Model Building with Intersecting D-branes

Gabriele Honecker

JG|U Mainz

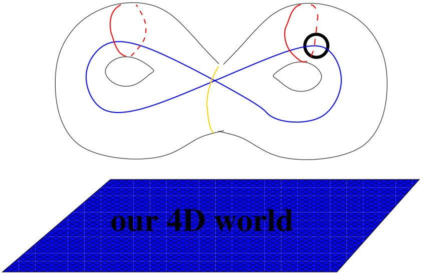
Bethe Forum, 8 November 2011

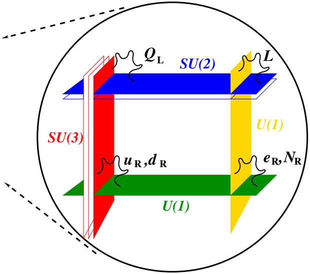
Contents

- **D-brane model building**
  - Global consistency conditions (RR tcc, K-theory)
  - Chiral spectrum & SUSY
- **Standard Model spectrum on Intersecting D-branes**
  - Spanish Quiver
  - Additional constraints (exotic matter, adjoints ...)
  - Example: the Standard Model on $T^6/\mathbb{Z}_6'$
- **Effective action on D-branes**
  - SUGRA methods (tree-level)
  - CFT methods (1-loop + D-instantons)
  - Example: Yukawas and masses for the SM on $T^6/\mathbb{Z}_6'$
- **Comparison with other string model building**
  - Orbifolds vs. smooth backgrounds
  - Symplectic vs. holomorphic techniques
  - $SO(32)$ vs. $E_8 \times E_8$
  - perturbative Type II & heterotic vs. non-perturbative F-theory
- **Further topics**,
  - Predictions for $M_{string} \sim$ TeV ...

Motivation: Model building with D6-branes on IIA/$\Omega\mathcal{R}$

- IIA orientifold background
- worldsheet parity Ω accompanied by anti-holomorphic involution $\mathcal{R}(z \to \bar{z})$ on compact 6D



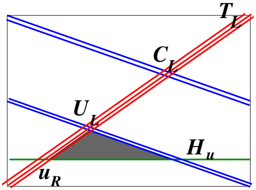


- D6$_a$-branes wrap 3-cycles Π_a and have $\mathcal{R}$ images Π'_a
- gauge group $U(N)$ on N identical D6-branes
- O6-planes wrap $\mathcal{R}$ invariant compact 3-cycles Π_{O6}
- *chiral* matter at *pointlike* intersections $\Pi_a \circ \Pi_b$ of D6-branes
  bifundamentals, (anti)symmetrics, adjoints of $U(N_a) \times U(N_b)$

Motivation cont'd: D6-branes

Advantage:

- geometrically very intuitive
  - gauge couplings $\frac{1}{g_a^2} \sim \mathrm{Vol}(\mathrm{D6}_a)$
  - D6-brane intersections $= \#$ particle generations
  - Yukawa couplings from closed triangles



Hurdles:

- supersymmetry
  - 3-cycles have to be *'special Lagrangian'*
  - *sLags* poorly studied by mathematicians
- how to compute the non-chiral spectrum?

Avoid hurdles on well understood backgrounds: fractional/rigid D6-branes on **toroidal orbifolds** $\boxed{T^6/\mathbb{Z}_N}$ or $\boxed{T^6/\mathbb{Z}_N \times \mathbb{Z}_M}$ (with/without discrete torsion)

Motivation: Orbifold vs. smooth Calabi-Yaus

strings on **smooth** CY_3:

- SUGRA approximation

a global *stringy* consistency ($\checkmark$)

b gauge groups & massless matter spectrum $\checkmark$

c charge selection rules on existence of matter interactions $\checkmark$

- strength and moduli dependence of matter interactions **???** [CY_3 metric unknown]

strings on **toroidal orbifolds**:

- small number of choices

a-c $\checkmark$

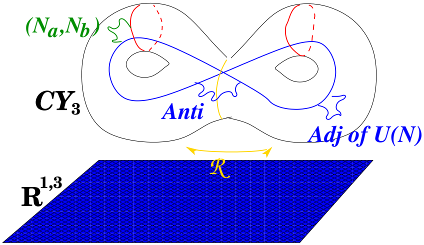
- complete *massive* spectrum
- all interactions computable $\Rightarrow$ complete action $\checkmark$

Some Reviews on Intersecting D6-branes

- A. M. Uranga, *"Chiral four-dimensional string compactifications with intersecting D-branes,"* Class. Quant. Grav. **20** (2003) S373 [arXiv:hep-th/0301032].
- R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, *"Toward realistic intersecting D-brane models,"* Ann. Rev. Nucl. Part. Sci. **55** (2005) 71 [arXiv:hep-th/0502005].
- E. Dudas, *"Orientifolds and model building,"* J. Phys. Conf. Ser. **53** (2006) 567.
- R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, *"Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes,"* Phys. Rept. **445** (2007) 1 [arXiv:hep-th/0610327].
- F. Marchesano, *"Progress in D-brane model building,"* Fortsch. Phys. **55** (2007) 491 [arXiv:hep-th/0702094].
- D. Lüst, *"String Landscape and the Standard Model of Particle Physics,"* arXiv:0707.2305 [hep-th].

. . . and many others

Model building with D6-branes



- D6-branes wrap cycles Π_a, Π_b; $\Omega\mathcal{R}$ images Π'_a, Π'_b generate $U(N_a) \times U(N_b)$
- O6-plane wraps $\mathcal{R}$ invariant cycle Π_{O6}
- *chiral* matter $(\mathbf{N}_a, \mathbf{N}_b)$ at intersections

- **RR tadpole** cancellation:

$$\boxed{\sum_a N_a\left(\Pi_a + \Pi'_a\right) = 4\,\Pi_{O6}}$$

- **chiral spectrum:**

rep.	net chirality
$(\mathbf{N}_a, \overline{\mathbf{N}}_b)$	$\Pi_a \circ \Pi_b$
$(\mathbf{N}_a, \mathbf{N}_b)$	$\Pi_a \circ \Pi'_b$
$(\mathbf{Anti}_a)$	$\frac{1}{2}\left(\Pi_a \circ \Pi'_a + \Pi_a \circ \Pi_{O6}\right)$
$(\mathbf{Sym}_a)$	$\frac{1}{2}\left(\Pi_a \circ \Pi'_a - \Pi_a \circ \Pi_{O6}\right)$

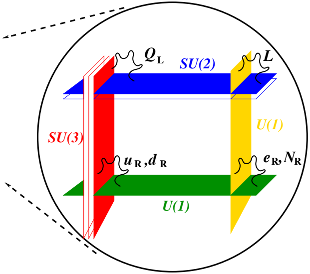
- **bulk SUSY**: hol.3-form Ω

$$\boxed{\int_{\Pi_a} \mathrm{Im}(\Omega) = 0} \quad \text{and}$$

$$\boxed{\int_{\Pi_a} \mathrm{Re}(\Omega) > 0}$$

Model building cont'd - Spanish Quiver

Ibáñez, Marchesano, Rabadan '01



- start with gauge group $U(3)_a \times \; U/Sp\;(2)_b \times U(1)_c \times U(1)_d$
- search for 3 left-handed quark generations: $\Pi_a \circ (\Pi_b + \Pi'_b) = \pm 3$
- ensure absence of chiral exotics: no symmetric of $U(3)_a$: $\Pi_a \circ \Pi'_a = \Pi_a \circ \Pi_{O6}$
- continue wih right-handed quarks, leptons
- Higgs at intersections of $SU(2)_b \times U(1)_c$

- **massless hyper charge $\mathbf{U(1)}_Y$:** $\boxed{\Pi_Y \stackrel{!}{=} \Pi'_Y}$ with $U(1)_Y \sim U(1)_a/3 + U(1)_c + U(1)_d$
- $SU(5)$ GUTs or Pati-Salam $SU(4) \times SU(2)_L \times SU(2)_R$ models have similar constructions

Status of supersymmetric model building

Gauge groups

- $SO(10)$ or E_6 GUTs not possible in *perturbative* II theories

  [no spinor reps, no exceptional groups $\rightsquigarrow$ F-theory or het. $E_8 \times E_8$]
- $SU(5) \subset U(5) \times U(1)$ *a priori* not excluded, but all known models have exotics in **15** of $SU(5)$
- left-right symmetric and Pati-Salam $SU(4) \times SU(2)_L \times SU(2)_R$ models arise naturally

Supersymmetry

- Classification of special Lagrangians needed Joyce '01

  Semi-local models by Palti '09
- Torus breaks $\mathcal{N} = 4$ SUSY in open string sector $\rightsquigarrow$ instability

  tachyons expected to drive back to global minimum at maximal SUSY
- Orbifolds can preserve $\mathcal{N} = 1$ SUSY

Effective action @ tree level

Grimm, Louis '04; ...; Grimm, Vieira-Lopes '11; Kerstan, Weigand '11

- uses tree-level **10D SUGRA** action for closed strings

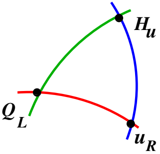
$$\frac{1}{2\kappa_{10}^2}\int_{10D} e^{-2\Phi}\sqrt{-G_{10}}\,R \rightarrow \boxed{\mathrm{Vol}_6}\int_{4D}\sqrt{-G_4}$$

  - Einstein-Hilbert term plus some closed string moduli dynamics
- plus **Born-Infeld & Chern-Simons** action for open string

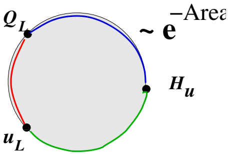
$$\mu_p\int_{p+1} e^{-\Phi}\,\mathrm{tr}F_{\mu\nu}F^{\mu\nu} \rightarrow \boxed{\mathrm{Vol}_{p-3}}\int_{4D}\mathrm{tr}F_{\mu\nu}F^{\mu\nu}$$

  - useful for tree-level gauge couplings (here: $p=6$)
  - problem to incorporate charged matter $\rightsquigarrow$ only selection rules for Yukawa couplings & higher interactions
- works for any $\mathbb{R}^{1,3}\times$ Calabi-Yau/orbifold background **at tree level**, where 2- and 3-forms can be expanded in a basis

Conformal field theory strategy @ 1-loop



$\Rightarrow$

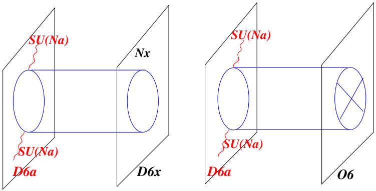


Scattering amplitudes are *in principle* computable at any loop-order

Lüst, Mayr, Richter, Stieberger '04; Cvetic, Papadimitriou '03; Abel, Schofield '03; Abel, Goodsell '05; ...

- work on torus - same techniques as for heterotic orbifolds: heterotic twist sector $\rightsquigarrow$ intersection angle of D-branes
- loop order determined by worldsheet with/without boundaries
- vertex operator insertions at boundary $\Leftrightarrow$ open string (matter)
  in bulk $\Leftrightarrow$ closed string (moduli)

Conformal field theory @ 1-loop cont'd



$$\sim \frac{b_a}{2} \ln \left(\frac{M_{\text{string}}}{\mu} \right)^2 + \frac{\Delta_a}{2}$$

Gauge thresholds can be calculated by magnetic background field method (gauging the partition function) developped by Abouelsaood, Callan, Nappi, Yost '87 & Bachas, Porrati '92 ...; intersecting D-branes: Lüst, Stieberger '03; Akerblom, Blumenhagen, Lüst, Schmidt-Sommerfeld '07; Gmeiner, G.H. '09; G.H. arXiv:1109.3192 [hep-th]; magnetised D9/D5-systems e.g. by Billo, Frau, Pesando, Di Vecchia, Lerda, Marotta'07 Angelantonj, Condeescu, Dudas, Lennek '09 ...

- gives **perturbatively exact** holomorphic **gauge kinetic function**
- leading order result for open string **Kähler metrics**
- includes all numerical factors

Gauge couplings in string & field theory

for D6-branes on torus: Akerblom, Blumenhagen, Lüst, Schmidt-Sommerfeld '07

- $\mathcal{N} = 1$ SUSY result of **1-loop string** calculation:

$$\boxed{\frac{8\pi^2}{g_a^2(\mu)} = \frac{8\pi^2}{g_{a,\mathrm{string}}^2} + \frac{b_a}{2} \ln\left(\frac{M_{\mathrm{string}}^2}{\mu^2}\right) + \frac{\Delta_a}{2}}$$

needs to be matched with ...

- $\mathcal{N} = 1$ SUSY **field theory**:

$$\boxed{\begin{aligned}\frac{8\pi^2}{g_a^2(\mu)} =& 8\pi^2\ \Re(\mathrm{f}_a) + \frac{b_a}{2} \ln\left(\frac{M_{\mathrm{Planck}}^2}{\mu^2}\right) + \frac{b_a + 2\,C_2(G_a)}{2}\,\mathcal{K} \\ &+ C_2(G_a)\ \ln[g_a^{-2}(\mu^2)] - \sum_a C_2(R_a)\ \ln\det K_{R_a}(\mu^2)\end{aligned}}$$

- derive *holomorphic* **gauge kinetic function** f_a, **Kähler metrics** K_{R_a} with representations $R_a \in \{(\mathbf{N}_a, \overline{\mathbf{N}}_b), (\mathbf{N}_a, \mathbf{N}_b), \mathbf{A}_a, \mathbf{S}_a\}$, Kähler potential $\mathcal{K}$ by matching string & field theory expressions

Tree-Level Gauge Couplings & Kähler potential

- **tree-level**: G.H. 1109.3192 & 1109.6533 [hep-th]

$$\frac{1}{g_a^2} = \frac{e^{-\phi_4}}{2\pi} \sqrt{\prod_{i=1}^3 V_{aa}^{(i)}} \overset{\text{SUSY}}{=} \begin{Bmatrix} S\,X_a^0 - \sum_{i=1}^3 U_i X_a^i & T^6 \\ S\,X_a^0 - U\,X_a^1 & T^6/\mathbb{Z}_6' \\ S\,X_a^0 & T^6/\mathbb{Z}_6 \end{Bmatrix} = \Re(\mathrm{f}_a^{\mathrm{tree}})$$

  - with definition of **dilaton & complex structure moduli**

$$(S, U_i)_{\mathrm{tree}} \equiv \frac{e^{-\phi_4}}{2\pi} \times \begin{cases} \left(1/\sqrt{\prod_{i=1}^3 r_i}\,,\ \sqrt{r_j r_k/r_i}\right) & T^6 & h_{21}^{\mathrm{bulk}} = 3 \\ \left(\frac{c_{\mathrm{lattice}}}{\sqrt{r}}\,,\ \frac{4}{3^{3/2} c_{\mathrm{lattice}}}\sqrt{r}\right) & T^6/\mathbb{Z}_6' & 1 \\ (c_{\mathrm{lattice}}\,,\ \emptyset) & T^6/\mathbb{Z}_6 & 0 \end{cases}$$

- **1-loop massless strings**:

$\frac{b_a}{2} \ln\left(\frac{M_{\mathrm{string}}}{\mu}\right)^2 \leftrightarrow \frac{b_a}{2}\left[\ln\left(\frac{M_{\mathrm{Planck}}}{\mu}\right)^2 + \mathcal{K}\right]$ provided by

$\left(\frac{M_{\mathrm{string}}}{M_{\mathrm{Planck}}}\right)^2 = e^{2\phi_4} \equiv \frac{e^{2\phi_{10}}}{\mathrm{Vol}_{CY_3}}$ and

$$\mathcal{K} = -\ln(S)^\alpha - \sum_k \ln(U_k)^\alpha - \sum_{i=1}^3 \ln T_i + \ldots \quad \text{with } \alpha = \begin{cases} 1 & T^6 \\ 2 & T^6/\mathbb{Z}_6' \\ 4 & T^6/\mathbb{Z}_6 \end{cases}$$

- **1-loop massive strings** provide
  - Kähler metrics & correction to *hol.* gauge kinetic functions
  - field redefinitions of moduli

Holomorphic gauge kinetic functions

G.H. 1109.3192 & 1109.6533 [hep-th]

- 1-loop correction from sum over all D6-branes and O6-planes

$$\delta^{\text{loop}}_{\text{total}} \mathrm{f}_a = \sum_b \delta_b^{\text{loop},\mathcal{A}} \mathrm{f}_a + \delta_{a'}^{\text{loop},\mathcal{M}} \mathrm{f}_a$$

- Kähler moduli dependence proportional to $b_{ab}^{\mathcal{A},(i)}$
- Complex structure moduli dependence at $\mathbb{Z}_2$ fixed points

1-loop contributions to holomorphic gauge kinetic function: Annulus	
$(\phi^{(1)}_{ab}, \phi^{(2)}_{ab}, \phi^{(3)}_{ab})$	$\delta_b^{\text{loop},\mathcal{A}} \mathrm{f}_{SU(N_a)}$
$(0,0,0)$	$-\sum_{i=1}^3 \frac{b_{ab}^{\mathcal{A},(i)}}{4\pi^2} \ln \eta(iv_i)$ $-\sum_{i=1}^3 \frac{\tilde{b}_{ab}^{\mathcal{A},(i)}(1-\delta_{\sigma^i_{ab},0}\delta_{\tau^i_{ab},0})}{8\pi^2} \ln\left(e^{-\frac{\pi(\sigma^i_{ab})^2 v_i}{4}} \frac{\vartheta_1(\frac{\tau^i_{ab}-i\sigma^i_{ab}v_i}{2}, iv_i)}{\eta(iv_i)}\right)$
$(0^{(i)}, \phi^{(j)}, \phi^{(k)})$ $\phi^{(j)} = -\phi^{(k)}$	$-\frac{b_{ab}^{\mathcal{A}}}{4\pi^2} \ln \eta(iv_i) - \frac{\tilde{b}_{ab}^{\mathcal{A}}(1-\delta_{\sigma^i_{ab},0}\delta_{\tau^i_{ab},0})}{8\pi^2} \ln\left(e^{-\frac{\pi(\sigma^i_{ab})^2 v_i}{4}} \frac{\vartheta_1(\frac{\tau^i_{ab}-i\sigma^i_{ab}v_i}{2}, iv_i)}{\eta(iv_i)}\right)$ $+\sum_{l=j,k} \frac{N_b\, I_{ab}^{\mathbb{Z}_2^{(l)}}}{8\pi^2\, c_a} \left(\frac{\text{sgn}(\phi^{(l)}_{ab})}{2} - \phi^{(l)}_{ab}\right)$
$(\phi^{(1)}, \phi^{(2)}, \phi^{(3)})$ $\sum_{i=1}^3 \phi^{(i)} = 0$	$\sum_{i=1}^3 \frac{N_b\, I_{ab}^{\mathbb{Z}_2^{(i)}}}{8\pi^2\, c_a} \left(\frac{\text{sgn}(\phi^{(i)}_{ab})+\text{sgn}(I_{ab})}{2} - \phi^{(i)}_{ab}\right)$

Holomorphic gauge kinetic functions cont'd

G.H. 1109.3192 & 1109.6533 [hep-th]

- Möbius strip contributions depend also on relative position to the O6-planes $$\delta^{\rm loop}_{\rm total}\mathrm{f}_a = \sum_b \delta_b^{{\rm loop},\mathcal{A}}\mathrm{f}_a + \delta_{a'}^{{\rm loop},\mathcal{M}}\mathrm{f}_a$$
- constant terms from intersections with O6-planes
- three kinds of 1-loop corrections: Kähler moduli ones proportional to b_a, independent complex structure moduli ones, constant terms - only first kind listed before -

1-loop contributions to holomorphic gauge kinetic function: Möbius strip on $T^6/\mathbb{Z}_{2N}$			
$(\phi^{(1)}_{aa'},\phi^{(2)}_{aa'},\phi^{(3)}_{aa'})$	$\delta_{a'}^{{\rm loop},\mathcal{M}}\mathrm{f}_{SU(N_a)}$	$(\phi^{(1)}_{aa'},\phi^{(2)}_{aa'},\phi^{(3)}_{aa'})$	$\delta_{a'}^{{\rm loop},\mathcal{M}}\mathrm{f}_{SU(N_a)}$
$(0,0,0)$ or $(\phi,0,-\phi)$ $\uparrow\uparrow \Omega\mathcal{R}$ on $T^2_{(2)}$	$-\frac{b^{\mathcal{M}}_{aa'}}{4\pi^2}\ln\eta(i\tilde{v}_2)+\tilde{c}_\phi\,\frac{\ln(2)}{2\pi^2}$ $-\frac{\tilde{b}^{\mathcal{M}}_{aa'}\,(1-\delta_{\sigma^2_{aa'},0}\delta_{\tau^2_{aa'},0})}{8\pi^2}\times$ $\ln\left(e^{-\frac{\pi(\sigma^2_{aa'})^2\tilde{v}_2}{4}}\,\frac{\vartheta_1(\frac{\tau^2_{aa'}-i\sigma^2_{aa'}\tilde{v}_2}{2},i\tilde{v}_2)}{\eta(i\tilde{v}_2)}\right)$	$i=2$ and $\perp\Omega\mathcal{R}$ on $T^2_{(2)}$ $(0^{(i)},\phi^{(j)},\phi^{(k)})_{\phi^{(j)}=-\phi^{(k)}}$ $i=1$ or 3	$\frac{\ln(2)}{16\pi^2}\left(\vert\tilde{I}^{\Omega\mathcal{R}}_a\vert+\vert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2^{(2)}}_a\vert\right)$ $-\frac{b^{\mathcal{M},(i)}_{aa'}}{4\pi^2}\ln\eta(i\tilde{v}_i)$ $+\frac{\ln(2)}{16\pi^2}\left(\vert\tilde{I}^{\Omega\mathcal{R}}_a\vert+\vert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2^{(2)}}_a\vert\right)$
$(0,0,0)$ $\uparrow\uparrow\Omega\mathcal{R}\mathbb{Z}_2^{(k)},\ k=1 \text{ or } 3$	$-\sum_{i=1,3}\frac{b^{\mathcal{M},(i)}_{aa'}}{4\pi^2}\ln\eta(i\tilde{v}_i)$ $+\frac{1}{8\pi^2}\ln\left(2^4\,\frac{v_1v_3V^{(1)}_{aa'}V^{(3)}_{aa'}}{(v_2V^{(2)}_{aa'})^2}\right)$	$(\phi^{(1)},\phi^{(2)},\phi^{(3)})$ $\sum_{i=1}^3\phi^{(i)}=0$	$\frac{\ln(2)}{16\pi^2}\left(\vert\tilde{I}^{\Omega\mathcal{R}}_a\vert+\vert\tilde{I}^{\Omega\mathcal{R}\mathbb{Z}_2^{(2)}}_a\vert\right)$

Kähler metrics

G.H. 1109.3192 & 1109.6533 [hep-th]

- Kähler metrics are independent of the orbifold background, but depend on the number of non-vanishing angles
- universal prefactor $f(S, U_I) = \left(S \prod_{I=1}^{h_{21}^{\text{bulk}}} U_I\right)^{-\frac{\alpha}{4}}$ wit $h_{21}^{\text{bulk}} = 3, 1, 0$ and $\alpha = 1, 2, 4$

Kähler metrics for bifundamental matter $(\mathbf{N}_a, \overline{\mathbf{N}}_b)$ on various orbifolds			
$(\phi_{ab}^{(1)}, \phi_{ab}^{(2)}, \phi_{ab}^{(3)})$	T^6 and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ without torsion	$T^6/\mathbb{Z}_{2N}$	$T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with discrete torsion
$(0, 0, 0)$	–	$f(S, U_I)\sqrt{\frac{2\pi V_{ab}^{(2)}}{v_1 v_3}}$	$f(S, U_I)\sqrt{\frac{2\pi V_{ab}^{(i)}}{v_j v_k}}$ $(ijk) \simeq (1, 2, 3)$ cyclic
$(0^{(i)}, \phi^{(j)}, \phi^{(k)})$ $\phi^{(j)} = -\phi^{(k)}$	$f(S, U_I)\sqrt{\frac{2\pi V_{ab}^{(i)}}{v_j v_k}}$		
$(\phi^{(1)}, \phi^{(2)}, \phi^{(3)})$ $\sum_{i=1}^3 \phi^{(i)} = 0$	$f(S, U_I)\sqrt{\prod_{i=1}^3 \frac{1}{v_i}\left(\frac{\Gamma(\lvert\phi_{ab}^{(i)}\rvert)}{\Gamma(1-\lvert\phi_{ab}^{(i)}\rvert)}\right)^{-\frac{\text{sgn}(\phi_{ab}^{(i)})}{\text{sgn}(I_{ab})}}}$		

Beyond perturbation theory: D-instantons

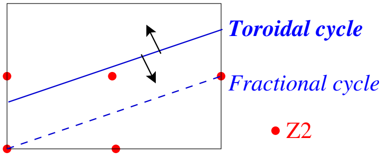
Blumenhagen, Cvetic, Weigand '06; Ibáñez, Uranga '06; ...

- **Euclidean D2-branes** wrapping 3-cycles provide
  - **Gauge instantons** if D6-brane in model wraps same 3-cycle
  - **'Stringy instantons'** if no D6-brane wrapped on this cycle
- **Charged zero-modes** of strings between D6-branes and Euclidean D2s **break** left-over **global U(1)** symmetries
- D-instantons can **generate** perturbatively forbidden **superpotential couplings**
- existence requires minimal set of zero-modes $\rightsquigarrow$ $O(1)$ instantons only on **rigid** D-branes
- generation of a **hierarchy** between perturbative and non-perturbative couplings:
  instantons suppressed by $e^{-\mathcal{S}_{\rm instanton}}$

D6-branes on toroidal orbifolds

- all known **globally consistent** (SUSY) SM & GUT examples in IIA orientifolds with D6-branes are on **toroidal orbifolds**
- *fractional D6-branes* on $T^6/\mathbb{Z}_6$ and $\boxed{T^6/\mathbb{Z}_6'}$ have proven fertile - not yet optimal due to some amount of **Adj** matter

with Ott '03 & Gmeiner '07-'09 $\rightsquigarrow$ *see later*



- expect improved models on *rigid D6-branes* on orbifolds $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6$ and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ with discrete torsion

work in progress $\rightsquigarrow$ *see later*

Intermezzo: Comparison with Other Approaches

- **Heterotic $E_8 \times E_8$ orbifolds** cf. talks by Vaudrevange, Groot Nibbelink
  - same CFT techniques as D-branes on orbifolds
  - blow-up generically breaks gauge group due to mixing of moduli and matter
- **Heterotic smooth $E_8 \times E_8$ models**
  - holomorphic bundles under good control (compared to sLags)
- **Type IIB models** cf. talk by Krippendorf
  - holomorphicity
  - no known *global* orbifold model
- **F-theory**
  - uses geometric brane picture
  - enhancements to E_8 or spinor representations
  - non-perturbative set-up $\rightsquigarrow$ loss of control over 'effective action'?

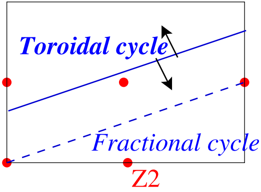
Fractional and Rigid D6-branes on orbifolds of $(T^2)^3$

Bulk branes on the six-torus $(T^2)^3$: $\boxed{\Pi^{\text{torus}}}$

- can be displaced continuously
- have continuous Wilson lines
- **three adjoint** matter multiplets

Fractional branes: $\boxed{\Pi^{\text{frac}} = \frac{1}{2}\left(\Pi^{\text{torus}} + \Pi^{\mathbb{Z}_2}\right)}$

- are stuck at $\mathbb{Z}_2$ orbifold singularities on T^4
- discrete displacements on T^4
- discrete Wilson lines on T^4
- $\mathbb{Z}_2$ eigenvalue ± 1
- **one adjoint** multiplet

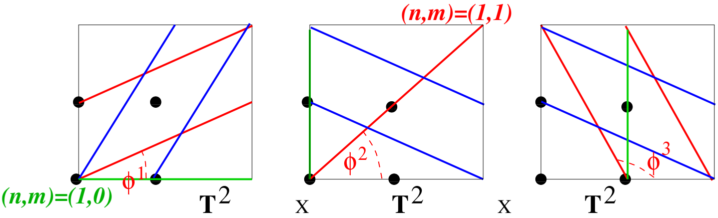


$$\boxed{\Pi^{\mathbb{Z}_2^{(i)}} = e^{(i)}_{\text{2-cycle}} \otimes \text{1-cycle}^{(i)}}$$

Rigid branes: $\boxed{\Pi^{\text{frac}} = \frac{1}{4}\left(\Pi^{\text{torus}} + \sum_{i=1}^{3} \Pi^{\mathbb{Z}_2^{(i)}}\right)}$

- stuck at $\mathbb{Z}_2 \times \mathbb{Z}_2$ singularities on T^6
- **no adjoint** matter

Supersymmetry on T^6 and orbifolds



- SUSY on the six-torus: $\sum_{i=1}^{3} \phi^i = 0$ Berkooz, Douglas, Leigh '96
- can be rewritten as

$$\int_{\Pi_a} \mathrm{Im}(\Omega) = 0 \quad \Leftrightarrow \quad \sum_{i=1}^{3} \tan(\phi^i) - \prod_{i=1}^{3} \tan(\phi^i) = 0$$

- SUSY on exceptional 3-cycles:
  - combinatorics of points & signs
  - 2 out of 4 points wrapped per two-torus
  - 2 $\mathbb{Z}_2$ eigenvalues
  - 1 Wilson line & 1 displacement per two-torus

$T^6/\mathbb{Z}_N$ orbifolds and Fractional D6-Branes

- number of 3-cycles: $b_3 = 2 + 2h_{21}$

$T^6/$	lattice Hodge numbers	U	$\vec{w}$	$2\vec{w}$	$3\vec{w}$	total
$\mathbb{Z}_3$	$SU(3)^3$		$(-\frac{2}{3},\frac{1}{3},\frac{1}{3})$			
	h_{11}	9	27			36
	h_{21}	0	0			0
$\mathbb{Z}_4$	$SU(2)^2 \times SO(5)^2$		$(-\frac{1}{2},\frac{1}{4},\frac{1}{4})$	$(0,\frac{1}{2},-\frac{1}{2})$		
	h_{11}	5	16	10		31
	h_{21}	1	0	6		7
$\mathbb{Z}_6$	$SU(3)^3$		$(-\frac{1}{3},\frac{1}{6},\frac{1}{6})$	$(-\frac{2}{3},\frac{1}{3},\frac{1}{3})$	$(0,\frac{1}{2},-\frac{1}{2})$	
	h_{11}	5	3	15	6	29
	h_{21}	0	0	0	5	5
$\mathbb{Z}_6'$	$SU(2)^2 \times SU(3)^2$		$(-\frac{1}{2},\frac{1}{3},\frac{1}{6})$	$(0,-\frac{1}{3},\frac{1}{3})$	$(\frac{1}{2},0,-\frac{1}{2})$	
	h_{11}	3	12	12	8	35
	h_{21}	1	0	6	4	$5+6$

- $\boxed{\mathbb{Z}_3}$: SUSY excludes chirality
- $\boxed{\mathbb{Z}_4}$: only one or two SUSY generations Blumenhagen, Görlich, Ott '02
- $\boxed{\mathbb{Z}_6}$: *one* kind of chiral 3-gen. SM spectrum G.H., Ott '04
- $\boxed{\mathbb{Z}_6'}$: *three* chiral 3-generation SMs with different numbers of Higgses Gmeiner, G.H. '07-'09

- $\boxed{\mathbb{Z}_3}$: 3 adjoints on parallel D6-branes
- $\boxed{\mathbb{Z}_{2N}}$: 1 adjoint on parallel D6-branes
- more adjoints at intersections with D6-orbifold images

Intermezzo: Moduli on $T^6/(\mathbb{Z}_N \times \Omega\mathcal{R})$ Orientifolds

Closed string spectrum: Grimm, Louis '05

- $\mathcal{N} = 1$ SUGRA & dilaton-axion
- h_{21} chiral (complex structures)
- h_{11}^+ vectors
- h_{11}^- chiral (Kähler moduli)
- (h_{11}^+, h_{11}^-) depends on background geometry

Example: $T^6/(\mathbb{Z}_6' \times \Omega\mathcal{R})$ on **ABa** Förste, G.H. '10

- $h_{11}^+ = 0_U + 4_{\mathbb{Z}_6} + 4_{\mathbb{Z}_3}$
- $h_{11}^- = 3_U + 8_{\mathbb{Z}_6} + 8_{\mathbb{Z}_3} + 8_{\mathbb{Z}_2}$ with 3_U the volumes of $(T^2)^3$
- $h_{21} = 1_U + 6_{\mathbb{Z}_3} + 4_{\mathbb{Z}_2}$ with 1_U the shape of the third torus **a**

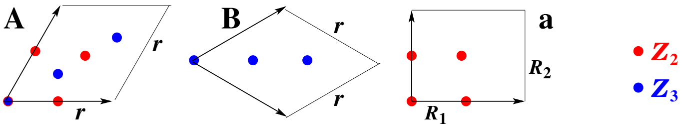
Open string spectrum:

- gauge groups & charged matter
- displacement and Wilson line moduli

Example: The Standard Model on $T^6/\mathbb{Z}_6'$

Gmeiner and G.H. '07-'09

$\boxed{T^6/\mathbb{Z}_6' : \ \vec{v}' = \frac{1}{6}(1, 2, -3)}$ on the $SU(3)^2 \times SU(2)^2$ lattice



- eight inequivalent backgrounds **AAa**, **ABa**, **BAa** ...
- 3 different types of *chiral* SM spectra on **ABa** and **BBa**
- maximal total rank 16
- maximal hidden sector: $Sp(6)$

  $\rightsquigarrow$ potential for ~~SUSY~~ via gaugino condensate
- *a priori* no constraint on $SU(5)$ GUTs, but: only 2 or 4 generations

Technical Intermezzo: $T^6/\mathbb{Z}_6'$ on the **ABa** lattice

Gmeiner and G.H. '07-'09

- **bulk RR tadpole** cancellation $\boxed{\sum_a N_a(P_a+Q_a)=8}$, $\boxed{\sum_a N_a(U_a-V_a)=24}$ $\rightsquigarrow$ *naive* maximal rank $=32/$*actual* $=16$
- exceptional RR tadpoles split into four independent eqs.
- **bulk SUSY**: $\boxed{\frac{1}{2\varrho}(P_a-Q_a)-(U_a+V_a)=0}$ and $\boxed{(P_a+Q_a)-\frac{2\varrho}{3}(V_a-U_a)>0}$ with one complex structure modulus ϱ
- exhaustive computer search:
  - $n=1$ particle generation for three different ϱ
  - $n=2$ generations for one ϱ
  - $n=3$ generations for five different ϱ
    - with three different 'chiral' Higgs sectors
    - ten different values $(b_{SU(3)}, b_{SU(2)}, b_{U(1)_Y}, b_{U(1)_{B-L}})$
    - 196 different models when including 1-loop gauge thresholds
  - never $n>3$ generations **!!!**

with $P=Xn_3$, $Q=Yn_3$, $U=Xm_3$, $Y=Ym_3$ and $X=n_1n_2-m_1m_2$, $Y=n_1m_2+m_1n_2+m_1m_2$

A SM spectrum with hidden sector on **ABa** on $T^6/\mathbb{Z}_6'$

- gauge group: $SU(3)_a \times SU(2)_b \times U(1)_Y \times U(1)_{B-L} \times Sp(6)_h$
- 'chiral' spectrum: *(from non-zero intersection #)*

$$[C] = 3 \times \Big[(\mathbf{3},\mathbf{2})_{1/6,1/3} + \left(\overline{\mathbf{3}},\mathbf{1}\right)_{1/3,-1/3} + \left(\overline{\mathbf{3}},\mathbf{1}\right)_{-2/3,-1/3} + (\mathbf{1},\mathbf{1})_{1,1} + (\mathbf{1},\mathbf{1})_{0,1}$$
$$+ 2 \times (\mathbf{1},\mathbf{2})_{-1/2,-1} + (\mathbf{1},\mathbf{2})_{1/2,1} + 3 \times \left(\mathbf{1},\overline{\mathbf{2}}\right)_{-1/2,0} + 3 \times \left(\mathbf{1},\overline{\mathbf{2}}\right)_{1/2,0} \Big]$$
$$\equiv 3 \times \Big[Q_L + d_R + u_R + e_R + \nu_R + 2 \times L + \overline{L} \Big] + 9 \times \Big[H_d + H_u \Big]$$

- 'truly non-chiral' spectrum: *(from zero intersection #)*

$$[V] = 2 \times (\mathbf{8},\mathbf{1})_{0,0} + 10 \times (\mathbf{1},\mathbf{3})_{0,0} + (25 + 1_m) \times (\mathbf{1},\mathbf{1})_{0,0}$$
$$+ \Big[(\mathbf{3},\mathbf{2})_{1/6,1/3} + 3 \times \left(\overline{\mathbf{3}},\mathbf{1}\right)_{1/3,2/3} + 3 \times \left(\overline{\mathbf{3}},\mathbf{1}\right)_{-2/3,-4/3} + 3 \times (\overline{\mathbf{3}}_A,\mathbf{1})_{1/3,2/3} + 6 \times (\mathbf{1},\mathbf{3}_S)_{0,0}$$
$$+4_m \times (\mathbf{1},\mathbf{1}_A)_{0,0} + 2_m \times \left(\mathbf{1},\overline{\mathbf{2}}\right)_{1/2,1} + 1_m \times (\mathbf{1},\mathbf{2})_{1/2,1} + 1_m \times (\mathbf{1},\mathbf{2})_{1/2,0} + 1_m \times \left(\mathbf{1},\overline{\mathbf{2}}\right)_{1/2,0}$$
$$+1_m \times (\mathbf{1},\mathbf{1})_{1,0} + 1_m \times (\mathbf{1},\mathbf{1})_{0,-1} + 1_m \times (\mathbf{1},\mathbf{1})_{1,1} + \ c.c. \Big]$$
$$+ 2 \times (\mathbf{1},\mathbf{1};\mathbf{15})_{0,0} + \left[(\mathbf{1},\mathbf{2};\mathbf{6})_{0,0} + (\mathbf{1},\mathbf{1};\mathbf{6})_{1/2,0} + \ c.c. \right]$$

- two massive $U(1)$s
- μ-term *perturbatively* forbidden by $U(1)_b \subset U(2)_b$

The SM example cont'd

- 3 right-handed neutrinos — Gmeiner, G.H. '07-'09
- extended Higgs sector:
  - charge selection rules: all Yukawas perturbatively allowed
  - leading order: one Higgs pair (H_u, H_d) per generation
- 3 parallel displacements (D6-branes $x \in \{a, b, c\}$): vevs $\langle (\mathbf{1}, \mathbf{1})_{\mathbf{0},\mathbf{0}} \rangle \subset \mathbf{Adj}_{U(N_x)}$ make most vector like matter massive

  $m^2 \sim \langle (\mathbf{1}, \mathbf{1})_{\mathbf{0},\mathbf{0}} \rangle^2$
- small amount of additional vector-like matter
  - one more vev $\langle (\mathbf{1}, \mathbf{1})_{\mathbf{0},\mathbf{0}} \rangle \subset \mathbf{Adj}_{U(3)_a}$ renders all vector-like quarks massive (3-point coupling)

    to appear soon with Joris Vanhoof
  - use of higher n-point functions under consideration
- hidden $Sp(6)$ strongly coupled
  - possibly gaugino condensation
- in addition Polonyi term (?)

The SM cont'd: field theory @ 1-loop

G.H. 1109.3192 and 1109.6533 [hep-th]; G.H., Joris Vanhoof to appear

- each torus cycle x has two orbifold images $(\theta^k x)$
- closed triangles with matter at apexes give wordsheet instanton contributions W_{xyz} to superpotential
- unsuppressed e.g. $E^i L^i H^i$ with $i = 1, 2, 3$ and $Q^3_L Q^1_R H^1$
- physical Yukawas $Y_{xyz} = \frac{W_{xyz}}{\sqrt{K_{xy} K_{yz} K_{zx}}} = c \left(\frac{v_1 v_2 v_3}{f(S,U)^2} \right)^{3/4}$

with $c = 1/(2(50)^{1/4})$ and $c = 3^{1/8}/(2(25\pi v_1)^{1/4})$, resp.

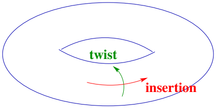
Matter localisations and Kähler metrics					
particle	sector	Kähler metric	particle	sector	Kähler metric
$Q_L^{1,2}$	ab'	$f(S,U)\sqrt{\frac{10}{v_1 v_2 v_3}}$	$L^{1\ldots 6}$	$b(\theta d)$	$f(S,U)\sqrt{\frac{8}{v_1 v_2 v_3}}$
$Q_L^{3,4}$	$a(\theta b')$	$f(S,U)\sqrt{\frac{10}{v_1 v_2 v_3}}$	$\overline{L}^{1\ldots 3}$	$b(\theta^2 d)$	$f(S,U)\sqrt{\frac{8}{v_1 v_2 v_3}}$
$\overline{Q}_L$	$a(\theta^2 b')$	$f(S,U)\sqrt{\frac{25}{2 v_1 v_2 v_3}}$	$E^{1,2,3}$	$c(\theta d)$	$f(S,U)\sqrt{\frac{10}{v_1 v_2 v_3}}$
$Q_R^{1,2}$	ac	$f(S,U)\sqrt{\frac{4\pi}{\sqrt{3} v_2 v_3}}$	$H^{1\ldots 6}$	bc	$f(S,U)\sqrt{\frac{10}{v_1 v_2 v_3}}$
Q_R^3	$a(\theta^2 c)$	$f(S,U)\sqrt{\frac{10}{v_1 v_2 v_3}}$	$H^{7\ldots 9}$	$b(\theta c)$	$f(S,U)\sqrt{\frac{25}{2 v_1 v_2 v_3}}$

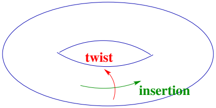
Interim Result

- D6-brane SM examples on $\boxed{T^6/\mathbb{Z}_6}$ and $\boxed{T^6/\mathbb{Z}_6'}$ have adjoint matter
  - continuous breaking of gauge groups
  - some couplings perturbatively absent
- field theory understanding @ 1-loop
  - D6-branes at angles *on the torus* use identical techniques as twisted sectors of heterotic orbifolds
  - exceptional contributions from $\mathbb{Z}_2$ fixed points *on orbifolds* have no heterotic match: new terms in $\delta^{\text{loop}}_{\text{total}} \mathrm{f}_a$
- **rigid D6-branes** on $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6}$ and $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'}$ **with discrete torsion** look promising for model building without adjoints

work in progress

discrete torsion: phase factors (under $\mathbb{Z}_N$) in twisted sectors (of $\mathbb{Z}_M$)





Hodge numbers on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2N}$ with(out) discrete torsion

$T^6/$ torsion	lattice Hodge numbers	U	$\vec{w}$	$2\vec{w}$	$3\vec{w}$	$\vec{v}$	$(\vec{v}+\vec{w})$	$(\vec{v}+2\vec{w})$	$(\vec{v}+3\vec{w})$	total
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$SU(2)^6$		$(0,\frac{1}{2},-\frac{1}{2})$			$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{2},0,-\frac{1}{2})$			
$\eta=1$	h_{11}	3	16			16	16			51
	h_{21}	3	0			0	0			3
$\eta=-1$	h_{11}	3	0			0	0			3
	h_{21}	3	16			16	16			51
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$SU(2)^2 \times SO(5)^2$		$(0,\frac{1}{4},-\frac{1}{4})$	$(0,\frac{1}{2},-\frac{1}{2})$		$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{2},-\frac{1}{4},-\frac{1}{4})$	$(\frac{1}{2},0,-\frac{1}{2})$		
$\eta=1$	h_{11}	3	8	10		12	16	12		61
	h_{21}	1	0	0		0	0	0		1
$\eta=-1$	h_{11}	3	0	10		4	0	4		21
	h_{21}	1	8	0		0	0	0		1+8
$\mathbb{Z}_2 \times \mathbb{Z}_6$	$SU(2)^2 \times SU(3)^2$		$(0,\frac{1}{6},-\frac{1}{6})$	$(0,\frac{1}{3},-\frac{1}{3})$	$(0,\frac{1}{2},-\frac{1}{2})$	$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{2},-\frac{1}{3},-\frac{1}{6})$	$(\frac{1}{2},-\frac{1}{6},-\frac{1}{3})$	$(\frac{1}{2},0,-\frac{1}{2})$	
$\eta=1$	h_{11}	3	2	8	6	8	8	8	8	51
	h_{21}	1	0	2	0	0	0	0	0	$1+2$
$\eta=-1$	h_{11}	3	0	8	0	0	4	4	0	19
	h_{21}	1	2	2	6	4	0	0	4	$15+4$
$\mathbb{Z}_2 \times \mathbb{Z}_6'$	$SU(3)^3$		$(-\frac{1}{3},\frac{1}{6},\frac{1}{6})$	$(-\frac{2}{3},\frac{1}{3},\frac{1}{3})$	$(0,\frac{1}{2},-\frac{1}{2})$	$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{6},-\frac{1}{3},\frac{1}{6})$	$(-\frac{1}{6},-\frac{1}{6},\frac{1}{3})$	$(\frac{1}{2},0,-\frac{1}{2})$	
$\eta=1$	h_{11}	3	2	9	6	6	2	2	6	36
	h_{21}	0	0	0	0	0	0	0	0	0
$\eta=-1$	h_{11}	3	1	9	0	0	1	1	0	15
	h_{21}	0	0	0	5	5	0	0	5	15

- $\mathbb{Z}_2$ sectors 'see' discrete torsion only for $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2N}$ with N odd $\rightsquigarrow$ potential for new SM or GUT vacua for $2N \in \{2, 6, 6'\}$

Orbifolds with Discrete Torsion: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

- most investigated case: Blumenhagen, Cvetic, Marchesano, Shiu '05; ...

$\vec{v} = \frac{1}{2}(1,-1,0) \quad (\mathbb{Z}_2^{(3)})$ $\qquad \vec{w} = \frac{1}{2}(0,1,-1) \quad (\mathbb{Z}_2^{(1)})$

Torus orbits of $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

tw. sector $\theta^k\omega^l$ / projector	00	01	10	11
00	$\times$	$\times$	$\times$	$\times$
01	$\times$	$\times$	$\bullet$	$\bullet$
10	$\times$	$\bullet$	$\times$	$\bullet$
11	$\times$	$\bullet$	$\bullet$	$\times$

$\times = \eta^0 = 1$

$\bullet = \eta = \pm 1$

- sectors split into an untwisted and a twisted orbit
- $\boxed{\eta = 1}$ preserves exceptional $\boxed{\text{2-cycles}}$ **'no torsion'**
- $\boxed{\eta = -1}$ preserves exceptional $\boxed{\text{3-cycles}}$ **'discrete torsion'**
- $(h_{11}, h_{21}) = (51, 3) \Leftrightarrow (3, 51)$
- preliminary SM searches gave no results

Orbifolds with Discrete Torsion: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6^{(\prime)}$

- orbifold generators:

Förste, G.H. '10

$$\vec{v} = \frac{1}{2}(1,-1,0), \qquad \vec{w} = \frac{1}{6}(0,1,-1) \text{ or } \vec{w}' = \frac{1}{6}(-2,1,1)$$

- discrete torsion on twist sectors only depends on the order of the orbifold:

The three torus orbits of $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6^{(\prime)}$												
twist sector projector	00	01	02	03	04	05	10	11	12	13	14	15
00	×	×	×	×	×	×	×	×	×	×	×	×
01	×	×	×	×	×	×	•	•	•	•	•	•
02	×	×	×	×	×	×	×	×	×	×	×	×
03	×	×	×	×	×	×	○	•	•	○	•	•
04	×	×	×	×	×	×	×	×	×	×	×	×
05	×	×	×	×	×	×	•	•	•	•	•	•
10	×	•	×	○	×	•	×	•	×	○	×	•
11	×	•	×	•	×	•	•	×	•	×	•	×
12	×	•	×	•	×	•	×	•	×	•	×	•
13	×	•	×	○	×	•	○	×	•	×	•	×
14	×	•	×	•	×	•	×	•	×	•	×	•
15	×	•	×	•	×	•	•	×	•	×	•	×

$\times = \eta^0 = \eta^2 = 1$

$\bullet = \eta = \pm 1$

$\bigcirc = \eta^3 = \pm 1$

- $\boxed{\mathbb{Z}_2}$ and $\mathbb{Z}_6^{(\prime)}$ sectors 'see' discrete torsion – $\mathbb{Z}_3$ doesn't
$\rightsquigarrow$ **favourable for model building**

Maximal (hidden) rank for D6-branes on orbifolds

- **SUSY** condition on **bulk D6-branes** is **independent** of discrete torsion
  Förste, G.H. arXiv:1010.6070[hep-th]
- sum **RR tadpole** contributions from O6-planes

$T^6/$	$2^{\frac{1+\eta}{2}} \sum_i L^i$ $\star\, \mathbb{Z}_2 \times \mathbb{Z}_2$: not a stringent bound	=naive maximal rank **discrete torsion**	 no torsion
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$16 \sum_{i=0}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}$	$32^\star$	$32^\star$
$\mathbb{Z}_2 \times \mathbb{Z}_6$	$24\eta_{\Omega\mathcal{R}} + 8 \sum_{i=1}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}$	32	24
$\mathbb{Z}_2 \times \mathbb{Z}_6'$	$4\eta_{\Omega\mathcal{R}} + 12 \sum_{i=1}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}$	32 or 16	20
	$12\eta_{\Omega\mathcal{R}} + 4 \sum_{i=1}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}$	16	12

- slightly bigger ranks with discrete torsion, but not doubled!
- Ansatz for SM gauge group: $U(3) \times U(2) \times U(1)^2$ = rank 7
  $\Rightarrow$ **maximal hidden rank** with discrete torsion
  - $= 25$ $(\mathbb{Z}_2 \times \mathbb{Z}_2, \mathbb{Z}_2 \times \mathbb{Z}_6, \mathbb{Z}_2 \times \mathbb{Z}_6')$
  - $= 9$ $(\mathbb{Z}_2 \times \mathbb{Z}_6')$

Some facts on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

$$\vec{v} = \frac{1}{2}(1,-1,0) \qquad \vec{w}' = \frac{1}{6}(-2,1,1) \qquad \text{on } SU(3)^3$$

- bulk part as for known $T^6/\mathbb{Z}_6$

  $T^6/\mathbb{Z}_6$: see G.H. & T. Ott hep-th/0404055; F. Gmeiner, D. Lüst & M. Stein hep-th/0703011
- all untwisted complex structures fixed by $\mathbb{Z}_3$ symmetry
- all SUSY bulk cycles parallel: $\boxed{\Pi_a \circ \Pi_{O6} = 0}$
  - **chirality** only in the presence of **discrete torsion**
  - SUSY models restricted to $(\mathbf{N}_a, \bar{\mathbf{N}}_b)$ and $(\mathbf{N}_a, \mathbf{N}_b)$ (no **Anti**!), i.e. no $SU(5)$ GUT, but Pati-Salam $SU(4) \times SU(2)_L \times SU(2)_R$ possible

with discrete torsion:

- three identical $\mathbb{Z}_2$ twisted sectors: $h_{21}^{\mathbb{Z}_2^{(i)}} = 5$ for $i = 1,2,3$
  - copies of $\mathbb{Z}_2 \subset T^6/\mathbb{Z}_6$
  - a stack of D6-branes passes through four $\mathbb{Z}_2^{(i)}$ fixed points, but only contributes to *three* $\mathbb{Z}_2^{(i)}$ twisted RR tcc

RR tadpoles and SUSY on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

$\boxed{\Pi^{\text{bulk}} = X\rho_1 + Y\rho_2}$ with $X, Y \in \mathbb{Z}$ Förste, G.H. arXiv:1010.6070[hep-th]

$$X \equiv n^1 n^2 n^3 - m^1 m^2 m^3 - \sum_{i \neq j \neq k \neq i} n^i m^j m^k, \quad Y \equiv \sum_{i \neq j \neq k \neq i} \left(n^i n^j m^k + n^i m^j m^k \right)$$

- Bulk SUSY on **AAB**: $\boxed{Y - X = 0}$ and $\boxed{X + Y > 0}$
- Orientifold projection on bulk cycles: $\rho_1 \stackrel{\Omega\mathcal{R}}{\leftrightarrow} \rho_2$
- Bulk RR tcc on **AAB** with discrete torsion:

$$\boxed{\sum_a N_a(X_a + Y_a) = 4 \left(\sum_{i=0}^2 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} + 3\, \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(3)}} \right)}$$

- for special choice of exotic O6-plane, $\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(3)}} = -1$, **no D6-branes** needed: $\mathcal{N} = 1$ SUSY on purely closed strings
  - $(h_{11}^+, h_{11}^-) = (0,3)_U + (0,9)_{\mathbb{Z}_3} + \sum_{i=1}^3 (1,0)_{\mathbb{Z}_6^{(i)}}$ (vectors, Kähler moduli) have no $\mathbb{Z}_2^{(i)}$ contributions!

Towards Model Building on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

work in progress with Wieland Staessens & Martin Ripka

- *A priori* four different lattices **AAA**, **AAB**, **ABB**, **BBB**
- Bulk parts for **AAA** and **ABB** arise via non-supersymmetric rotation $\pi/3$ along some lattice, **ABB** and **BBB** similarly
- Does this symmetry extend to exceptional cycles?
- On **AAA** only 3 bulk-cycles consistent with no adjoints, on **ABB** 9 different cycles
- combinatorics with exceptional cycles around 100 $\times$ (two $\mathbb{Z}_2$ eigenvalues, 3 Wilson lines, 3 displacements)
- no chiral symmetric expected to be very constraining
  $\rightsquigarrow$ expect small number of possible QCD stacks

Intermezzo: TeV scale strings

Anchordoqui, Goldberg, Lüst, Nawata, Stieberger, Taylor '08; ...

- give up the idea of SUSY for $M_{string} \sim$ 1-10 TeV
- estimate the scales: $\mathrm{Vol}_6 M_{string}^6 = \mathcal{O}(10^{32})$
- $M_{string} \sim$ TeV possible if $V_\perp$ is large
- need to assume **weak string coupling**

  (no microscopic black hole production or other strong gravity effects)
- near the string mass threshold: new signatures:
  - model independent Regge recurrences of massice gluons $g*$
  - model dependent charged Kaluza-Klein and winding states
  - plus neutral particles
- resonance at production of $g*$ computed via scattering amplitudes on the torus @ tree-level
- how generic is the result?
- Any *globally consistent* string model with the general features?
  - D6-branes *on orbifolds* don't work (no $V_\perp$ possible)

Conclusions I: Model Building with Intersecting D-Branes

- SM & GUT building on D6-branes is *conceptually* clear
- need **explicit examples** to check 'predicted properties'
- 3-generation SUSY models on orbifolds $\supset \mathbb{Z}_3$ symmetry:
  - $\boxed{T^6/\mathbb{Z}_6}$: *one* chiral 3-generation SM spectrum
    with exotic Higgs sector (composites) G.H., Ott '04
  - $\boxed{T^6/\mathbb{Z}_6'}$: *three* chiral 3-generation SM
    with different number of Higgs- and vector-like lepton pairs
    $\rightsquigarrow$ 10 different sets of beta functions @ 1-loop
    $\rightsquigarrow$ 196 different models including gauge thresholds Gmeiner, G.H. '07-09
  - $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'}$: no $SU(5)$ GUTs
    $\mathcal{O}(10)$ bulk cycles admitting rigid D-branes
    $\rightsquigarrow$ approx. 100 distinct rigid D-branes per bulk cycle
    sizable constraint 'chiral symmetrics' (?) G.H., Ripka, Staessens in progress
  - $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6}$: *a priori* SM, Pati-Salam, $SU(5)$ GUTs possible
    additional complex structure d.o.f. compared to $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

Conclusions II: Field Theory on D6-Branes

- @ tree-level: **dimensional reduction** of 10D SUGRA + Born-Infeld/Chern-Simons action along D-branes
- @ tree-level: **worldsheet instanton** sums for 3-point interactions on **torus**
- @ 1-loop: on **orbifolds** gauge threshold corrections
  - perturbatively exact result for **holomorphic gauge kinetic function**
  - leading order for **Kähler metrics**
- @ any loop: on torus (orbifolds?) scattering amplitudes
  - functional dependences
- non-perturbatively: D-instantons

Outlook

- ongoing **search** for $\boxed{\text{SM \& GUTs}}$ on $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6}$ and $\boxed{T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'}$ with discrete torsion G.H., Ripka, Staessens in progress
- if SM or GUT spectra are found, need **field theory**
  - numerical values of gauge couplings @ electro-weak scale
  - Yukawa couplings - *also needed for $T^6/\mathbb{Z}_6$ and $T^6/\mathbb{Z}_6'$*
    exact formal expressions for fractional/rigid D6-branes not known, in particular for chirality at zero angle - but feasible!
  - single instantons, in particular if μ term or some Yukawa doesn't exist perturbatively
  - hidden sector gaugino condensation possible?
  - ...
- **Moduli stabilised** at the orbifold point? *Fluxes needed?*
- **M-theory duality** to heterotic $E_8 \times E_8$ orbifolds?
  - Identical particle physics from different string models?