Blow–ups of heterotic orbifolds and Gauged Linear Sigma Models

Stefan Groot Nibbelink

Arnold Sommerfeld Center,
Ludwig-Maximilians-University Munich

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Overview

Heterotic toroidal orbifolds

Overview:

- Geometry of toroidal orbifolds
- Shift embedding and discrete Wilson lines
- Twisted orbifold states
- MSSM–like models

Geometry of toroidal orbifolds

A toroidal $T^6/\mathbb{Z}_N$ orbifold is defined by:

- Some complex coordinates $u = (u_1, u_2, u_3)$,
- A torus lattice $\Lambda_T := \{n_i\, e_i, n_i \in \mathbb{Z}\}$ defining periodicity conditions:

$$u \sim u + \ell\ , \qquad \ell \in \Lambda_T\ .$$

  (We will take Λ_T factorized, e.g. A_2^3 for $T^6/\mathbb{Z}_3$.)

- Orbifold twist θ:

$$\theta:\ (u_1, u_2, u_3) \mapsto \Big(e^{2\pi i v_1}\, u_1, e^{2\pi i v_2}\, u_2, e^{2\pi i v_3}\, u_3\Big)$$

  where the v_a are quantized in units of $1/N$, i.e. $v_a = n_a/N$; $n_a, \in \mathbb{Z}$.

- To preserve target space supersymmetry: $\sum_a v_a = 0 \mod 2$.

Shift embedding and discrete Wilson lines

To define a heterotic orbifold we have to specify the gauge degrees of freedom; we take them to be 16 complex fermions λ^I, $I = 1, \ldots, 16$.

Their orbifold boundary conditions are defined by:

- A gauge shift embedding $V = (V^1, \ldots, V^{16})$:

$$\theta : \ (\lambda^1, \ldots, \lambda^{16}) \mapsto \left(e^{2\pi i V^1} \lambda^1, \ldots, e^{2\pi i V^{16}} \lambda^{16} \right)$$

- Some Wilson lines $W_i = (W_i^1, \ldots, W_i^{16})$:

$$e_i : \ (\lambda^1, \ldots, \lambda^{16}) \mapsto \left(e^{2\pi i W_i^1} \lambda^1, \ldots, e^{2\pi i W_i^{16}} \lambda^{16} \right)$$

All the entries of V and W_i are quantized in units of $1/N$.

Modular invariance

The one loop partition function has to be invariant under modular transformations.

This results in a set of stringent consistency conditions on the gauge shift and Wilson lines:

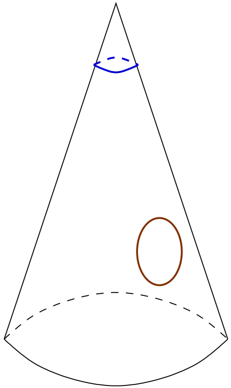
$$\frac{N}{2}\Big(V^2 - v^2\Big) \equiv 0\,, \qquad \frac{N}{2}\,W_i^2 \equiv 0\,,$$

where "$\equiv$" means equal up to integers.

Twisted orbifold states

An orbifold supports untwisted and twisted states.

Dixon,Harvey,Vafa,Witten'85, Ibanez,Nilles,Quevedo'87



An r–twisted state, e.g. $|T_r\rangle = |p_r, P_r\rangle, \tilde{\alpha}^a_{-\tilde{v}^a_r}|p_r, P_r\rangle$, is characterized by:

- a right-moving shifted momentum:

$$p_r = p + r\,v\,, \quad p \in SO(8) \text{ vector weight lattice;}$$

- a left-moving shifted momentum:

$$P_r = P + r\,V\,, \quad P \in E_8 \times E_8 \text{ root lattice;}$$

- and possible oscillators, e.g. $\tilde{\alpha}^a_{-\tilde{v}^a_r}$, with $\tilde{v}^a_r = rv^a$ mod integers.

MSSM–like models

Given that the input data of heterotic orbifolds is rather limited, it is possible to perform systematic searches for interesting models:

- The $T^6/\mathbb{Z}_{6\text{–II}}$ orbifold gives rise to a large pool of possible so–called mini–landscape MSSMs. Buchmuller,Hamaguchi,Lebedev,Ratz'04, Lebedev,Nilles,Raby,Ramos-Sanchez,Ratz,Vaudrevange,Wingerter'07
- On the $T^6/\mathbb{Z}_{12\text{–I}}$ also MSSM–like models were constructed. Kim^2,Kyae'07
- The MSSM–like models on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ break the GUT via freely acting Wilson line. Blaszczyk,SGN,Ratz,Rühle,Trapletti,Vaudrevange'09

Calabi–Yau compactifications of the heterotic string

Overview:

- Smooth Calabi–Yau spaces
- Vector bundles
- Torsional manifolds

Smooth Calabi–Yau spaces

Smooth compactifications requires the target space to be a Calabi–Yau (CY) space to preserve $\mathcal{N} = 1$ 4D supersymmetry.

A CY space X is defined by the following properties:

- a complex manifold, i.e. admit global complex coordinates,
- which is Kähler, i.e. its fundamental form J is closed:

$$J_2 = G_{a\underline{a}}\, \mathrm{d}z^a \wedge \mathrm{d}\bar{z}^{\underline{a}} \;, \qquad \mathrm{d}J_2 = 0 \;,$$

- and with vanishing first Chern class, i.e. $c_1(X) = [\mathrm{tr} R_2] = 0$.

Smooth Calabi–Yau spaces

The topology of X is uniquely specified by its cohomology:

- the number, $h_{1,2}$, of harmonic (2,1)–forms,
- the number, $h_{1,1}$, of harmonic (1,1)–forms $\omega^A_{(1,1)}$,
- and their intersections numbers:

$$d_{ABC} := \int_X \omega^A_{(1,1)} \wedge \omega^B_{(1,1)} \wedge \omega^C_{(1,1)} \,.$$

The fundamental (Kähler) form J_2 can be expanded as

$$J_2 = \sum_A a_A \, \omega^A_{(1,1)} \,,$$

where a_A are called the Kähler moduli.

Divisors and curves

A CY space may possess various subspaces:

- Divisors (four–cyles), D, are 4D real closed hyper surfaces of X.
- Curves, C, are 2D real closed hyper surfaces of X.

Poincaré duality:

$$\text{divisor } D \quad \longleftrightarrow \quad (1,1)\text{–form } \omega^{D}_{(1,1)},$$

$$\text{curve } C = DD' \quad \longleftrightarrow \quad (2,2)\text{–form } \omega^{C}_{(2,2)} = \omega^{D}_{(1,1)} \wedge \omega^{D'}_{(1,1)}.$$

Therefore, we often write $D = \omega^{D}_{(1,1)}$ and let the context decide whether the hyper surface or the (1,1)–form is meant.

Kähler cone

The Kähler cone of X is the subspace of the Kähler moduli space such that the volumes

$$\text{vol}(X) = \frac{1}{3!}\int_X J_2^3 > 0\;,$$

$$\text{vol}(D) = \frac{1}{2!}\int_D J_2^2 = \frac{1}{2!}\int_X J_2^2\,\omega^D_{(1,1)} > 0\;,$$

$$\text{vol}(C) = \int_C J_2 = \int_X J_2\,\omega^C_{(2,2)} > 0\;,$$

of X, all its divisors D and all its curves C are positive.

Vector bundles

In order that a gauge background is compatible with $\mathcal{N} = 1$ SUSY in 4D, it has to satisfy the Hermitean Yang–Mills (HYM) equations

$$F_{ab} = F_{\underline{a}\,\underline{b}} = 0\,, \qquad G^{a\underline{a}} F_{a\underline{a}} = 0\,.$$

By the Donaldson-Uhlenbeck-Yau theorem solutions to these equations exist for any vector bundle V that is

- holomorphic,
- stable, i.e. for any subsheaf with $\mathrm{rk}(\mathcal{F}) < \mathrm{rk}(V)$: $\mu_J(\mathcal{F}) < \mu_J(V)$,
- and has slope zero: $\mu_J(V) = 0$.

The slope $\mu_J(V)$ is defined as $\mu_J(V) = \dfrac{1}{rk(V)} \displaystyle\int_X J^2\, c_1(V)\,.$

Examples of stable vector bundles

The construction of stable holomorphic vector bundles is in general very complicated. But there are two simple examples:

- Standard embedding: $F_2 = R_2$:

  This $SU(3)$ bundle is obtained by setting the gauge connection A_1 equal to the spin–connection ω_1.

- Line bundles: $F_2 = (V_A)^I \, \omega^A_{(1,1)} \, \mathcal{H}_I$:

  These Abelian bundles are parameterized by line bundle vectors V_A that encode the embedding in Cartan subalgebra $\{\mathcal{H}_I\}$.

For the latter one needs to ensure that it is slope zero, i.e.

$$\mu_J(F) = \frac{1}{2} \int_X J_2^2 \, F_2 = 0 \ .$$

Torsional manifolds

The non–integrated Bianchi identity for the gauge invariant field strength H_3 of the Kalb–Ramond two–form B_2 reads: Strominger'86

$$\mathrm{d}H_3 = \alpha' \Big(\mathrm{tr} R_2^2 - \mathrm{tr} F_2^2 \Big) \,, \qquad H_3 = i(\bar\partial - \partial) J_2 \,.$$

Integrated over a four–cycle D:

$$\int_D \Big(\mathrm{tr} R_2^2 - \mathrm{tr} F_2^2 \Big) = 0 \,.$$

Hence, unless $\mathrm{tr} R_2^2 = \mathrm{tr} F_2^2$, as in the standard embedding, any vector bundle in a heterotic compactification

- leads to torsion, i.e. $H_3 \neq 0$,
- and X is no longer Kähler: $\mathrm{d}J_2 \neq 0$.

Resolutions of orbifold singularities

Overview:

- Toric geometry
- Non–compact toric resolutions
- Compact orbifold resolutions
- Twisted state VEVs and line bundle fluxes
- Blow–ups of MSSM–like orbifold models

Toric geometry

A toric space $X = (\mathbb{C}^N - Z_{\text{ex}})/(\mathbb{C}^*)^n$ is defined by

- complex coordinates $z = (z_1, \ldots, z_N) \in \mathbb{C}^N$,
- some exclusion set Z_{ex},
- and n independent $\mathbb{C}^* = \mathbb{C} - \{0\}$ scalings:

$$\mathbb{C}^*_r : \ (z_1, \ldots, z_N) \sim \Big(\lambda_r^{(q_r)_1} z_1, \ldots, \lambda_r^{(q_r)_N} z_N \Big) , \qquad \lambda_r \in \mathbb{C}^* .$$

The divisors of X are identified by the equations $D_i := \{z_i = 0\}$, with linear equivalence relations among them.

Many topological properties are encoded in the exclusion set Z_{ex}:

In particularly it determines which curves exist.

Non–compact toric resolutions

The idea of toric resolutions of $\mathbb{C}^3/\mathbb{Z}_N$ singularities is to replace the orbifold action θ by one or more $\mathbb{C}^*$–scalings.

To keep the dimension the same we need to introduce for each $\mathbb{C}^*_r$ an exceptional coordinate $x_r \in \mathbb{C}$.

We can then distinguish between:

- ordinary divisors: $D_a := \{z_a = 0\}$,
- and exceptional divisors: $E_r := \{x_r = 0\}$.

The their intersections and exclusion set Z_{ex} are determined from the so–called toric diagram.

Example: Resolution of $\mathbb{C}^3/\mathbb{Z}_3$

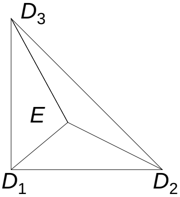
The resolution of $\mathbb{C}^3/\mathbb{Z}_3$ is obtained by replacing

$$\theta: \ (z_1, z_2, z_3) \mapsto \left(e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{2\pi i/3} z_3\right)$$

by

$$\mathbb{C}^*: \ (z_1, z_2, z_3, x) \mapsto \left(\lambda z_1, \lambda z_2, \lambda z_3, \lambda^{-3} x\right), \quad \lambda \in \mathbb{C}^*.$$

Hence we have ordinary divisors $D_a := \{z_a = 0\}$ and single exceptional one $E := \{x = 0\}$.



- Exclusion set:
  $Z_{\text{ex}} := \{z_1 = z_2 = z_3 = 0\}$,
- Non–zero intersections:

$$D_1 D_2 E = D_1 D_3 E = D_2 D_3 E = 1.$$

Example: Resolutions of $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$

In this case we need to replace three orbifold actions

$$\theta_1 :\ (z_1, z_2, z_3) \mapsto (z_1, -z_2, -z_3)\ ,$$

$$\theta_2 :\ (z_1, z_2, z_3) \mapsto (-z_1, z_2, -z_3)\ ,$$

$$\theta_3 :\ (z_1, z_2, z_3) \mapsto (-z_1, -z_2, z_3)\ ,$$

by $\mathbb{C}^*$–scalings:

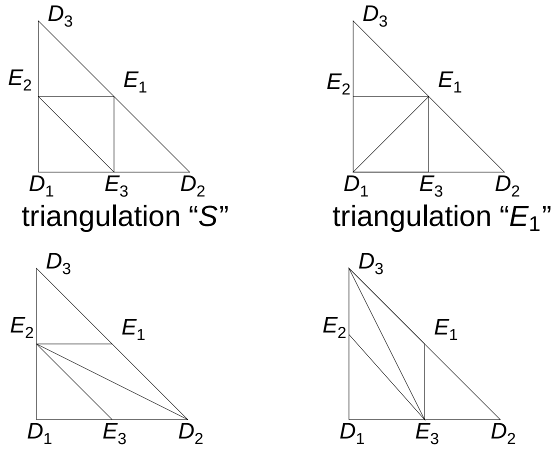
$$\mathbb{C}^*_1 :\ (z_1, z_2, z_3, x_1, x_2, x_3) \mapsto \Big(z_1, \lambda z_2, \lambda z_3, \lambda^{-2} x_1, x_2, x_3\Big)\ ,$$

$$\mathbb{C}^*_2 :\ (z_1, z_2, z_3, x_1, x_2, x_3) \mapsto \Big(\lambda z_1, z_2, \lambda z_3, x_1, \lambda^{-2} x_2, x_3\Big)\ ,$$

$$\mathbb{C}^*_3 :\ (z_1, z_2, z_3, x_1, x_2, x_3) \mapsto \Big(\lambda z_1, \lambda z_2, z_3, x_1, x_2, \lambda^{-2} x_3\Big)\ .$$

Exceptional divisors $E_r := \{x_r = 0\}$, $r = 1, 2, 3$.

$\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ toric diagrams



triangulation "S"

triangulation "E_1"

triangulation "E_2"

triangulation "E_3"

Flop transitions:

"S" $\longrightarrow$ "E_1":
curve E_2E_3 removed;
curve D_1E_1 appeared.

"S" $\longrightarrow$ "E_2":
curve E_1E_3 removed;
curve D_2E_2 appeared.

"S" $\longrightarrow$ "E_3":
curve E_1E_2 removed;
curve D_3E_3 appeared.

The different triangulations have different exclusion sets and lead to different intersection numbers.

Compact orbifold resolutions

The formal construction of compact orbifold resolutions is done on the level of abstract divisors: Denef,Douglas,Florea,Grassi,Kachru'05, Lust,Reffert,Scheidegger,Stieberger'06

- Identify the global set of divisors:

R_a	inherited divisors from the original torus T^6,
E_r	exceptional divisors inside the orbifold singularities,
D_{ai}	ordinary divisors needed in the integral cohomology.

- Determine the set of linear equivalence relations, schematically:

$$n_a D_{ai} \sim R_a + \sum_r n_{a,i;r} E_r \ .$$

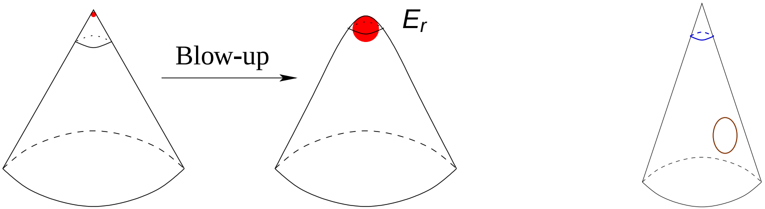
- Determine the intersection numbers from auxiliary polyhedra.

Line bundle fluxes and twisted state VEVs

To build blow–ups of heterotic models we also have to specify which gauge bundle we want to use. SGN,Klevers,Ploger,Trapletti,Vaudrevange'08, SGN, Held,Ruehle,Michele Trapletti,Vaudrevange'09

Only for line bundles we have a systematic characterization:

$$F = (V_r)^I \, E_r \, \mathcal{H}_I \qquad \longleftrightarrow \qquad P_r = rV + P$$



The bundle vectors V_r are determined by the left–moving shifted momenta P_r of twisted state VEVs $\langle T_r \rangle$ that generate the blow–up.

Blow–ups of MSSM–like orbifold models

Along these lines one can for a given heterotic orbifold model we can select a number of twisted states with non–vanishing VEVs and construct the corresponding heterotic resolution models.

We constructed line bundle blow–ups for

- mini–landscape MSSMs based on $T^6/\mathbb{Z}_{6\text{–II}}$,
  SGN,Held,Rühle,Trappletti,Vaudrevange'08
- heterotic MSSMs based on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$.
  Blaszczyk,SGN,Rühle,Trappletti,Vaudrevange'10

There are various physical, practical and conceptual issues with these constructions...

Fate of hyper charge in full blow–up

In the mini-landscape MSSMs the hyper charge gets broken in full blow–up: SGN,Held,Rühle,Trappletti,Vaudrevange'08

- From the smooth Calabi-Yau perspective, because the hyper charge is *not* perpendicular to all the bundle vectors.

  ($U(1)_Y$ is part of the structure group. Distler,Greene'88)

- From the orbifold perspective in full blowup, because there are fixed points with only SM charged twisted states.

Two possible ways to avoid this:

- Do not blow–up the singularities with only SM charged states.
- Use orbifold models in which the GUT breaking is performed by a freely acting Wilson line, e.g. the heterotic $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ MSSMs.

  Blaszczyk,SGN,Ratz,Rühle,Trappletti,Vaudrevange'09

Triangulation dependence

The intersection numbers of the divisors affect e.g.

- the Bianchi identities
- the spectrum of massless states
- the volumes of divisors and curves

The intersection numbers are extremely sensitive to the triangulation chosen. And the number of possible triangulations is huge:

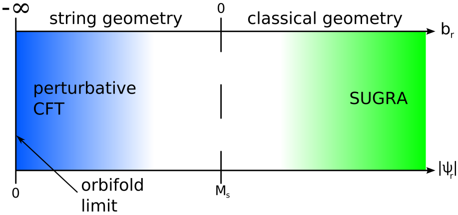
- $T^6/\mathbb{Z}_{6\text{–II}}$: almost 2 million triangulations,
- $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$: order of 10^{33} triangulations.

Is there an appropriate choice for triangulation of the blow–up? What decides this choice? What physics is behind this?

Difference regions of moduli space

The matching of orbifold and resolution models is difficult: we are comparing different descriptions at very different moduli space regions:

Aspinwall,Greene,Morrison'93



- In the orbifold regime we can study perturbations of the CFT.
- In the SUGRA regime we can perform a large volume analysis.
- But in the overlapping region neither of them is reliable.

We need a worldsheet description that smoothly interpolate between the different regimes...

(2,2) Gauged Linear Sigma Models (GLSMs)

Overview:

- (2,2) Superspace in two dimensions
- Superpotential and Fayet–Iliopoulos terms
- Non–compact GLSM resolutions

Supersymmetries on the heterotic worldsheet

The heterotic string has at least $(1,0)$ worldsheet supersymmetry.

When the worldsheet possesses: Hull,Witten'85

- $(2,2)$ supersymmetry:
  - The target space is a Kähler manifold,
  - and describes standard embedding only.
- $(2,0)$ supersymmetry:
  - The target space is a complex manifold,
  - which is generically equipped with torsion,
  - and can describe generic holomorphic vector bundles.

(2,2) Superspace in two dimensions

2D $(2,2)$ Superspace is the dimensional reduced version of 4D $\mathcal{N}=1$ Superspace. Witten'93

I.e. it describes

- worldsheet coordinates $\sigma = \frac{\sigma_0+\sigma_3}{2}$, $\bar{\sigma} = \frac{\sigma_0-\sigma_3}{2}$,
- and two complex Grassmann variables θ^+, θ^-.

Essentially, the only difference between (2,2) superspace in 2D and $\mathcal{N}=1$ superspace in 4D is that the Lorentz group is reduced from $SO(1,3)$ to $SO(1,1)$:

- This allows new types of superfields,

  e.g. twisted–chiral superfields.

(2,2) Superfields

We can recycle the 4D $\mathcal{N} = 1$ multiplet in (2,2) superspace:

- Chiral multiplet: $\overline{D}_+ \mathcal{Z} = \overline{D}_- \mathcal{Z} = 0$:

  components: $\mathcal{Z} = (z, \psi_+, \psi_-, F_z)$ and charge q.

- Vector multiplet: $V^\dagger = V$:

  components: $V = (A, A_\sigma, A_{\bar{\sigma}}, \lambda_+, \lambda_-, D)$ with $A = A_1 + iA_2$.

The reduced Lorentz group allows for novel multiplets:

- Twisted–chiral multiplet: $\overline{D}_+ \Sigma = D_- \Sigma = 0$:
- E.g. super field strength: $\Sigma = \overline{D}_+ D_- V$.

What is linear in GLSM

In GLSMs one take take the kinetic action to be quadratic in the chiral superfield $\mathcal{Z}$:

$$S_{\text{kin}, \mathcal{Z}} = \int \mathrm{d}^2\sigma \mathrm{d}^4\theta\, \overline{\mathcal{Z}} e^{2q\,V} \mathcal{Z}\ ,$$

and vector superfield V:

$$S_{\text{kin}, V} = \int \mathrm{d}^2\sigma \mathrm{d}^4\theta\, \frac{1}{e^2}\, \overline{\Sigma}\Sigma\ .$$

This can of course be easily extended to multiplet chiral superfields $\mathcal{Z}_i$, $\mathcal{X}$, $\mathcal{C}$ and vector multiplets V_r, etc.

Superpotential

We can introduce a superpotential term:

$$S_{\text{super}} = \int \mathrm{d}^2\sigma \mathrm{d}^2\theta\, \mathcal{C}\, P(\mathcal{Z}) + \text{h.c.}$$

Here $P(\mathcal{Z})$ is a homogeneous polynomial of the chiral superfields $\mathcal{Z} = (\mathcal{Z}_i)$.

Gauge invariance of the superpotential demands: $-q_c + q(P(\mathcal{Z})) = 0$. (We take the charges of $\mathcal{Z}_i$ positive and of $\mathcal{C}$ negative.)

The F–term of $\mathcal{C}$ leads to the constraint

$$F_c = P(z) = 0\ .$$

Hence using a superpotential we can implement hyper surface constraints in the effective target space geometry.

Fayet–Iliopoulos term

We can introduce a Fayet–Iliopoulos(FI) term:

$$S_{\text{FI}} = \int \mathrm{d}^2\sigma \mathrm{d}\bar{\theta}^- \mathrm{d}\theta^+ \,\rho\,\Sigma + \text{h.c.}$$

The complex FI–parameter $\rho = b + i\beta$ corresponds to

- an axion β,
- and a real Kähler parameter b.

The D–term constraint, e.g.

$$D = \sum_i q_i \, |z_i|^2 - b = 0$$

results in various phases, i.e. target space topologies.

Example: $\mathbb{C}^3/\mathbb{Z}_3$ GLSM resolution

The resolution of $\mathbb{C}^3/\mathbb{Z}_3$ is obtained by promoting

$$\theta: \ (z_1, z_2, z_3) \mapsto \left(e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{2\pi i/3} z_3\right)$$

to superfields $\mathcal{Z}_a, \mathcal{X}$ with charges:

$U(1)$ charge	$\mathcal{Z}_1$	$\mathcal{Z}_2$	$\mathcal{Z}_3$	$\mathcal{X}$
E	1	1	1	-3

The D–term

$$|z_1|^2 + |z_2|^2 + |z_3|^2 - 3\,|x|^2 = b$$

leads two phases:

- Orbifold phase: $b < 0$: $x \neq 0$ leaves a $\mathbb{Z}_3$ action on the z_a's.
- Blow–up phase: $b > 0$: there is a four–cycle with a radius set by b.

Phases of $\mathbb{C}^3/\mathbb{Z}_2\times\mathbb{Z}_2$ GLSM resolutions

Similarly the $\mathbb{C}^3/\mathbb{Z}_2\times\mathbb{Z}_2$ orbifold leads to the charge assignment:

$U(1)$ charge	$\mathcal{Z}_1$	$\mathcal{Z}_2$	$\mathcal{Z}_3$	$\mathcal{X}_1$	$\mathcal{X}_2$	$\mathcal{X}_3$
E_1	0	1	1	-2	0	0
E_2	1	0	1	0	-2	0
E_3	1	1	0	0	0	-2

This leads the D–term potential:

$$V = \frac{e_1^2}{2}\Big(|z_2|^2+|z_3|^2-2\,|x_1|^2-b_1\Big)^2 + \frac{e_2^2}{2}\Big(|z_1|^2+|z_3|^2-2\,|x_2|^2-b_2\Big)^2 + \frac{e_3^2}{2}\Big(|z_1|^2+|z_2|^2-2\,|x_3|^2-b_3\Big)^2$$

The divisor $E_1{:=}\{x_1{=}0\}$ exists if:

$$V\Big|_{E_1}=0 \quad\Rightarrow\quad b_1\geq 0$$

The curve D_1E_1 exists when:

$$V\Big|_{D_1E_1}=0 \quad\Rightarrow\quad b_1\geq 0\,,\quad b_1-b_2-b_3\geq 0$$

Phases of $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ GLSM resolutions

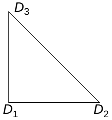
Divisor exists when	
D_1	always
D_2	always
D_3	always
E_1	$b_1 \geq 0$
E_2	$b_2 \geq 0$
E_3	$b_3 \geq 0$
Curve exists when	
$D_1 D_2$	$b_3 \leq 0$
$D_1 D_3$	$b_2 \leq 0$
$D_2 D_3$	$b_1 \leq 0$

Curve exists when	
$E_1 E_2$	$b_1, b_2 \geq 0\,, \quad b_3 \leq b_1 + b_2$
$E_2 E_3$	$b_2, b_3 \geq 0\,, \quad b_1 \leq b_2 + b_3$
$E_1 E_3$	$b_1, b_3 \geq 0\,, \quad b_2 \leq b_1 + b_3$
$D_1 E_1$	$b_1 \geq 0\,, \quad b_1 \geq b_2 + b_3$
$D_2 E_2$	$b_2 \geq 0\,, \quad b_2 \geq b_1 + b_3$
$D_3 E_3$	$b_3 \geq 0\,, \quad b_3 \geq b_1 + b_2$
$D_1 E_{2,3}$	$b_2 \geq 0\,, \quad b_3 \geq 0$
$D_2 E_{1,3}$	$b_1 \geq 0\,, \quad b_3 \geq 0$
$D_3 E_{1,2}$	$b_1 \geq 0\,, \quad b_2 \geq 0$

Phases of $\mathbb{C}^3/\mathbb{Z}_2\times\mathbb{Z}_2$ GLSM resolutions

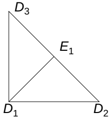
orbifold phase:
no exceptional divisors

$$b_1 \leq 0$$
$$b_2 \leq 0$$
$$b_3 \leq 0$$



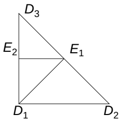
partial resolution:
one exceptional divisor

$$b_1 \geq 0$$
$$b_2 \leq 0$$
$$b_3 \leq 0$$

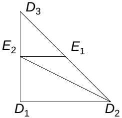


partial resolution:
two exceptional divisors

$$b_1 \geq b_2 \geq 0$$
$$b_3 \leq 0$$

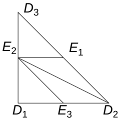


$$b_2 \geq b_1 \geq 0$$
$$b_3 \leq 0$$

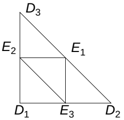


full resolution: three exceptional divisors

$$b_1,\ b_3 \geq 0$$
$$b_2 \geq b_1 + b_3$$



$$b_1 + b_2 \geq b_3 \geq 0$$
$$b_1 + b_3 \geq b_2 \geq 0$$
$$b_2 + b_3 \geq b_1 \geq 0$$



Toroidal orbifold resolution GLSMs

Overview:

- Global toroidal orbifold resolutions
- Different types of resolution GLSMs
- Moduli space of orbifold resolution models
- Partially resolvable GLSMs

Global toroidal orbifold resolutions

To obtain GLSM resolutions for compact orbifolds $T^6/\mathbb{Z}_N$ we proceed as follows: Blaszczyk,SGN,Rühle'11

- Start from a factorized $T^6 = T^2_1 \times T^2_2 \times T^2_3$.
- Describe each two–torus T^2_a as a hyper surface in a weighted projective space with homogeneous coordinates z_{ai}.
- Use the local GLSM resolution procedure to resolve the singularities at the fixed points $z_{1i} = z_{2j} = z_{3k} = 0$.

We will illustrate this in some detail for $T^6/\mathbb{Z}_3$ next.

Two–torus $T^2(\mathbb{Z}_3)$ possessing a $\mathbb{Z}_3$ symmetry

A two–torus possessing a $\mathbb{Z}_3$ orbifold symmetry can be described as an algebraic torus, i.e. a hyper surface in the projective space $\mathbb{P}^2_{111}[3]$:

$U(1)$ charge	$\mathcal{Z}_1$	$\mathcal{Z}_2$	$\mathcal{Z}_3$	$\mathcal{C}$
R	1	1	1	-3

$$W = \mathcal{C}(\mathcal{Z}_1^3 + \mathcal{Z}_2^3 + \mathcal{Z}_3^3)$$

Their D– and F–terms

$$|z_1|^2 + |z_2|^2 + |z_3|^2 - 3\,|c|^2 = a\,, \quad c\,z_i^2 = 0\,, \quad z_1^3 + z_2^3 + z_3^3 = 0\,,$$

leads two phases:

- Non–geometrical phase: the target space is a point.
  $a < 0$: $c \neq 0 \Rightarrow z_i = 0$
- Geometrical phase: the target space is a two–torus of radius $\sqrt{a}$.
  $a > 0$: $\exists z_i \neq 0 \Rightarrow c = 0$

Two–tori as hyper surfaces in projective spaces

Torus	Projective hyper surface / Superpotential	$U(1)$ charges of $\mathcal{Z}_1$	$\mathcal{Z}_2$	$\mathcal{Z}_3$	$\mathcal{Z}_4$	$\mathcal{C}$	$\mathcal{C}'$
$T^2(\mathbb{Z}_3)$	$\mathbb{P}^2_{1,1,1}[3]$	1	1	1	–	-3	–
	$W = \mathcal{C}(\mathcal{Z}_1^3 + \mathcal{Z}_2^3 + \mathcal{Z}_3^3)$						
$T^2(\mathbb{Z}_4)$	$\mathbb{P}^2_{1,1,2}[4]$	1	1	2	–	-4	–
	$W = \mathcal{C}(\mathcal{Z}_1^4 + \mathcal{Z}_2^4 + \mathcal{Z}_3^2)$						
$T^2(\mathbb{Z}_6)$	$\mathbb{P}^2_{1,2,3}[6]$	1	2	3	–	-6	–
	$W = \mathcal{C}(\mathcal{Z}_1^6 + \mathcal{Z}_2^3 + \mathcal{Z}_3^2)$						
$T^2(\mathbb{Z}_2)$	$\mathbb{P}^3_{1,1,1,1}[2,2]/\mathbb{Z}_2^2$	1	1	1	1	-2	-2
	$W = \mathcal{C}(\kappa\, \mathcal{Z}_1^2 + \mathcal{Z}_2^2 + \mathcal{Z}_3^2) + \mathcal{C}'(\mathcal{Z}_1^2 + \mathcal{Z}_2^2 + \mathcal{Z}_4^2)$						

Minimal fully resolvable GLSM for $T^6/\mathbb{Z}_3$

The GLSM description of $T^6 = T^2_1(\mathbb{Z}_3) \times T^2_2(\mathbb{Z}_3) \times T^2_3(\mathbb{Z}_3)$ can be extended with one exceptional gauging to: Aspinwall,Plesser'11

$U(1)$ charge	$\mathcal{Z}_{1i}$	$\mathcal{Z}_{2j}$	$\mathcal{Z}_{3k}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{X}_{ijk}$
R_1	1	0	0	-3	0	0	0
R_2	0	1	0	0	-3	0	0
R_3	0	0	1	0	0	-3	0
E_{111}	δ_{1i}	δ_{1j}	δ_{1k}	0	0	0	$-3\,\delta_{1i}\delta_{1j}\delta_{1k}$

with the superpotential:

$$W = \sum_a \mathcal{C}_a \left(\mathcal{Z}_{a1}^3\, \mathcal{X}_{111} + \mathcal{Z}_{a2}^3 + \mathcal{Z}_{a3}^3 \right) .$$

This GLSM has phases describing both the $T^6/\mathbb{Z}_3$ and its resolution.

Minimal fully resolvable GLSM

In the orbifold and blow–up regimes (where $c_a = 0$) the D– and F–terms reduce to: Aspinwall,Plesser'11, Blaszczyk,SGN,Rühle'11

$$|z_{a1}|^2 + |z_{a2}|^2 + |z_{a3}|^2 = a_a\,, \qquad z_{a1}^3 x_{111} + z_{a2}^3 + z_{a3}^3 = 0\,,$$

$$|z_{11}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3\,|x_{111}|^2 = b_{111}\,, \qquad a = 1,2,3\,.$$

- Orbifold phase: $b_{111} < 0 < a_a$: $x_{111} \neq 0$ induces a $\mathbb{Z}_3$ action:

  $\theta_{111}:\ (z_{11}, z_{21}, z_{31}) \mapsto (\zeta\, z_{11}, \zeta\, z_{21}, \zeta\, z_{31})$ which has fixed points, $z_{11} = z_{21} = z_{31} = 0$, at the 27 roots of

  $$z_{a2}^3 + z_{a3}^3 = 0\,,\ a = 1,2,3\,.$$

- Blow–up phase: $0 < b_{111} < a_a$: fixed points are gone;

  $E_{111} = \{x_{111} = 0\}$ has 27 components with size b_{111}.

GLSM phases correspond to target space topologies

The following topology changes can be described by GLSM phase transitions:

i) Modification of the intersection properties of divisors:

E.g. the so–called flop transitions, where in one phase two divisors intersect, while in the next they do not anymore.

ii) Appearance or disappearance of divisors:

E.g. exceptional cycles that appear in blow–up process of orbifold singularities.

iii) Alteration of the target space dimension:

E.g. the transition of the algebraic torus from the geometrical to the non–geometrical regime.

Phases of resolution GLSMs:

1. Non–geometrical regime ($a, b < 0$):

   The target space is just a point.

2. Orbifold regime ($b < 0 < a$):

   Conventional orbifold: Torus modded by discrete rotations.

3. Blow–up regime ($0 < b < a$):

   All exceptional cycles have finite size, but smaller than the torus cycles.

4. Critical blow–up regime ($0 < a < b < 3\,a$):

   The exceptional and torus cycles have comparable sizes.

5. Over–blow–up regime ($0 < 3\,a < b$):

   The roles of blow–up and torus cycles have become interchanged.

6. Singular over–blow–up regime ($a < 0 < b$):

   The torus cycles disappeared inducing target space singularities.

Maximal fully resolvable GLSM

In the minimal fully resolvable model we had one Kähler parameter that sets the sizes of all 27 exceptional cycles simultaneously.

In the maximal fully resolvable model:

$U(1)$ charges	$\mathcal{Z}_{1i}$	$\mathcal{Z}_{2j}$	$\mathcal{Z}_{3k}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{X}_{ijk}$
R_1	1	0	0	-3	0	0	0
R_2	0	1	0	0	-3	0	0
R_3	0	0	1	0	0	-3	0
$E_{i'j'k'}$	$\delta_{i'i}$	$\delta_{j'j}$	$\delta_{k'k}$	0	0	0	$-3\,\delta_{i'i}\delta_{j'j}\delta_{k'k}$

each fixed point gets its own $U(1)_{E_{ijk}}$ and coordinate x_{ijk} for all $i,j,k = 1,2,3$.

$$W = \mathcal{C}_1 \sum_i \mathcal{Z}_{1i}^3 \prod_{j,k} \mathcal{X}_{ijk} + \mathcal{C}_2 \sum_j \mathcal{Z}_{2j}^3 \prod_{i,k} \mathcal{X}_{ijk} + \mathcal{C}_3 \sum_k \mathcal{Z}_{3k}^3 \prod_{i,j} \mathcal{X}_{ijk} \,.$$

Partially resolvable and non–factorized GLSMs

We can construct many other GLSMs associated to the $T^6/\mathbb{Z}_3$ orbifold by only using subsets of the maximal number of gaugings

$U(1)$ charges	$\mathcal{Z}_{1i}$	$\mathcal{Z}_{2j}$	$\mathcal{Z}_{3k}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{X}_{ijk}$
R_1	1	0	0	-3	0	0	0
R_2	0	1	0	0	-3	0	0
R_3	0	0	1	0	0	-3	0
$E_{i'j'k'}$	$\delta_{i'i}$	$\delta_{j'j}$	$\delta_{k'k}$	0	0	0	$-3\,\delta_{i'i}\delta_{j'j}\delta_{k'k}$

This allows us to build:

- Partially resolvable GLSMs:

  in which subsets of fixed points cannot be resolved

- Resolutions of non–factorized orbifolds:

  on non–factorized torus lattices

Partially resolvable and non–factorized GLSMs

Overview of all partially resolvable and non–factorized $T^6/\mathbb{Z}_3$ we can obtain by using a subset of all possible exceptional gaugings:

Exceptional coordinates	Discrete Group	FP Sets: Groups	FP Sets: Singular	Lattice	Fully resolvable by adding
x_{111}	–	1×27	0	$A_2 \times A_2 \times A_2$	–
x_{111}, x_{211}	$\mathbb{Z}_3$	3×9	9	$A_2 \times A_2 \times A_2$	x_{311}
x_{111}, x_{221}				$F_4 \times A_2$	x_{331}
x_{111}, x_{222}				E_6	x_{333}
$x_{111}, x_{211}, x_{121}$	$\mathbb{Z}_3^2$	9×3	18	$A_2 \times A_2 \times A_2$	$x_{131}, x_{221}, x_{231}, x_{311}, x_{321}, x_{331}$
$x_{111}, x_{221}, x_{112}$				$F_4 \times A_2$	$x_{113}, x_{222}, x_{223}, x_{331}, x_{332}, x_{333}$
$x_{111}, x_{221}, x_{212}$				no Lie lattice	$x_{123}, x_{133}, x_{232}, x_{313}, x_{322}, x_{331}$
$x_{111}, x_{211}, x_{121}, x_{112}$	$\mathbb{Z}_3^3$	27×1	23	$A_2 \times A_2 \times A_2$	rest

GLSM resolvable toroidal $T^6/\mathbb{Z}_N$ orbifolds

Using our GLSM methods we treat the following orbifolds:

Point group	Orbifold twist vector	T^6 torus lattice	Exceptional gaugings	Invisible moduli $h^{1,1}_{\text{off-diag}}$	Invisible moduli $h^{1,2}_{\text{twisted}}$	Indistinguishable fixed points/tori
$\mathbb{Z}_3$	$\frac{1}{3}(1,1,-2)$	A_2^3	27	6	0	0
$\mathbb{Z}_4$	$\frac{1}{4}(1,1,-2)$	$D_2^2 \times A_1^2$	23	2	6	1×2 FT
		$D_2 \times A_1 \times A_3$	6	2	2	2×8 FP, 2×2 FT
		A_3^2	8	2	0	4×4 FP
$\mathbb{Z}_{6-\text{I}}$	$\frac{1}{6}(1,1,-2)$	$G_2^2 \times A_2$	17	2	5	1×3 FT , 1×2 FP
$\mathbb{Z}_{6-\text{II}}$	$\frac{1}{6}(1,2,-3)$	$G_2 \times A_2 \times A_1^2$	32	0	10	0

as they can all defined starting from factorized six–tori.

But none of the $\mathbb{Z}_7, \mathbb{Z}_{8\text{–I}}, \mathbb{Z}_{8\text{–II}}, \mathbb{Z}_{12\text{–I}}$ and $\mathbb{Z}_{12\text{–II}}$ orbifolds and the $\mathbb{Z}_{6\text{–II}}$ orbifold on non–factorizable lattices.

In addition...

GLSM resolvable toroidal $T^6/\mathbb{Z}_M \times \mathbb{Z}_N$ orbifolds

Point group	Orbifold twist vectors	T^6 torus lattice	Exceptional gaugings	Invisible moduli $h^{1,1}_{\text{off–diag}}$	Invisible moduli $h^{1,2}_{\text{twisted}}$	Indistinguishable fixed points/tori
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\frac{1}{2}(1,-1,0)$, $\frac{1}{2}(0,1,-1)$	A_1^6	48	0	0	0
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\frac{1}{2}(0,1,-1)$, $\frac{1}{4}(1,-1,0)$	$D_2^2 \times A_1^2$	57	0	0	$1 \times 2\,$FT
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{I}}$	$\frac{1}{2}(0,1,-1)$, $\frac{1}{6}(1,1,-2)$	$G_2 \times A_2 \times A_1^2$	26	0	0	$3 \times 3\,$FT , 1×2FP
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{II}}$	$\frac{1}{6}(1,-1,0)$, $\frac{1}{2}(0,1,-1)$	$G_2^2 \times A_2$	46	0	2	$1 \times 3\,$FT
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$\frac{1}{3}(1,-1,0)$, $\frac{1}{3}(0,1,-1)$	A_2^3	81	0	0	0
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$\frac{1}{3}(0,1,-1)$, $\frac{1}{6}(1,-1,0)$	$G_2^2 \times A_2$	65	0	1	$2 \times 2\,$FT , $3 \times 2\,$FP
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$\frac{1}{4}(1,-1,0)$, $\frac{1}{4}(0,1,-1)$	D_2^3	87	0	0	0
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$\frac{1}{6}(1,-1,0)$, $\frac{1}{6}(0,1,-1)$	G_2^3	80	0	0	$1 \times 2\,$FP

(2,0) Gauged Linear Sigma Models (GLSMs)

Overview:

- (2,0) Superspace in two dimensions
- Worldsheet gauge anomaly conditions
- GLSMs corresponding to twisted state VEVs

(2,0) Superspace in two dimensions

(2,0) supersymmetric worldsheet theories are needed to describe

- generic holomorphic vector bundles
- on a complex manifold with torsion.

2D $(2,0)$ Superspace is spanned by: Dine,Seiberg'86

- worldsheet coordinates $\sigma = \frac{\sigma_0+\sigma_3}{2}, \bar{\sigma} = \frac{\sigma_0-\sigma_3}{2}$,
- and a single complex Grassmann variable θ^+.

I.e. (2,0) superspace can be obtained from (2,2) superspace by dimensional reducing the Grassmann variable θ^-.

Matter multiplets: Chiral & chiral-Fermi superfields

Superfield type	Superfield symbol	$U(1)$ charge	Bos. DOF on	Bos. DOF off	Ferm. DOF on	Ferm. DOF off
chiral	Ψ_a	$(q_r)_a$	z_a	-	ψ_a	-
chiral-Fermi	Λ^I	$(Q_r)^I$	-	h^I	λ^I	-

- $z_a, \quad a = 0, \ldots, 3$ are the complex target space coordinates,
- $\psi_a, \quad a = 0, \ldots, 3$ are their right–moving superpartners,
- $\lambda^I, \quad I = 1, \ldots, 16$ are the left–moving fermions that generate the target space gauge degrees of freedom (DOF)

The worldsheet action of the free heterotic string reads

$$S_{\text{het}} = \int \mathrm{d}^2\sigma \mathrm{d}^2\theta^+ \left\{ \frac{i}{2} \overline{\Psi}_a \bar{\partial} \Psi_a - \frac{1}{2} \overline{\Lambda}^I \Lambda^I \right\}$$

Bosonic gaugings & Fayet-Iliopoulos terms

Superfield type	Superfield symbol	charge	Bos. DOF on	Bos. DOF off	Ferm. DOF on	Ferm. DOF off
gauge	$(V,A)^r$	0	$A^r_\sigma, A^r_{\bar\sigma}$	$\widetilde{D}^r$	ϕ^r	-

The actions of the gauge multiplets Witten'93, Distler,Kachru'93

$$S_{\text{gauge}} = \frac{1}{2e^2}\int \mathrm{d}^2\sigma \mathrm{d}^2\theta^+\,\overline{F}F$$

$$S_{\text{FI}} = \int \mathrm{d}^2\sigma \mathrm{d}\theta^+\,\rho(\Psi)\,F + \text{h.c.}$$

are expressed in terms of

- the (2,0) gauge superfield strength $F = -\frac{1}{2}\overline{D}_+\big(A - i\bar\partial V\big)$
- and the complex FI–parameter ρ.

Non–Abelian bundles as fermionic gaugings

Superfield type	Superfield symbol	$U(1)$ charge	Bos. DOF on	Bos. DOF off	Ferm. DOF on	Ferm. DOF off
Fermi-gauge	Σ^i	0	s^i	-	φ^i	-

The holomorphic functions $M^I{}_i(\Psi)$ define the monad bundle $\mathbb{V}$:
Witten'93,Blumenhagen,Wisskirchen'96

$$0 \longrightarrow \mathbb{V} \longrightarrow \underset{\Xi}{\overset{}{\mathcal{O}^{N_\Sigma}}} \overset{M}{\longrightarrow} \bigotimes_I \mathcal{O}(Q_1^I, \ldots, Q_{N_V}^I) \longrightarrow 0 \,.$$

$$\mathcal{O}^{N_\Sigma} \ni \Xi \,, \qquad \bigotimes_I \mathcal{O}(Q_1^I, \ldots, Q_{N_V}^I) \ni \Lambda$$

This can realized by fermionic gaugings Distler'92

$$\Lambda^I \rightarrow \Lambda^I + M^I{}_i(\Psi)\,\Xi^i \,, \qquad \Sigma^i \rightarrow \Sigma^i + \Xi^i \,,$$

with chiral–Fermi parameters Ξ^i, $i = 1, \ldots, N_\Sigma$.

The standard embedding: $M^a{}_r(\Psi) = (Q_r)^a{}_b \Psi_b$.

Some consistency requirements on a (2,0) GLSM

No pure or mixed gauge anomalies:

$$\forall r, s: \sum_I (Q_r)^I (Q_s)^I = \sum_a (q_r)_a (q_s)_a \qquad \rightsquigarrow \qquad c_2(\mathbb{V}) = c_2(TX)$$

No divergent FI-terms on the worldsheet:
(Otherwise the FI–parameters flow to $\pm\infty$ in the IR)

$$\forall r: \sum_a (q_r)_a = 0 \qquad \rightsquigarrow \qquad c_1(TX) = 0$$

Distler'93

Bianchi identities versus GLSM anomaly cancellation on resolutions of $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$

The Bianchi identities on triangulation “E_1” read: SGN,Ha,Trapletti’08

$$Q_2^2 + Q_3^2 = 3\,, \qquad Q_2^2 - 2\,Q_1 \cdot Q_3 = 1\,, \qquad Q_3^2 - 2\,Q_1 \cdot Q_2 = 1$$

and on triangulation “S”:

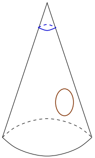
$$Q_1^2 + 2\,Q_2 \cdot Q_3 = 2\,, \qquad Q_2^2 + 2\,Q_1 \cdot Q_3 = 2\,, \qquad Q_3^2 + 2\,Q_1 \cdot Q_2 = 2$$

The pure and mixed GLSM anomaly cancellations require: SGN’10

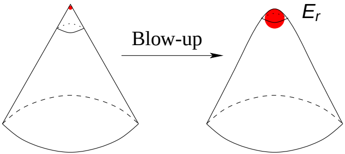
$$Q_1^2 = Q_2^2 = Q_3^2 = \frac{3}{2}\,, \qquad Q_1 \cdot Q_2 = Q_2 \cdot Q_3 = Q_3 \cdot Q_1 = \frac{1}{4}$$

Hence the GLSM anomaly conditions ensure that the Bianchi identities in any triangulation are fulfilled.

Twisted state VEVs generate the blow–up



$\langle T_r \rangle \neq 0 \Rightarrow$



An r–twisted state $|T_r\rangle = |p_r, P_r\rangle, \tilde{\alpha}^a_{-\tilde{v}^a_r}|p_r, P_r\rangle$, has

- right– and left–moving shifted momenta:

$$p_r = p + rv\,, \qquad p \in SO(8) \text{ vector weight lattice;}$$

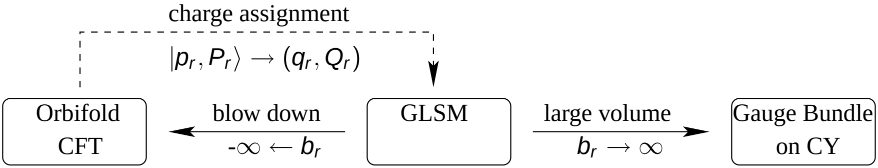
$$P_r = P + rV\,, \qquad P \in SO(32) \text{ root lattice;}$$

- and $\widetilde{N}_r$ excitations $\tilde{\alpha}^a_{-\tilde{v}^a_r}$. ($\widetilde{N}_r$ counted in units $\tilde{v}^a_r = rv^a$ mod integers.)

The level matching condition reads:

$$\boldsymbol{P_r^2 = 1 + p_r^2} - 2\,\widetilde{N}_r$$

Shifted momenta and GLSM charges



This charge assignment has to be such that:

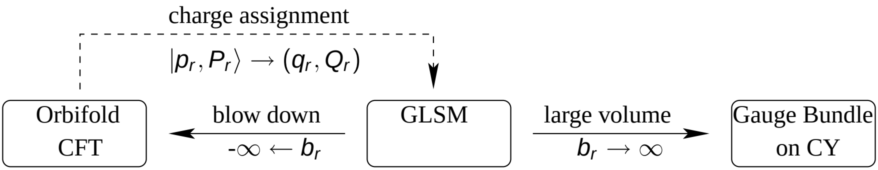
- the number of DOF is as in the free theory
- the sum of charges zero

The left-moving momenta define the charges of Ψ^a: SGN'10

$$(q_r)_a = (p_r)_a = \frac{(n_r)_a}{N} \qquad \sum_a (n_r)_a = N$$

And we introduce a new chiral superfield Ψ_r with charge $(q_r)_r = -1$.

Non–oscillatory blow–up modes



We set the charges $(Q_r)^I$ of the chiral-Fermi superfields Λ^I: SGN'10

$$(Q_r)^I = (P_r)^I$$

The level matching condition ensures pure anomaly cancellation:

$$Q_r^2 = P_r^2 = 1 + p_r^2 = q_r^2$$

for twisted states without oscillator excitations ($\widetilde{N}_r = 0$).

Oscillatory blow–up modes

When a twisted state that has oscillator excitations, e.g.:

$$|T_r\rangle = \tilde{\alpha}^a_{-\tilde{v}_r} |p_r, P_r\rangle \ , \qquad \tilde{\alpha}^a_{-\tilde{v}_r} \tilde{\alpha}^b_{-\tilde{v}_r} |p_r, P_r\rangle$$

Setting $Q_r = P_r$, we encounter a pure anomaly cancellation:

$$Q_r^2 = P_r^2 = 1 + p_r^2 - 2\,\widetilde{N}_r \neq 1 + p_r^2 = q_r^2 \ ,$$

as $\widetilde{N}_r \neq 0$. To remove the anomaly we propose to: SGN'10

- add some integers the entries P_r to define Q_r
- and include an extra chiral-Fermi multiplet Λ^r with charge -1

But this forces us to introduce a fermionic gauging: Distler'95

$$\delta_r \Lambda^I = \beta^I_{r\,a}\, \Psi_a\, \Xi_r + \beta^I_{r\,ab}\, \Psi_a \Psi_b\, \Xi_r + \ldots \ , \qquad \delta_r \Lambda^r = -\Psi_r\, \Xi_r \ ,$$

to preserve the number of fermionic DOF.

A $\mathbb{C}^3/\mathbb{Z}_4$ Model:

$v = (0, \frac{1}{4}^2, -\frac{1}{2})$

$V = (\frac{1}{4}^2, -\frac{1}{2}, 0^{13})$

$SO(26) \times U(2) \times U(1)$

Choi,SGN,Trapletti'04

Nilles et al'06

state	representation	in 4D	$Q(\Lambda^I, \Lambda^3, \Lambda^\alpha; \Lambda^{\text{-}1}, \Lambda^{\text{-}2})$
1st twisted sector: $\widetilde{N}_1 = 0,\ P_1^2 = \frac{11}{8}$			
$\vert p_1\ (\frac{1}{4}^2\ \frac{1}{2}\ \underline{0^{12}\ \pm 1})\rangle$	$(\mathbf{26},\mathbf{1})_{\text{-}\frac{1}{2},\text{-}\frac{1}{8}}$	✓	$(\frac{1}{4}^2\ \frac{1}{2}\ \underline{0^{12}\ \pm 1}; 0\ 0)$
$\vert p_1\ (\text{-}\frac{3}{4}^2\ \text{-}\frac{1}{2}\ 0^{13})\rangle$	$(\mathbf{1},\mathbf{1})_{1,\text{-}\frac{1}{8}}$	✓	$(\text{-}\frac{3}{4}^2\ \text{-}\frac{1}{2}\ 0^{13}; 0\ 0)$
$\widetilde{N}_1 = \frac{1}{4},\ P_1^2 = \frac{7}{8}$			
$\tilde{\alpha}^a_{-\frac{1}{4}}\vert p_1\ (\underline{\frac{1}{4}\ \text{-}\frac{3}{4}}\ \frac{1}{2}\ 0^{13})\rangle$	$\mathbf{2}_R(\mathbf{1},\mathbf{2})_{0,\text{-}\frac{3}{8}}$	✓	$(\frac{1}{4}^2\ \frac{1}{2}\ 0^{13}; \text{-}1\ 0)$
$\widetilde{N}_1 = \frac{1}{2},\ P_1^2 = \frac{3}{8}$			
$\tilde{\alpha}^3_{-\frac{1}{2}}\vert p_1\ (\frac{1}{4}^2\ \text{-}\frac{1}{2}\ 0^{13})\rangle$	$(\mathbf{1},\mathbf{1})_{0,\frac{3}{8}}$	✓	$(\frac{1}{4}^2\ \frac{1}{2}\ 0^{13}; \text{-}1\ 0)$
$\tilde{\alpha}^{\bar{3}}_{-\frac{1}{2}}\vert p_1\ (\frac{1}{4}^2\ \text{-}\frac{1}{2}\ 0^{13})\rangle$	$(\mathbf{1},\mathbf{1})'_{0,\frac{3}{8}}$	✓	$(\frac{1}{4}^2\ \frac{1}{2}\ 0^{13}; \text{-}1\ 0)$
$\tilde{\alpha}^a_{-\frac{1}{4}}\tilde{\alpha}^b_{-\frac{1}{4}}\vert p_1\ (\frac{1}{4}^2\ \text{-}\frac{1}{2}\ 0^{13})\rangle$	$\mathbf{3}_R(\mathbf{1},\mathbf{1})_{0,\frac{3}{8}}$	✓	$(\frac{1}{4}^2\ \frac{1}{2}\ 0^{13}; \text{-}1\ 0)$
2nd twisted sector: $\widetilde{N}_2 = 0,\ P_2^2 = \frac{3}{2}$			
$\vert p_2\ (\frac{1}{2}^2\ 0\ \underline{0^{12}\ \pm 1})\rangle$	$(\mathbf{26},\mathbf{1})_{\text{-}\frac{1}{2},\frac{1}{4}}$	✓	$(\frac{1}{2}^2\ 0\ \underline{0^{12}\ \pm 1}; 0\ 0)$
$\vert p_2\ (\text{-}\frac{1}{2}^2\ 1\ 0^{13})\rangle$	$(\mathbf{1},\mathbf{1})_{0,\text{-}\frac{3}{4}}$	✓	$(\text{-}\frac{1}{2}^2\ 1\ 0^{13}; 0\ 0)$
$\vert p_2\ (\text{-}\frac{1}{2}^2\ \text{-}1\ 0^{13})\rangle$	$(\mathbf{1},\mathbf{1})_{1,\frac{3}{4}}$	✓	$(\text{-}\frac{1}{2}^2\ \text{-}1\ 0^{13}; 0\ 0)$
$\vert p_2\ (\text{-}\frac{1}{2}^2\ 0\ \underline{0^{12}\ \pm 1})\rangle$	$(\mathbf{26},\mathbf{1})_{\frac{1}{2},\text{-}\frac{1}{4}}$	x	$(\text{-}\frac{1}{2}^2\ 0\ \underline{0^{12}\ \pm 1}; 0\ 0)$
$\vert p_2\ (\frac{1}{2}^2\ \text{-}1\ 0^{13})\rangle$	$(\mathbf{1},\mathbf{1})_{0,\frac{3}{4}}$	x	$(\frac{1}{2}^2\ \text{-}1\ 0^{13}; 0\ 0)$
$\vert p_2\ (\frac{1}{2}^2\ 1\ 0^{13})\rangle$	$(\mathbf{1},\mathbf{1})_{\text{-}1,\text{-}\frac{3}{4}}$	x	$(\frac{1}{2}^2\ 1\ 0^{13}; 0\ 0)$
$\widetilde{N}_2 = \frac{1}{2},\ P_2^2 = \frac{1}{2}$			
$\tilde{\alpha}^a_{-1/2}\vert p_2\ (\underline{\frac{1}{2}\ \text{-}\frac{1}{2}}\ 0\ 0^{13})\rangle$	$\mathbf{2}_R(\mathbf{1},\mathbf{2})_{0,0}$	✓	$(\frac{1}{2}^2\ 0\ 0^{13}; 0\ \text{-}1)$
$\tilde{\alpha}^{\bar{a}}_{-1/2}\vert p_2\ (\underline{\frac{1}{2}\ \text{-}\frac{1}{2}}\ 0\ 0^{13})\rangle$	$\overline{\mathbf{2}}_R(\mathbf{1},\mathbf{2})_{0,0}$	x	$(\frac{1}{2}^2\ 0\ 0^{13}; 0\ \text{-}1)$

$a, b, A, B = 1, 2$

$\alpha, \beta = 1, \ldots 13$

Non-Abelian Bundles from Oscillator States

We consider a GLSM associated to the $\mathbb{C}^3/\mathbb{Z}_4$ orbifold with charges

	Ψ^0	Ψ^a	Ψ^3	Ψ^{-1}	Ψ^{-2}	Λ^A	Λ^3	Λ^α	Λ^{-1}	Λ^{-2}
q_1, Q_1	0	$\frac{1}{4}^2$	$\frac{1}{2}$	-1	0	$\frac{1}{4}^2$	$\frac{1}{2}$	0	-1	0
q_2, Q_2	0	$\frac{1}{2}^2$	0	0	-1	$\frac{1}{2}^2$	0	0	0	-1

First twisted oscillatory blow-up modes give:

$$\delta_1 \Lambda^A = {\beta_1}^A{}_a \, \Psi^a \, \Xi_1 \,, \quad \delta_1 \Lambda^3 = (\beta_1 \, \Psi^3 + \tfrac{1}{2} \beta_{1\,ab} \, \Psi^a \Psi^b) \Xi_1 \,,$$

$$\delta_1 \Lambda^{-1} = -\Psi^{-r} \, \Xi_1 \,, \qquad \delta_1 \Lambda^\alpha = \delta_1 \Lambda^{-2} = 0$$

Second twisted oscillatory blow-up modes give:

$$\delta_2 \Lambda^A = {\beta_2}^A{}_a \, \Psi^a \, \Xi_2 \,, \quad \delta_2 \Lambda^{-2} = -\Psi^{-2} \, \Xi_2 \,, \quad \delta_2 \Lambda^\alpha = \delta_2 \Lambda^3 = \delta_2 \Lambda^{-1} = 0$$

Standard embedding: ${\beta_1}^A{}_a = \frac{1}{4} \, \delta^A_a$, $\beta_1 = \frac{1}{2}$, ${\beta_2}^A{}_a = \frac{1}{2} \, \delta^A_a$ and $\beta_{1\,ab} = 0$

Outlook: What is next?

- Precise mapping between compact orbifold CFTs with twisted and untwisted VEVs and (2,0) GLSMs
- Systematic construction of vector bundles in the GLSM language.
- Computational tools to determine the effective (massless) 4D spectrum in any GLSM phase.
- Model building on compact GSLM resolutions