 + i\chi_1^0}{\sqrt{2}} \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} \Phi_2^+ \\ \frac{v_2 + \phi_2^0 + i\chi_2^0}{\sqrt{2}} \end{pmatrix}$$

The doublets contain 8 real degrees of freedom

- 3 Goldstone bosons: G^0 and $G^\pm$
- 5 physical fields:
  - ✔ 2 $\mathcal{CP}$-even states h^0 and H^0
  - ✔ 1 $\mathcal{CP}$-odd state A^0
  - ✔ 2 charged states H^+ and H^-

Two-Higgs doublet model

7 free dimensionless real parameters introduced in the Higgs potential

- 6 couplings $\lambda_{1...6}$
- 2 vacuum expectation values $v_{1,2}$ $\quad v_1^2 + v_2^2 = v^2 = \frac{1}{\sqrt{2}\,G_F} \sim (248\text{ GeV})^2$

They could be related to physical quantities

- Masses of the Higgs bosons: M_h, M_H, M_A and $M_{H^\pm}$
- the ratio of the vevs: $\tan\beta \equiv \frac{\langle H_2^0\rangle}{\langle H_1^0\rangle} = \frac{v_2}{v_1}$
- the mixing angle α between the two $\mathcal{CP}$-even states
- the coupling λ_5

Two-Higgs doublet model

There are two possibilities to couple the Higgs doublets to fermions:

- **Type-I**: One Higgs doublet (Φ_2) couples to all fermions, whereas the other (Φ_1) does not couple to them at all
- **Type-II**: One Higgs doublet (Φ_1) couples only to down-like fermions and the other (Φ_2) only to up-like ones
  The MSSM is a type-II 2HDM

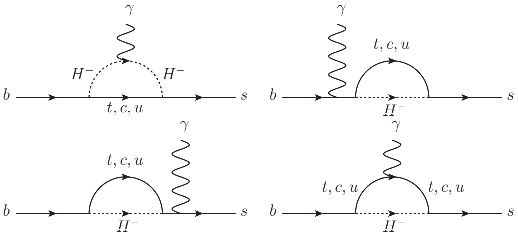
Outline

Restrictions: $\mathcal{B}(b \to s\gamma)$

- We have strong constraints coming from flavor physics
  $\mathcal{B}(\bar{B} \to X_s\gamma) \sim (3.55 \pm 0.25) \cdot 10^{-4}$ from BaBar and Belle
  $\mathcal{B}(\bar{B} \to X_s\gamma) \sim (3.15 \pm 0.23) \cdot 10^{-4}$ SM NNLO prediction

- The good agreement between the SM prediction and the experimental result puts severe constraints on the flavor structure of NP models.

  New charged-particles contribute to this rare decay.



Leading-order contributions due to the charged Higgs $H^{\pm}$

Restrictions: $\mathcal{B}(b \to s\gamma)$

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  New charged-particles contribute to this rare decay.

The charged Higgs bosons contribution:

- ✔ positive
- ✔ increases when $M_{H^\pm}$ decreases

Type-I 2HDM: Couplings $H^\pm qq' \propto 1/\tan\beta$
Couplings highly suppressed for $\tan\beta > 1$

Type-II 2HDM: Couplings $H^\pm qq' \propto \tan\beta$
Couplings enhanced for $\tan\beta > 1$
Restriction ➞ $M_{H^\pm} > 295$ GeV   Misiak et al., 2006

Restrictions: $\delta\rho$

- rho-parameter: $\rho = \rho_0 + \delta\rho$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

One-loop corrections induced by Higgs bosons   Barbieri & Maiani, 1983

$$\begin{aligned}\delta\rho_{2HDM} \quad = \quad & \frac{G_F}{8\sqrt{2}\,\pi^2}\left\{ M_{H^\pm}^2 \left[1 - \frac{M_{A^0}^2}{M_{H^\pm}^2 - M_{A^0}^2} \ln\frac{M_{H^\pm}^2}{M_{A^0}^2} \right] \right. \\ & + \cos^2(\beta-\alpha)\, M_{h^0}^2 \left[\frac{M_{A^0}^2}{M_{A^0}^2 - M_{h^0}^2} \ln\frac{M_{A^0}^2}{M_{h^0}^2} - \frac{M_{H^\pm}^2}{M_{H^\pm}^2 - M_{h^0}^2} \ln\frac{M_{H^\pm}^2}{M_{h^0}^2} \right] \\ & \left. + \sin^2(\beta-\alpha)\, M_{H^0}^2 \left[\frac{M_{A^0}^2}{M_{A^0}^2 - M_{H^0}^2} \ln\frac{M_{A^0}^2}{M_{H^0}^2} - \frac{M_{H^\pm}^2}{M_{H^\pm}^2 - M_{H^0}^2} \ln\frac{M_{H^\pm}^2}{M_{H^0}^2} \right] \right\}\end{aligned}$$

Experimental measurements: $|\delta\rho_{2HDM}| \lesssim 10^{-3}$

$\delta\rho_{2HDM}$ vanish for $M_A \to M_{H^\pm}$

We will demand $\longrightarrow$ $M_A \sim M_{H^\pm}$

Restrictions: Perturbativity-Unitarity

- Perturbativity on the Yukawas
  They could receive large enhancements at large or small $\tan\beta$.
  Yukawas with $H^{\pm}$: $Y_t \propto \frac{m_t}{v \tan\beta}$ $Y_b \propto \frac{m_b \tan\beta}{v}$
  $\rightarrow$ $0.3 < \tan\beta \lesssim 60$ El Kaffas, Osland & Greid, 2007

- Perturbative unitarity on the Higgs self-couplings
  They could receive large enhancements at low or large $\tan\beta$.
  We use a condition *à la Lee-Quigg-Thacker*
  - ✔ trilinear Higgs self coupling

    $$|C_{HHH}| \leq \left|\lambda^{(SM)}_{HHH}(M_{h_{SM}} \simeq 1\ TeV)\right| = 3\frac{M^2_{h_{SM}}}{v}\Bigg|_{M_{h_{SM}}=1\ TeV}$$
  - ✔ quartic Higgs self coupling

    $$|C_{HHHH}| \leq \left|\lambda^{(SM)}_{HHHH}(M_{h_{SM}} \simeq 1\ TeV)\right| = 3\frac{M^2_{h_{SM}}}{v^2}\Bigg|_{M_{h_{SM}}=1\ TeV}$$

  ✘ Note that there is no consensus on how to impose unitarity!
  Kanemura, Kubota & Takasugi, 1993; Akeroyd, Arhrib & Naimi, 2000; Horejsi & Kladiva, 1006

Restrictions: Vacuum stability

- Vacuum stability
  We assume that the quartic interaction terms in the potential do not give negative contribution for all directions of scalar fields at each energy scale up to Λ
  Require a Higgs potential bounded from below

$$\lambda_1 + \lambda_3 > 0 \qquad \lambda_2 + \lambda_3 > 0$$

$$2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} + 2\lambda_3 + \lambda_4 + \frac{1}{2}\,\mathrm{Min}\Big(0, \lambda_5 + \lambda_6 - 2\lambda_4 - \big|\lambda_5 - \lambda_6\big|\Big) > 0$$

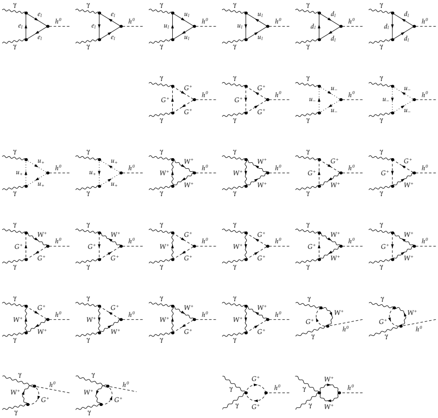
Kanemura, Kasai & Okada, 1999

Outline

One-loop Feynman diagrams

The $\gamma\gamma h^0$ interaction is generated at the quantum level

SM contributions:
* Heavy fermions t, b
* Vector bosons $W^{\pm}$
* Goldstone bosons $G^{\pm}$



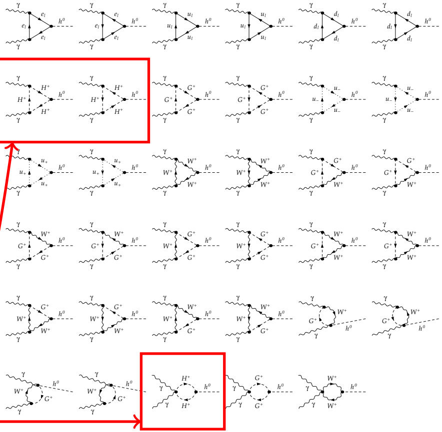
One-loop diagrams describing the process $\gamma\gamma \to h$, within the 2HDM

One-loop Feynman diagrams

The $\gamma\gamma h^0$ interaction is generated at the quantum level

SM contributions:
* Heavy fermions t, b
* Vector bosons $W^\pm$
* Goldstone bosons $G^\pm$

+ 2HDM contributions:
* Charged Higgs $H^\pm$



One-loop diagrams describing the process $\gamma\gamma \to h$, within the 2HDM

Cross section

$$\hat{\sigma}(\gamma\gamma \to h) = \sigma(\gamma\gamma \to h) \cdot M_h^2\, \delta\left(s - M_h^2\right)$$

s is the center-of-mass energy.
Dirac Delta is a trademark feature of the $2 \to 1$ phase space.

$$\sigma(\gamma\gamma \to h) = \frac{\pi}{M_h^4} \sum_{\eta_1\,\eta_2} \left|M(\gamma\gamma \to h)\right|^2$$

Sum performed over polarizations.

$$\delta\left(s - M_h^2\right) \to \frac{1}{\pi}\,\frac{s\,\Gamma_h/M_h}{\left(s - M_h^2\right)^2 + (s\,\Gamma_h/M_h)^2}$$

Substitution of the Breit-Wigner form of the Higgs width,
in place of the zero-width Delta distribution.

Calculations performed using *FeynArts, FormCalc* & *LoopTools* T. Hahn

✔ We have implemented the $2 \to 1$ phase space.

Trilinear coupling

The phenomenology of $\gamma\gamma \to h$ will be lead by the coupling H^+H^-h.
→ This coupling has not a fixed value in the 2HDM
→ It could receive large enhancements!

Trilinear coupling H^+H^-h

$$C_{H^+H^-h} = \frac{i}{v}\left[\sin(\beta-\alpha)\left(M_h^2 - 2\,M_{H^\pm}^2\right) - \frac{\cos(\beta-\alpha)}{\sin 2\beta}\left(2\,M_h^2 - \lambda_5\,v^2\right)\right]$$

Maximum enhancement for:

- low and high values of $\sin\alpha$
- low and high values of $\tan\beta$
  Remember that $\tan\beta < 1$ are disadvantaged

El Kaffas, Osland & Greid, 2007

Benchmarks

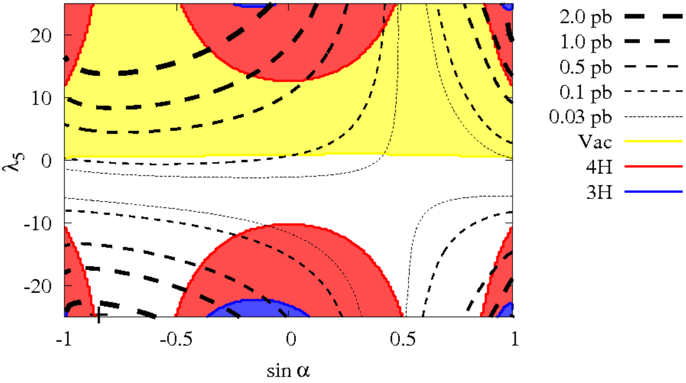
Higgs mass parameters used

	Set I	Set II	Set III	Set IV
M_h [GeV]	115	150	200	200
$M_{H^\pm}$ [GeV]	105	105	300	350
M_{H^0} [GeV]	165	200	250	250
M_A [GeV]	100	110	290	340
	Type I			

- The dynamics will be determined by M_h and $M_{H^\pm}$
- However, the constraints depend on all the mass parameters
- Set I and Set II only suitable for Type-I 2HDM ($M_{H^\pm} > 295$ GeV)

$\sigma(\gamma\gamma \to h)$

➤ Set I
$M_h = 115$ GeV
$M_{H^\pm} = 105$ GeV
$M_{H^0} = 165$ GeV
$M_A = 100$ GeV



The maximum takes place for $\tan\beta = 1.70$, $\sin\alpha = -0.86$ and $\lambda_5 = -25$

$\sigma_{\text{Max}}(\gamma\gamma \to h) \sim 2.53$ pb

$\sigma(\gamma\gamma \to h_{SM}) \sim 0.13$ pb for $M_{h_{SM}} = 115$ GeV

Sizable region where the production cross section stays high, bordering the range of some picobarns.

$\sigma(\gamma\gamma \to h)$

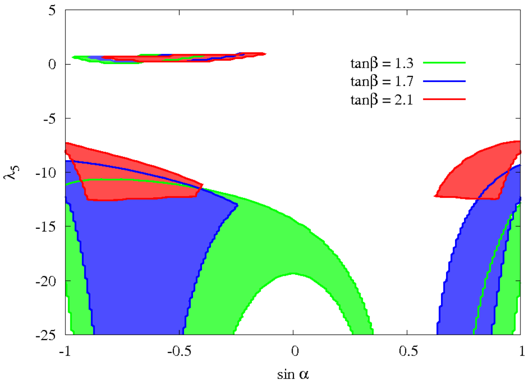
➤ Set I

$M_h = 115$ GeV

$M_{H^\pm} = 105$ GeV

$M_{H^0} = 165$ GeV

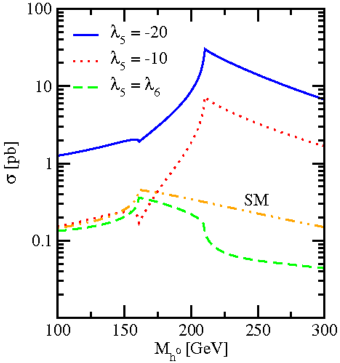
$M_A = 100$ GeV



Regions allowed by constraints and corresponding to a cross section 10% bigger than the SM one.

- Even if the cross-section should be greater for bigger values of $\tan\beta$, the available phase space will limit the growth
- Unitarity constraints limit $\tan\beta \lesssim 5$

$\sigma(\gamma\gamma \to h)$



The coupling H^+H^-h maximized for high $|\lambda_5|$

For high values of $|\lambda_5|$:

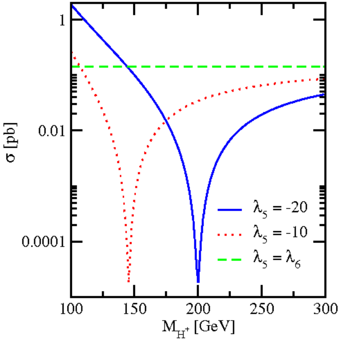
➤ threshold production of 2 real $H^\pm$ at $M_h = 2 \cdot M_{H^\pm} \sim 210$ GeV
→ enhancement

➤ threshold production of 2 real $W^\pm$ and $G^\pm$ at $M_h = 2 \cdot M_W \sim 160$ GeV
→ destructive interference

➤ enhancement could reach a factor $\mathcal{O}(100)$ in the most optimistic case

Using $\tan\beta = 1.70$, $\sin\alpha = -0.86$ and Set I.

$\sigma(\gamma\gamma \to h)$



- For low $M_{H^\pm}$, $\gamma\gamma h$ coupling dominated by the $H^\pm$ corrections
- Therefore, the increase of $M_{H^\pm}$ hampers the enhancement.
- Strong suppression effect due to destructive interference (fermion, gauge and Higgs bosons)

Ellis, Gaillard & Nanopoulos, 76

Using $\tan\beta = 1.70$, $\sin\alpha = -0.86$ and Set I.

$\sigma(\gamma\gamma \to h)$

Maximum cross section

	Set I	Set II	Set III	Set IV
M_h [GeV]	115	150	200	200
$M_{H^\pm}$ [GeV]	105	105	300	350
$\boldsymbol{\sigma_{Max}(\gamma\gamma \to h)}$ [pb]	**2.53**	**3.51**	**0.33**	**0.33**
$\sigma(\gamma\gamma \to h_{SM})$ [pb]	0.13	0.20	0.28	
	Type I			

For Set II and III, both type-I and type-II lead to the same σ (up to $\sim 1\%$)

Possible enhancement coming from the Yukawas $hq\bar{q}$

Could be important for low $\tan\beta$: $Y_t \propto m_t \frac{\cos\alpha}{\sin\beta}$ $Y_b \propto m_b \left(\frac{\cos\alpha}{\sin\beta}\right)^{\pm 1}$

but $\tan\beta < 1$ are disadvantaged... El Kaffas, Osland & Greid, 2007

→ For low $\tan\beta$, the enhancement is $\lesssim 15\%$

$\sigma(e^+e^- \to \gamma\gamma \to h)$

➤ Let us recall that a photon collider is an option of a lepton collider.

It is possible to take into account the conversion $e^+e^- \to \gamma\gamma$ by the convolution

$$\sigma(e^+e^- \to \gamma\gamma \to h)(s) = \sum_{\{ij\}} \int_0^1 d\tau \, \frac{d\mathcal{L}_{ij}^{ee}}{d\tau} \, \hat{\sigma}_{\eta_i \eta_j}(\gamma\gamma \to h)(\tau s)$$

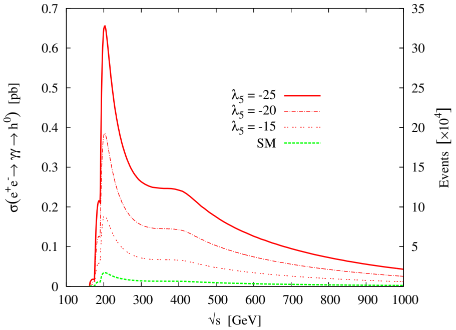
* $\hat{\sigma}_{\eta_i \eta_j}(\gamma\gamma \to h)$: *partonic* cross section
* τ: fraction of the energy carried by the photon
* $\mathcal{L}_{ij}^{ee}$ stands for the photon luminosity distribution

$$\frac{d\mathcal{L}_{ij}^{ee}}{d\tau} = \int_\tau^1 \frac{dx}{x} \, \frac{1}{1+\delta_{ij}} \, [f_{i/e_1}(x)\, f_{j/e_2}(\tau/x) + f_{j/e_1}(x)\, f_{i/e_2}(\tau/x)]$$

* f_{i/e_1} denotes the photon density functions.

➤ We use the ones provided by CompAZ Telnov, 2006 & Żarnecki, 2003

$\sigma(e^+e^- \to \gamma\gamma \to h)$



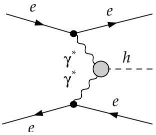
* Set I with $\sin\alpha = -0.86$, $\tan\beta = 1.7$

* Luminosity $\mathcal{L} = 500$ fb^{-1}

- The shape of the cross section lead by the parametrization of the photon energy spectrum.
- Huge number of events for low center-of-mass energy.
- Due to interference effects, the enhancement capabilities become partially reduced.

Outline

$\sigma(e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-h)$



- Cross-section for this fusion process grow with s up to very high values of s, *roughly* as:

$$\sigma \sim \frac{\alpha^4}{M^2} \log^2 \frac{s}{m_e^2} \log^n \frac{s}{M^2}$$

$n \geq 1$ given by high energy behavior of the *partonic* process

Weizsäcker-Williams equivalent photon approximation

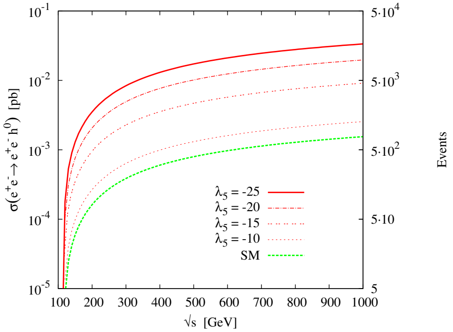
- Quasi-singular collinear behavior

$$\sigma(e^+e^- \to e^+e^-X) = \left[\frac{\alpha_{em}}{2\pi} \log \frac{s}{4\, m_e^2}\right]^2 \int_{M_X^2/s}^{1} d\tau\, f(\tau)\, \sigma_{\gamma\gamma\to X}(\tau\, s)$$

Weizsäcker-Williams distribution function

$$f(\tau) = \frac{1}{\tau}\left[(2+\tau)^2 \log\frac{1}{\tau} - 2\,(1-\tau)(3+\tau)\right]$$

$\sigma(e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-h)$



* Set I with
$\sin\alpha = -0.86$,
$\tan\beta = 1.7$
* Luminosity
$\mathcal{L} = 500$ fb^{-1}

- Logarithmic evolution of the cross section $\sigma(e^+e^- \to e^+e^-h)$
- After exceeding the threshold, σ increases up to $\sim 10^{-2}$ pb
  ~ 5000 events for $\mathcal{L} = 500$ fb^{-1}
- Possible enhancement of almost a factor 20

Outline

Conclusions

We analyse the production of a single Higgs boson within the 2HDM through the following mechanisms:

- the direct collision of real photons in $\gamma\gamma$ colliders
- the fusion of virtual photons in e^+e^- colliders

$\gamma\gamma h$ effective interaction generated by charged particle loops
In particular by $H^\pm$ loops and then possible enhancement due to the H^+H^-h trilinear coupling

We take into account the restrictions coming from:

- EW precision data: $\mathcal{B}(b \to s\gamma)$, $\delta\rho$
- perturbativity and unitarity bounds
- vacuum stability

Conclusions

In the most favorable scenarios $\sigma(\gamma\gamma \to h) \sim 2.5$ pb for $M_h = 115$ GeV, 20 times above the SM prediction.

- enhancement given by the H^+H^-h coupling
- large $|\lambda_5|$ and low $\tan\beta$ values needed
- light $H^\pm$ needed $\Longrightarrow$ Type-I 2HDM
- enhancement reduced by destructive interferences with $W^\pm$ and fermions
- for higher M_h, the enhancement could be more important

➤ For very low $\tan\beta$ and λ_5, no enhancement due to trilinear couplings, However possible enhancement ($\lesssim 15\%$) produce by Yukawa couplings.

➤ For $M_{H^\pm} > 300$ GeV, the differences between type-I and type-II $\mathcal{O}(1\%)$

In the chosen benchmarks, the H^0 and A^0 production cross-sections are of the order of $\mathcal{O}(10^{-2})$ pb

Conclusions

$\gamma\gamma$ option of a e^+e^- collider

We keep the track that $\gamma\gamma$ collisions are generated upon e^+e^- beams:
$e^+e^- \to \gamma\gamma \to h$

The expected number of events will fall above 10^4 per $\mathcal{L} = 100$ fb^{-1}
in the typical energy range of the ILC: 500 – 1000 GeV.

e^+e^- collider

$\gamma\gamma$ fusion: $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-h$

- ➤ Cross-section exhibits a logarithmic growing with s typical of vector boson fusion process
- ➤ Cross-section overcomes the value of 0.01 pb for $\sqrt{s} = 500$ GeV and a $M_h = 115$ GeV

and perspectives. . .

Higgs boson produced at rest (not boosted)
The Higgs decay will lead to back-to-back heavy-quark jets ($b\bar{b}$)
The analysis of the invariant mass distribution could lead to a very precise determination of the Higgs mass

Together with other production channels $e^+e^- \to HH$ and $e^+e^- \to HHH$,
the single Higgs boson production provides a strong insight into the structure of the EWSB

➤ Single Higgs-boson production through photon fusion within the MSSM
The trilinear couplings are no longer free but fixed by gauge symmetry
Possibilities to discern between the 2HDM and the MSSM. . .

Work in progress

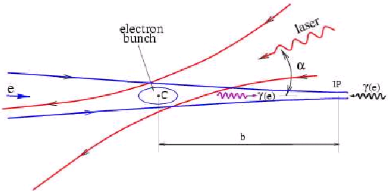
Benchmarks

Parameters used

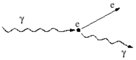
	Set I	Set II	Set III	Set IV
M_h [GeV]	115	150	200	200
$M_{H^\pm}$ [GeV]	105	105	300	350
M_{H^0} [GeV]	165	200	250	250
M_A [GeV]	100	110	290	340
$\tan\beta$	1.7	1.7	1	1
$\sin\alpha$	-0.86	-0.86	-0.82	-0.82
λ_5	-25	-25	0	0

Photon Collider

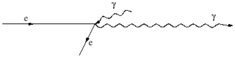
High energy, high intensity photon beam can be obtained using Compton backscattering of laser light off the high energy electrons



Compton scattering:



backscattering:



Natural extension of all e^+e- linear collider proyect

A.F.Żarnecki
12.01.2005

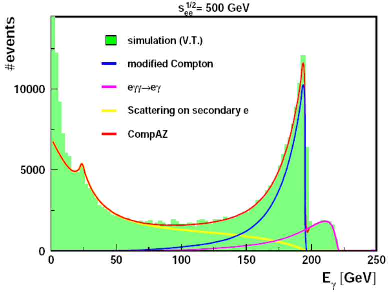
CompAZ

Parametrization of the photon energy spectrum

Compton formula

corrected for:

- nonlinear effects
- angular correlations
- two photon scattering
- electron rescattering



⇒ CompAZ

A.F.Żarnecki
12.01.2005

$\sigma(\gamma\gamma \to H)$ & $\sigma(\gamma\gamma \to A)$

Maximum cross section

	Set I	Set II	Set III	Set IV
M_{H^0} [GeV]	165	200	250	250
M_{A^0} [GeV]	100	110	290	340
$M_{H^\pm}$ [GeV]	105	105	300	350
$\sigma_{Max}(\gamma\gamma \to H^0)$ [pb]	0.076	0.067	0.012	0.012
$\sigma_{Max}(\gamma\gamma \to A^0)$ [pb]	0.011	0.011	0.058	0.12
	Type I			

Restrictions: Perturbativity-Unitarity

- Perturbative unitarity on the Higgs self-couplings
  They could receive large enhancements at large $\tan\beta$.
  We use a condition *à la Lee-Quigg-Thacker*
  - ✔ trilinear Higgs self coupling
    $$|C_{HHH}| \leq \left|\lambda_{HHH}^{(SM)}(M_{h_{SM}} \simeq 1\ TeV)\right| = 3\frac{M_{h_{SM}}^2}{v}\Bigg|_{M_{h_{SM}}=1\ TeV}$$
  - ✔ quartic Higgs self coupling
    $$|C_{HHHH}| \leq \left|\lambda_{HHHH}^{(SM)}(M_{h_{SM}} \simeq 1\ TeV)\right| = 3\frac{M_{h_{SM}}^2}{v^2}\Bigg|_{M_{h_{SM}}=1\ TeV}$$

Restrictions: Perturbativity-Unitarity

- Kanemura, Kasai & Okada, 1999
  Require that the running coupling constants of the Higgs self-couplings and the Yukawa couplings do not blow up below a certain energy scale Λ

$$
\begin{aligned}
\lambda_i(\mu) &< 8\pi \\
y_t(\mu) &< 4\pi
\end{aligned}
$$

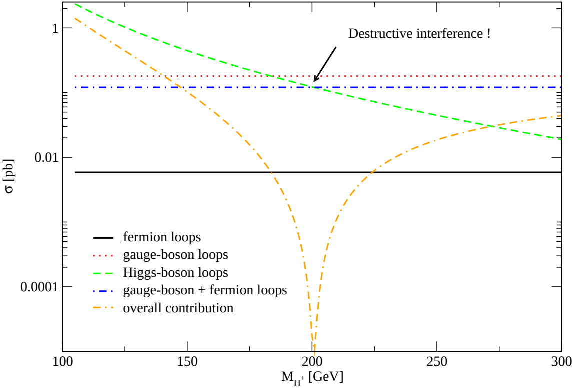
  for a renormalization scale μ less than Λ.

Restrictions: Perturbativity-Unitarity

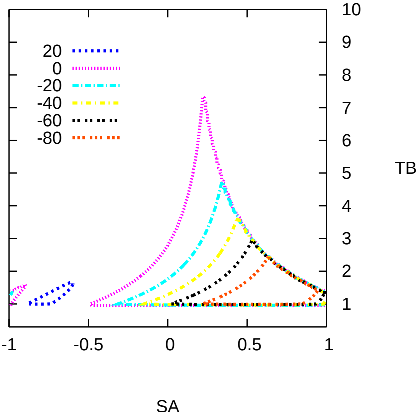
- Akeroyd, Arhrib & Naimi, 2000
  In very high energy collisions, it can be shown that the dominant contribution to the amplitude of the two-body scattering $S_1\, S_2 \to S_3\, S_4$ is the one which is mediated by the quartic coupling.
  Therefore the unitarity reduces to a constraint on the quartic coupling, $|C(S_1, S_2, S_3, S_4)| \lesssim 8\pi$

$$e_1 = 2\lambda_3 - \lambda_4 - \frac{\lambda_5}{2} + \frac{5}{2}\lambda_6$$
$$e_2 = 2\lambda_3 + \lambda_4 - \frac{\lambda_5}{2} + \frac{1}{2}\lambda_6$$
$$f_+ = 2\lambda_3 - \lambda_4 + \frac{5}{2}\lambda_5 - \frac{1}{2}\lambda_6$$
$$f_- = 2\lambda_3 + \lambda_4 + \frac{1}{2}\lambda_5 - \frac{1}{2}\lambda_6$$
$$f_1 = f_2 = 2\lambda_3 + \frac{1}{2}\lambda_5 + \frac{1}{2}\lambda_6$$

$$a_\pm = 3(\lambda_1 + \lambda_2 + 2\lambda_3) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + (4\lambda_3 + \lambda_4 + \frac{1}{2}(\lambda_5 + \lambda_6))^2}$$
$$b_\pm = \lambda_1 + \lambda_2 + 2\lambda_3 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{1}{4}(-2\lambda_4 + \lambda_5 + \lambda_6)^2}$$
$$c_\pm = \lambda_1 + \lambda_2 + 2\lambda_3 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{1}{4}(\lambda_5 - \lambda_6)^2}$$



Using $\tan\beta = 1.70$, $\sin\alpha = -0.86$, $\lambda_5 = -25$ and Set I.



Yukawas: Enhancement in %, for $\lambda_5 = \lambda_6$ and Set III