

320 Model building with branes @ singularities

H. BETHE

Forum, November 2011

Sven Krippendorf, Bethe Center for Theoretical Physics & PI Bonn



based on

1106.6039 Dolan, Quevedo, SK

1102.1973 Burgess, Maharana, Quevedo, SK

1002.1790 Dolan, Maharana, Quevedo, SK

... continuing the talks on brane model building
with yet another approach of getting towards the Standard Model

Outline

- Why bother with local model building in type IIB and branes@singularities?
- Gauge theories of toric singularities
- Systematic search for a realistic model (spectrum, couplings, unification)

String Phenomenology

- If String Theory is a model for the real world, we need a low-energy string model that satisfies all particle physics and cosmological observations.
- TOO MANY? Common answer: there is a whole landscape of string models (heterotic, IIA, IIB, I, ...) allowing you to model almost anything so there should be plenty of SM realisations...
- TOO FEW? Is there a single realistic model?

Challenges/Experimental Data for String Models

- Gauge and matter structure of SM
- Hierarchy of masses (including neutrinos)
- Flavour structure (CKM, PMNS, CP), absence of FCNC
- Hierarchy of gauge couplings (unification)
- Stable proton
- Inflation, dark energy
-

If one of them does not work, this rules out the model!!!

... there are a couple of approaches, some of which include

- heterotic
 - intersecting branes
 - F-theory
 - branes@singularities
- 
- local models

Bottom-Up Approach

Local Brane Properties

- Local Brane Properties
- Gauge group
- Chiral Spectrum
- Tree-level Yukawa couplings
- Gauge couplings
- Proton Stability
- Flavour symmetries

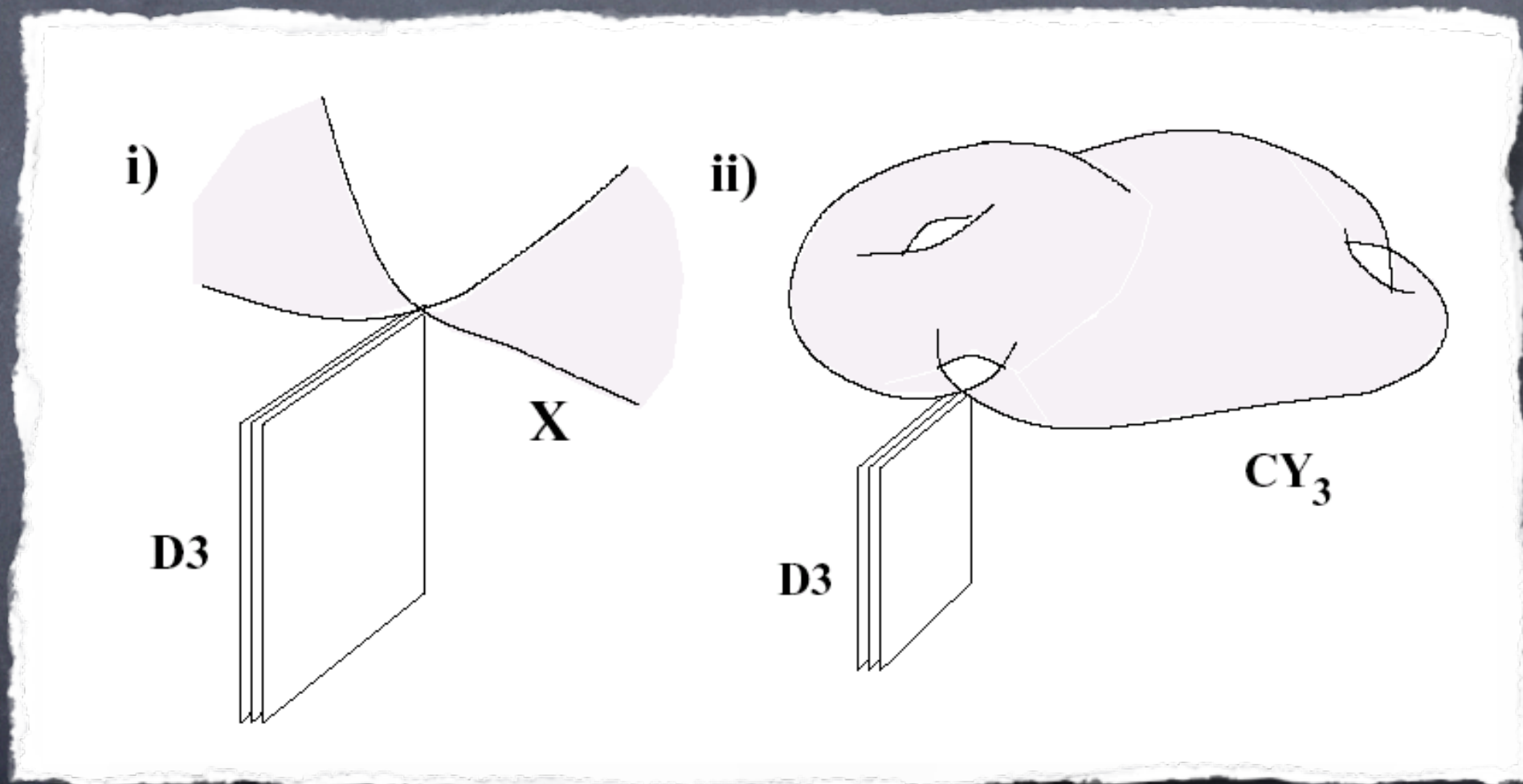
Global (bulk) Properties

- Moduli Stabilisation
- Cosmological Constant
- SUSY Breaking
- Scales (unification)
- Inflation, Reheating
- Cosmological Moduli Problem

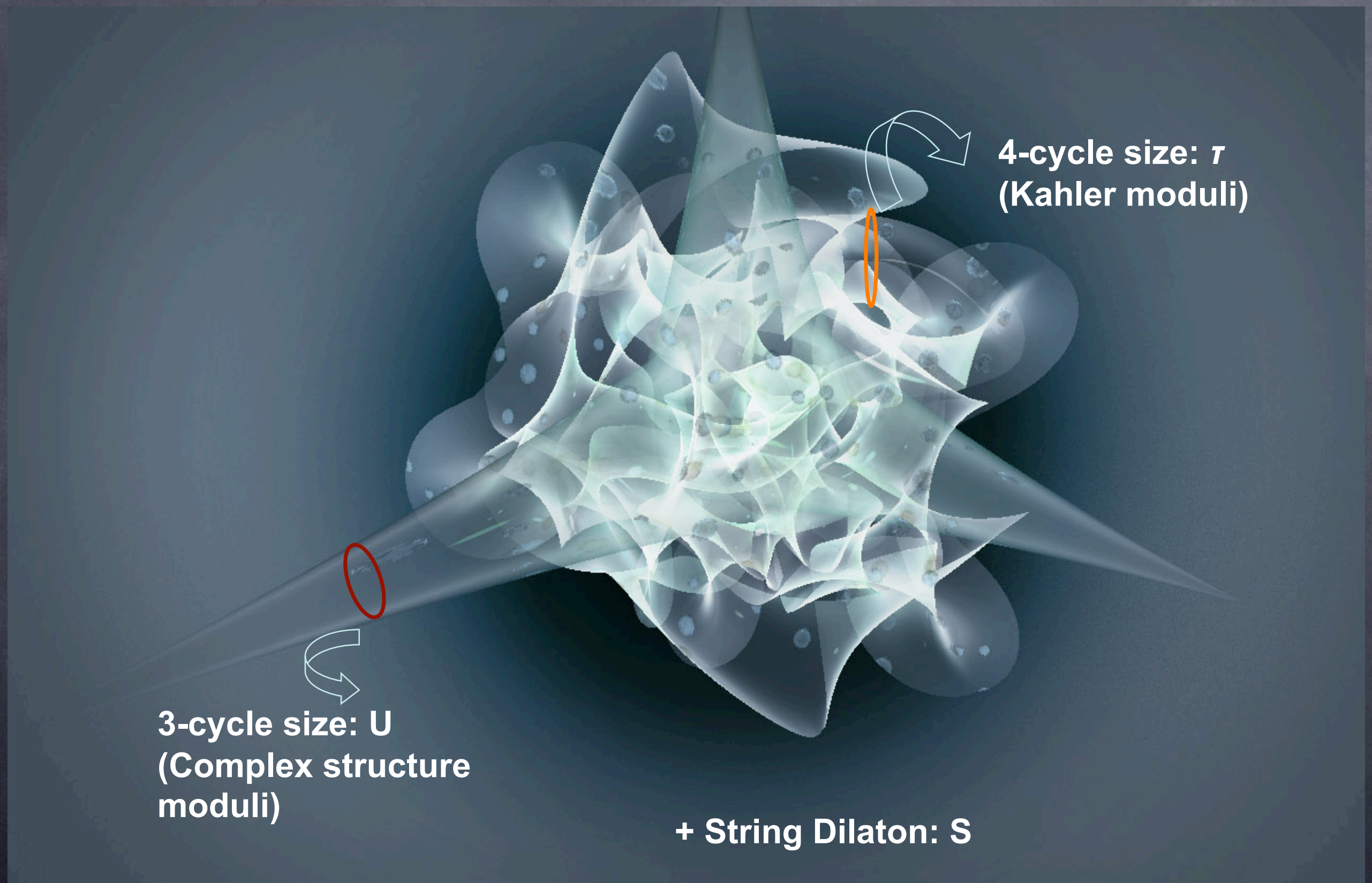
Aldazabal, Ibanez, Quevedo, Uranga 2000

Branes@singularities

(mostly) D3 branes at special point in CY. Study local geometry for gauge theory properties.

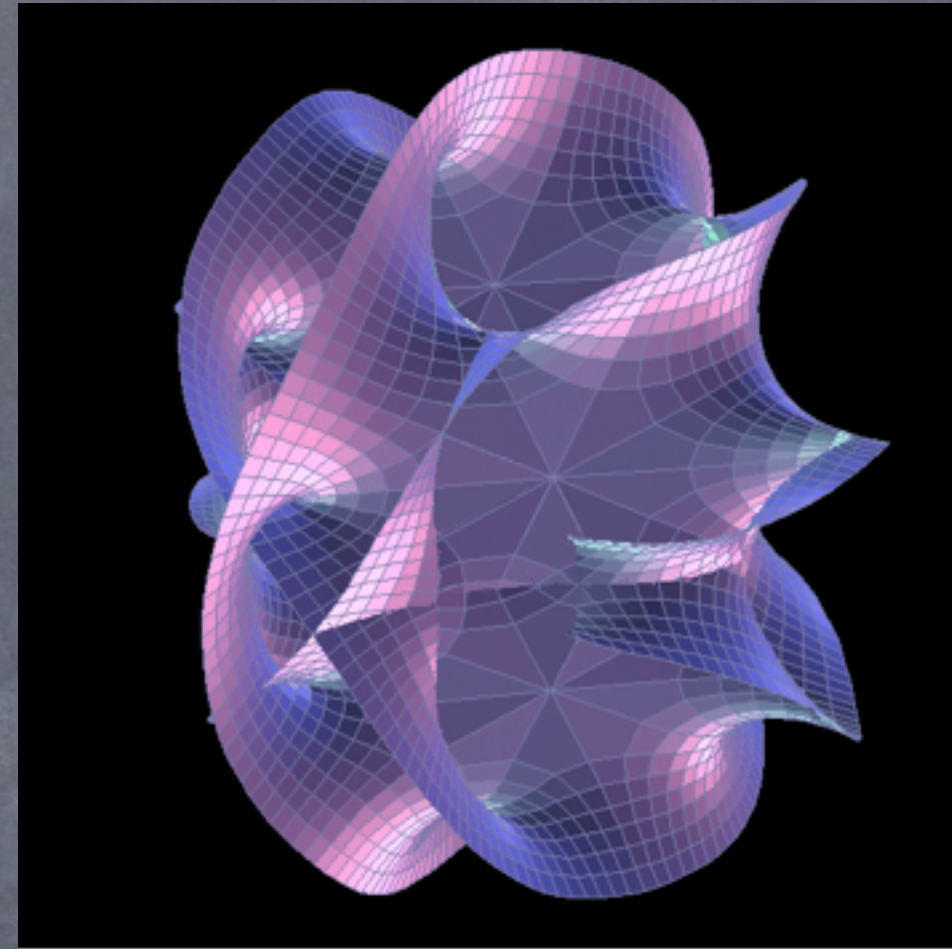


... local but having moduli stabilisation in mind



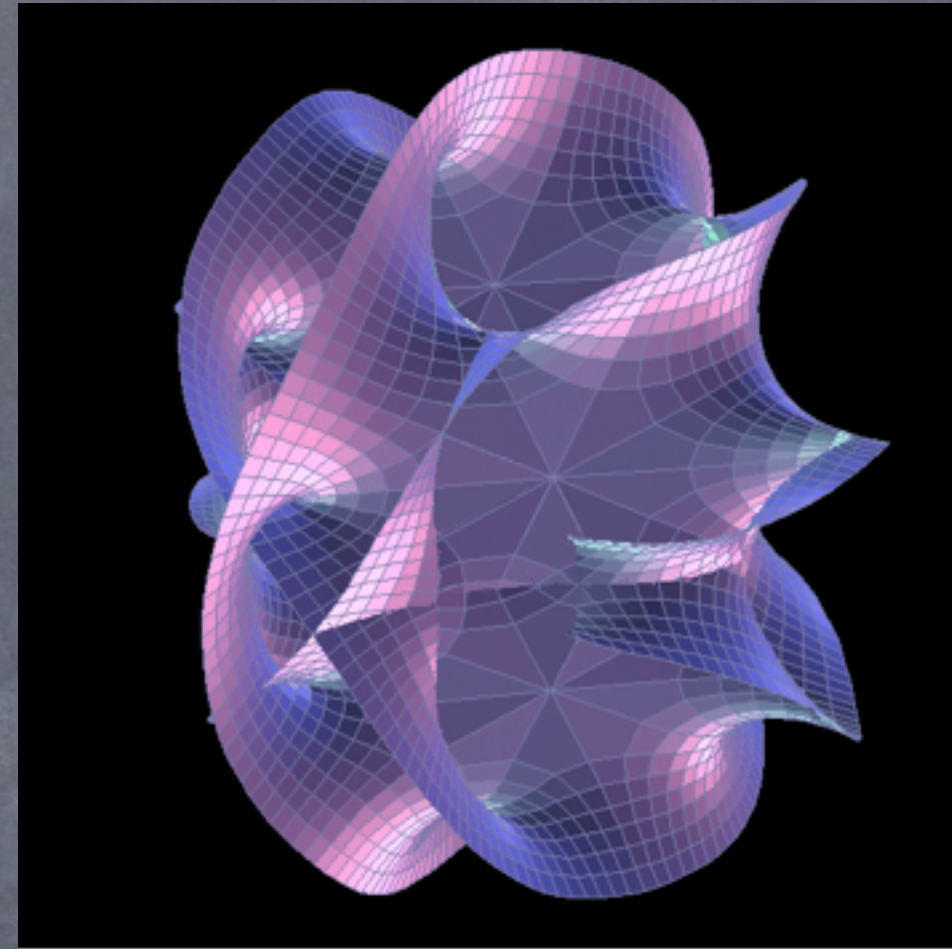
Moduli Stabilisation in type IIB

- CY: Kähler moduli (T_i, τ), complex structure moduli (U), dilaton (S)
- 4D $N=1$ supergravity leading order description.
- Potential vanishes due to no-scale structure.
- GKP: turn on fluxes (\rightarrow stabilise cs, dilaton at susy minimum $D_i W=0$)
- KKLT: non-perturbative effects (E3 branes, gaugino condensation on D7) stabilise Kähler moduli at susy AdS minimum, but $W_0=10^{-15}$.



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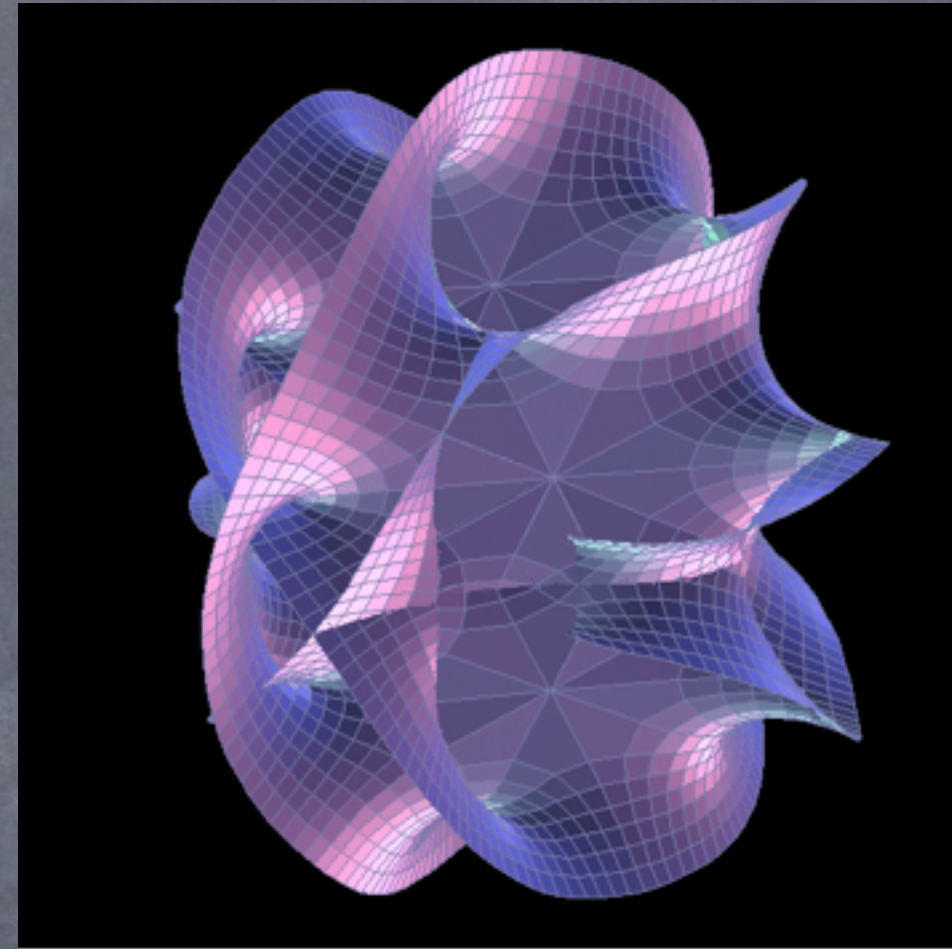
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$$K = -2 \log (\mathcal{V}(T_i, \bar{T}_i)) - \log (S + \bar{S}) - \log \left(-i \int_M \Omega \wedge \bar{\Omega} \right).$$
$$W = 0$$

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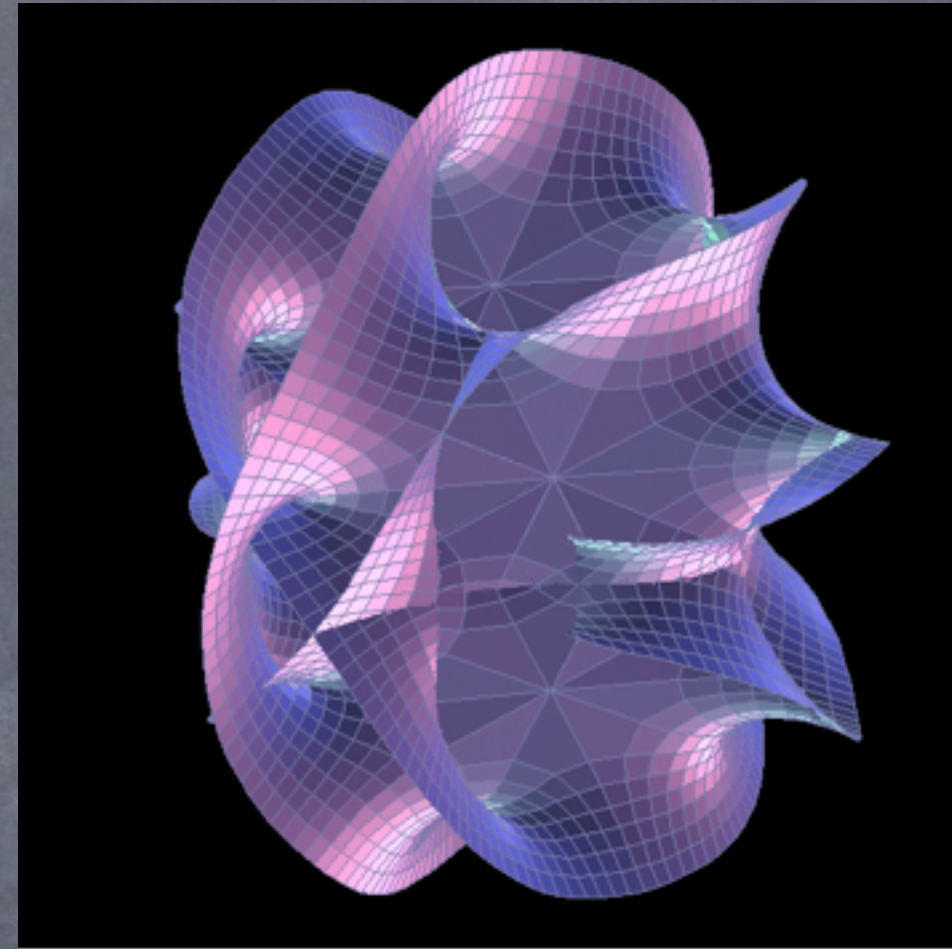


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$$W = \underbrace{\int_M G_3 \wedge \Omega}_{=W_0}$$

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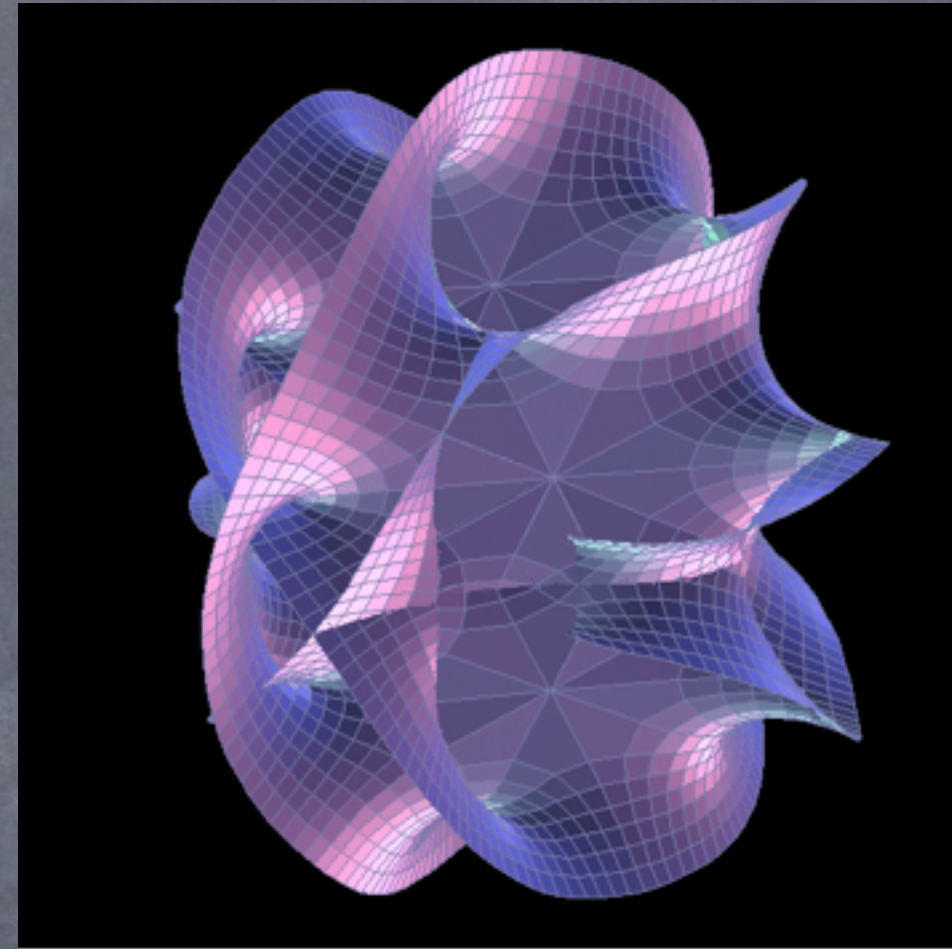


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$$W = W_0 + A_i e^{-a_i T_i}$$

Moduli Stabilisation in type IIB

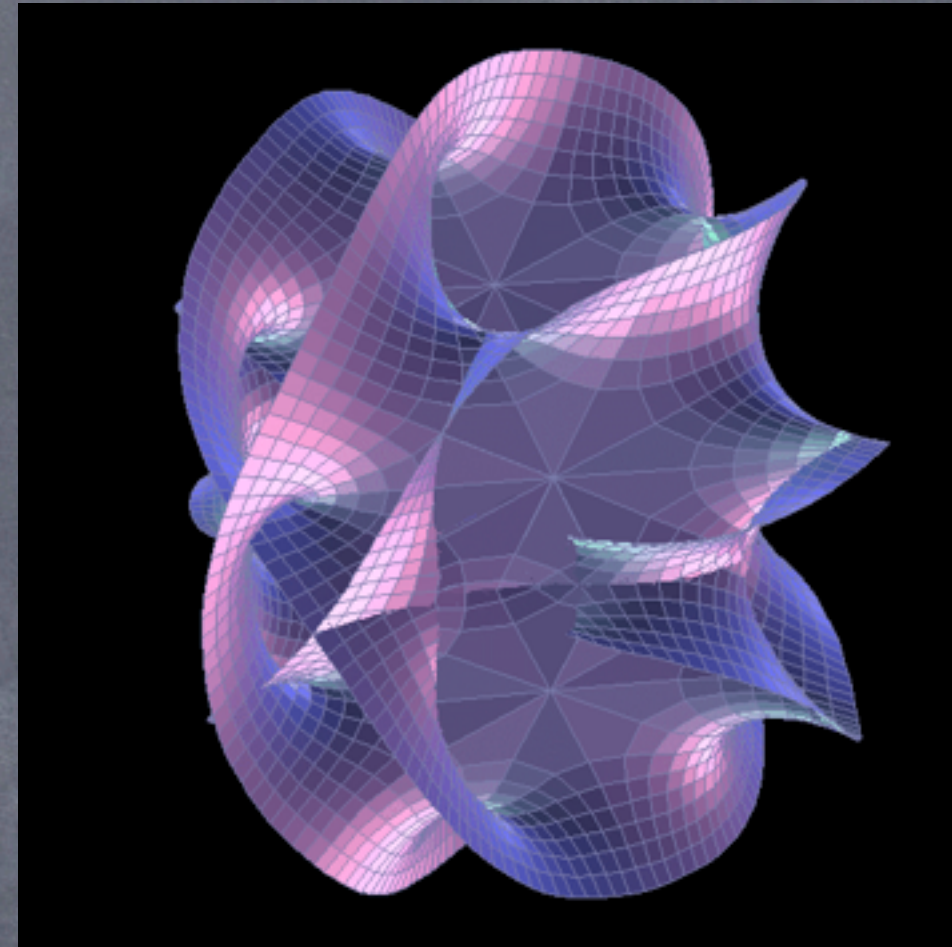
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$$K = -2 \log(\tau^{3/2}) + \tilde{K}(U, \bar{U}, S, \bar{S}).$$
$$W = W_0 + A e^{-aT}$$

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$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}$$

$$\mathbb{CP}_{[1,1,1,6,9]}$$



$$K = -2 \log(\tau^{3/2}) + \tilde{K}(U, \bar{U}, S, \bar{S}).$$

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LARGE volume type IIB Moduli Stabilisation

$$K = -2 \log \left(\mathcal{V} + \frac{\xi}{g_s^{3/2}} \right) + K(U, \bar{U}, S, \bar{S})$$

$$W = W_0 + A e^{-a T_s}$$



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LARGE volume type IIB Moduli Stabilisation

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$$W = W_0 + A e^{-a T_s}$$



$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}$$

$$V = \sum_{\Phi=S,U} \frac{K^{\Phi\bar{\Phi}} D_{\Phi} W \bar{D}_{\bar{\Phi}} \bar{W}}{\mathcal{V}^2} + \frac{\lambda |aA|^2 \sqrt{\tau_s} e^{-2a\tau_s}}{\mathcal{V}} - \frac{\mu W_0 a A \tau_s e^{-a\tau_s}}{\mathcal{V}^2} + \frac{\nu \xi |W_0|^2}{g_s^{3/2} \mathcal{V}^3}$$

Minimum:

$$\mathcal{V} \sim e^{a\tau_s} \gg 1$$

$$\tau_s \sim \frac{\xi^{2/3}}{g_s}$$

hierarchical suppressions in 1/Vol expansion:

$$M_{\text{String}} = \frac{M_{\text{Planck}}}{\sqrt{\mathcal{V}}}$$

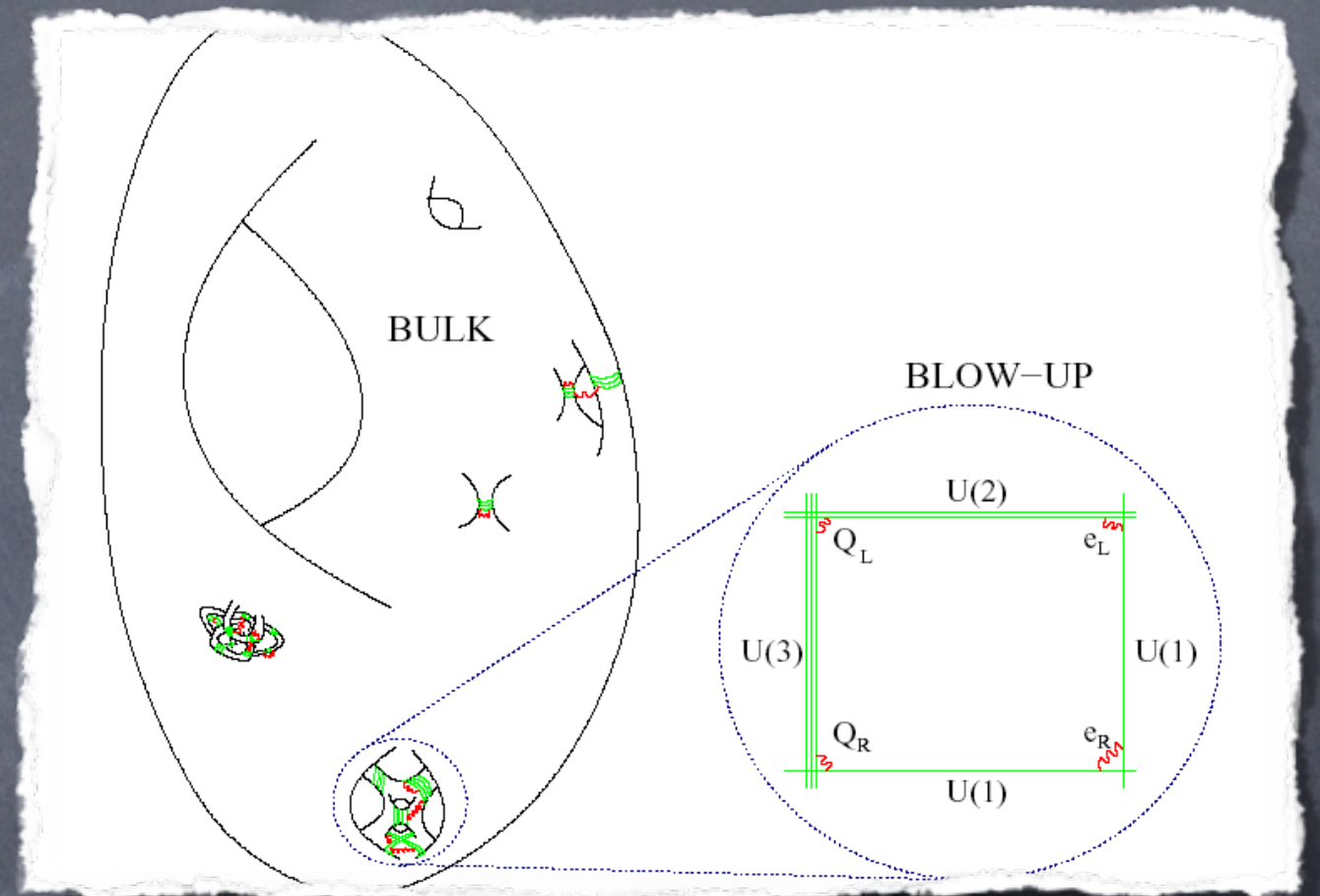
Berg, Haack, Pajer, Cicoli et al.

Balasubramanian, Berglund,
Conlon, Quevedo, Suruliz

... stable against string loop corrections (non-trivial)

Where can we build models?

- LVS implies SM is localised (cannot wrap exponentially large cycle) → Bottom Up
- options include in IIB: Magnetised D7 branes, local F-theory models
- Here: Fractional D3/D7 branes



$$K_{\text{matter}} = K_{\alpha\bar{\beta}}(T_i, \bar{T}_i, U, \bar{U}, S, \bar{S}) C_{\alpha} \bar{C}_{\bar{\beta}} = \frac{1}{V^{2/3}} (c + f(T_s, T_b)) \tilde{K}_{\alpha\bar{\beta}}(U, \bar{U}, S, \bar{S}) C_{\alpha} \bar{C}_{\bar{\beta}}$$

CCQ: Invariance of physical Yukawas on Kähler moduli

Gravity/Moduli mediated SUSY Breaking (classical scenario)

gravitino mass:

$$m_{3/2} = e^{K/2} |W| \sim \frac{M_P |W_0|}{\nu}$$

Assumptions: $V_0=0$,
no D-term contribution, no 1-loop
redefinitions of moduli

Gravity/Moduli mediated SUSY Breaking (classical scenario)

gravitino mass:

$$m_{3/2} = e^{K/2} |W| \sim \frac{M_P |W_0|}{\mathcal{V}}$$

non-vanishing F-terms: F_{Tb}, F_{Ts}, F_S

gauge kinetic function: $f = \text{Re}(S)$

0906.3297

... and after many no-scale cancellations, we obtain the following soft-masses

M_{gaugino}	$\frac{m_{3/2}}{\mathcal{V}}$
m_{scalar}	$\frac{m_{3/2}}{\sqrt{\mathcal{V}}}$ Or $\frac{m_{3/2}}{\mathcal{V}}$
M_{string}	$\frac{M_P}{\sqrt{\mathcal{V}}}$
\mathcal{V}	10^{6-7}

Assumptions: $V_0=0$,
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aside: lightest modulus ($m_{Tb} \sim M_P / \mathcal{V}^{3/2}$)
heavier than TeV soft-masses
[->cosmological moduli problem]

Gravity/Moduli mediated SUSY Breaking (redefined scenario)

gravitino mass:

$$m_{3/2} = e^{K/2} |W| \sim \frac{M_P |W_0|}{\nu}$$

M_{gaugino}	$m_{3/2}$
m_{scalar}	$m_{3/2}$
M_{string}	$\frac{M_P}{\sqrt{\nu}}$
ν	10^{15}

Gravity/Moduli mediated SUSY Breaking (redefined scenario)

gravitino mass:

$$m_{3/2} = e^{K/2} |W| \sim \frac{M_P |W_0|}{\mathcal{V}}$$

non-vanishing F-terms: $F_{TSM}, F_{Tb}, F_{Ts}, F_S$

gauge kinetic function: $f = \text{Re}(S) + T_{SM}$

1003.0388, 1011.0999

... including 1-loop redefinitions of
moduli ($t_s \rightarrow t_s + a \ln V$)

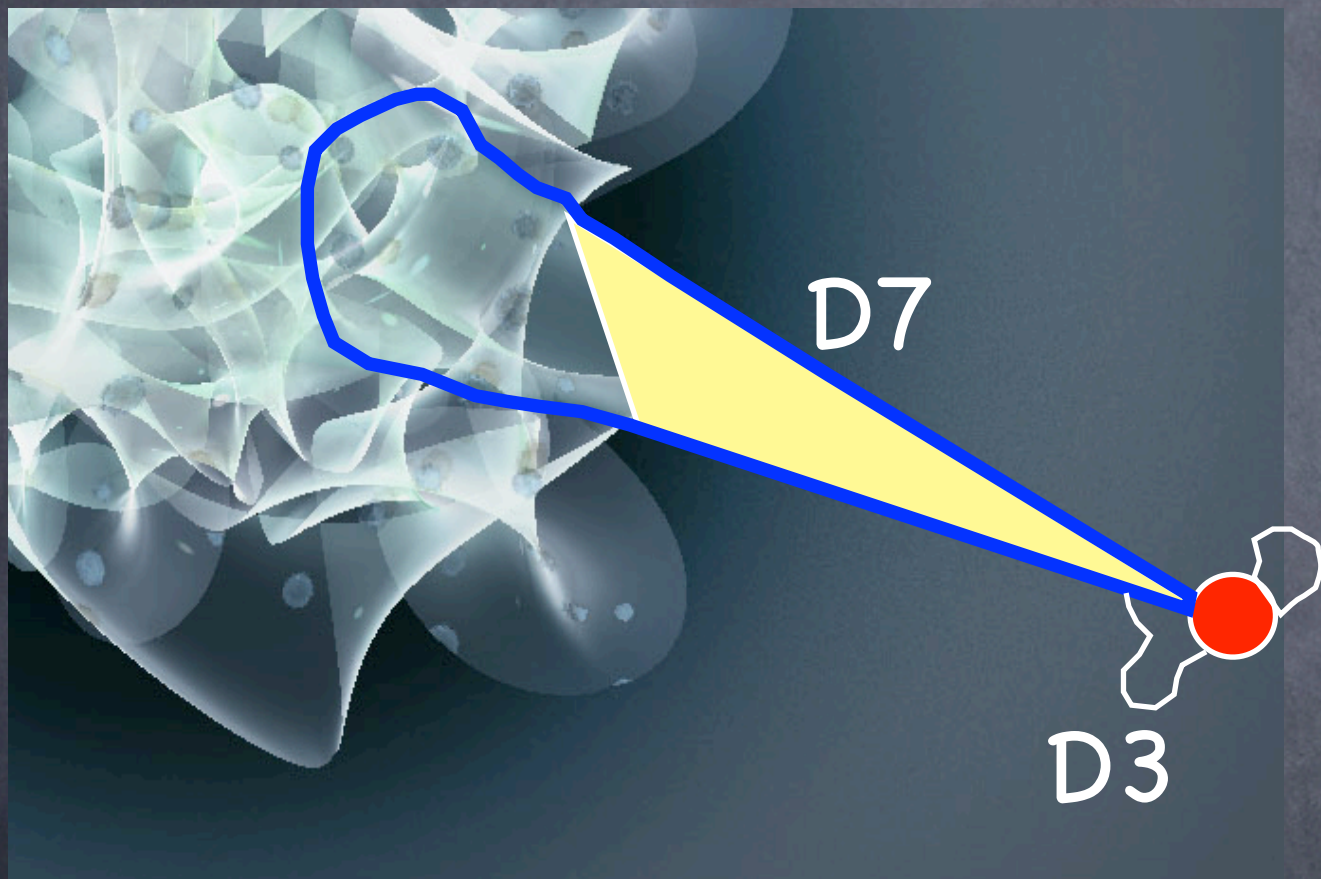
M_{gaugino}	$m_{3/2}$
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\mathcal{V}	10^{15}

string scale below GUT scale
no direct solution to cosmological
moduli problem

for now I mostly assume the classical scenario, keeping in mind that this ambiguity needs to be addressed in the near future...

...and let's look at today's topic model building

Branes@singularities



- Local 4-cycles can shrink (typical geometries are del-Pezzo surfaces).
- Gauge theory arises from D3 branes at singularities and D7 branes intersecting with singularities. SM gauge groups from D3 branes.
- Gauge theory studied in “decoupling” limit (i.e. long [infinite] throat), some bulk effects are known (e.g. leading to non commutative geometry \rightarrow different Yukawa couplings 0512122).

What types of singularities are there (suitable for model building)?

- Orbifold Singularities
- del Pezzo singularities (P^2 blown-up), Conifold

-> TORIC SINGULARITIES

- non-toric singularities

What types of singularities are there (suitable for model building)?

- Orbifold Singularities

few suitable for model building

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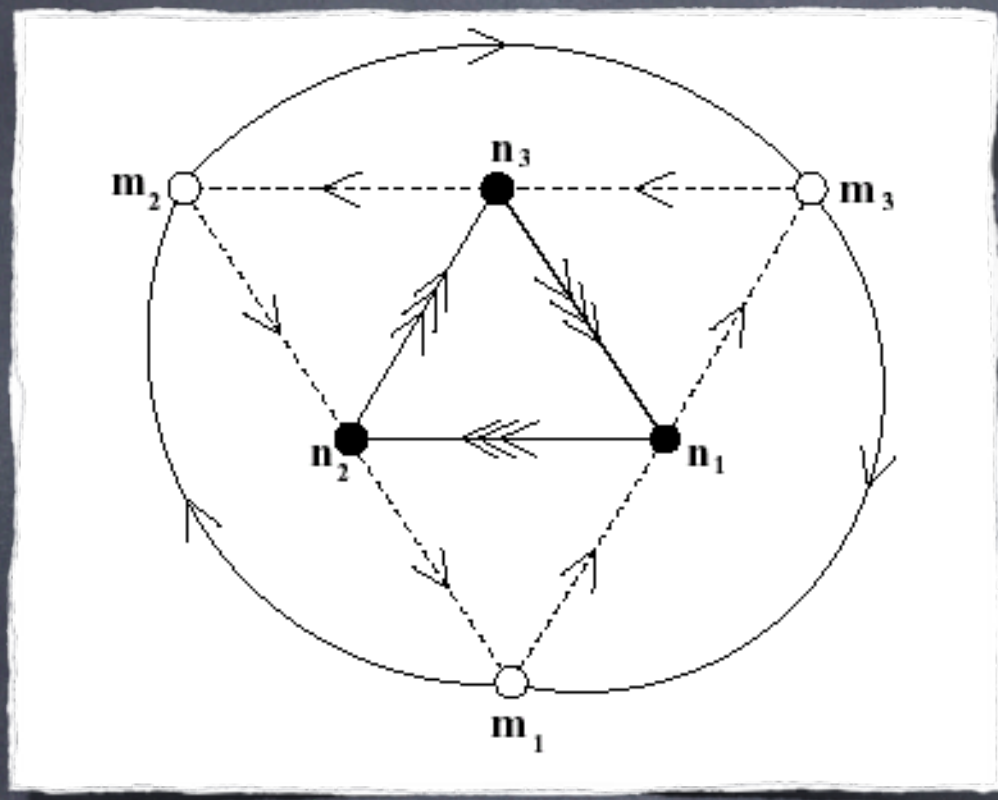
-> TORIC SINGULARITIES

infinite class, techniques

- non-toric singularities

limited techniques

Classic Example: $\mathbb{C}^3/\mathbb{Z}_3$



D3 matter content:

$$3[(n_1, \bar{n}_2, 1), (1, n_2, \bar{n}_3), (\bar{n}_1, 1, n_3)]$$

$$W_{D3D3} = \epsilon_{ijk} X_{12}^i Y_{23}^j Z_{31}^k$$

- n_i D3-branes: $U(n_1) \times U(n_2) \times U(n_3)$
- m_i D7-branes: $U(m_1) \times U(m_2) \times U(m_3)$
- Arrows: bi-fundamental matter
- Anomaly cancellation

$$m_2 = 3(n_3 - n_1) + m_1,$$

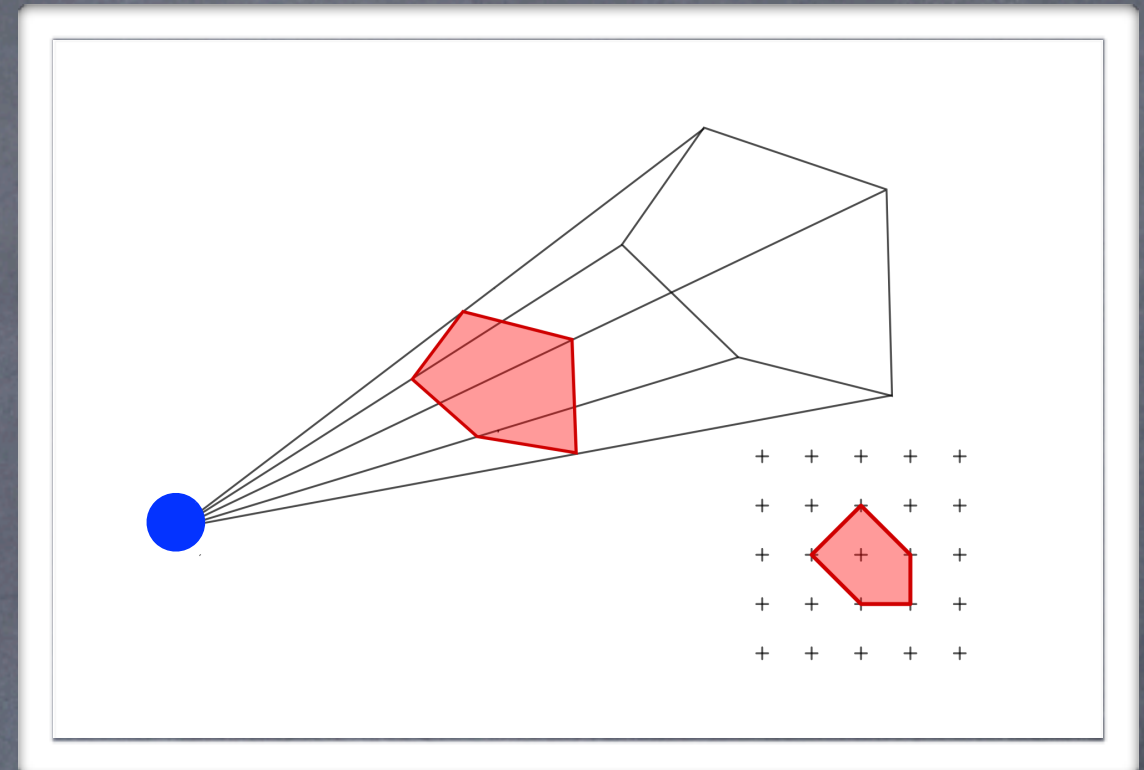
$$m_3 = 3(n_3 - n_2) + m_1$$

- Hypercharge:

$$Q_{\text{anomaly-free}} = \sum_i \frac{Q_i}{n_i}$$

Gauge theories probing toric singularities

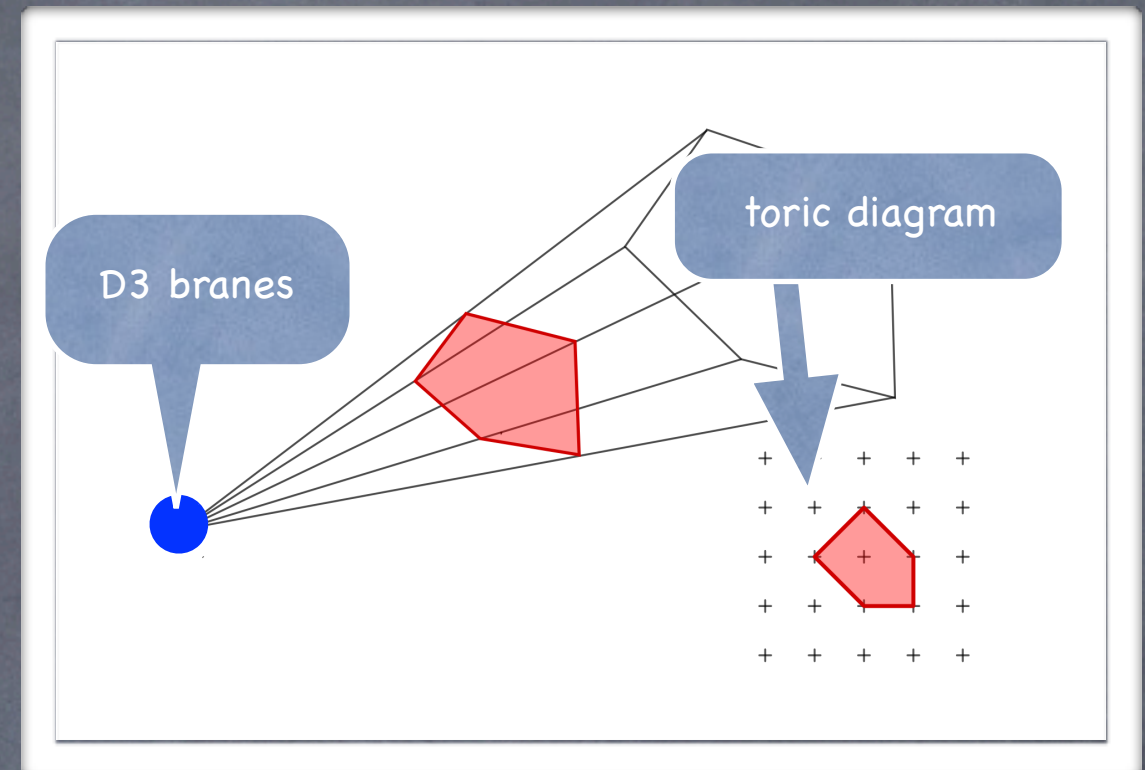
- Toric CY-cone: represented as T^3 fibration over rational polyhedral cone (\rightarrow toric diagram) [non-compact but embeddable in global manifold]
- Gauge theory of bound states is a quiver gauge theory (rank of gauge group only freedom).
- Gauge theory obtained via T-dual D5/NS5 brane system wrapping T^2 . This system on T^2 is the dimer and encodes the whole gauge theory.
- Gauge theory at the tip of the cone can be seen as bound states of D7, D5, D3 branes after transition from the geometric regime (marginal stability walls).



Hanany, Kennaway, Franco, Vegh, Wecht, Martelli, Sparks, Feng, He, Vafa, Yamazaki
0503149, 0504110, 0505211, 0511063, 0511287, 0604136, 0706.1660, 0803.4474

Gauge theories probing toric singularities

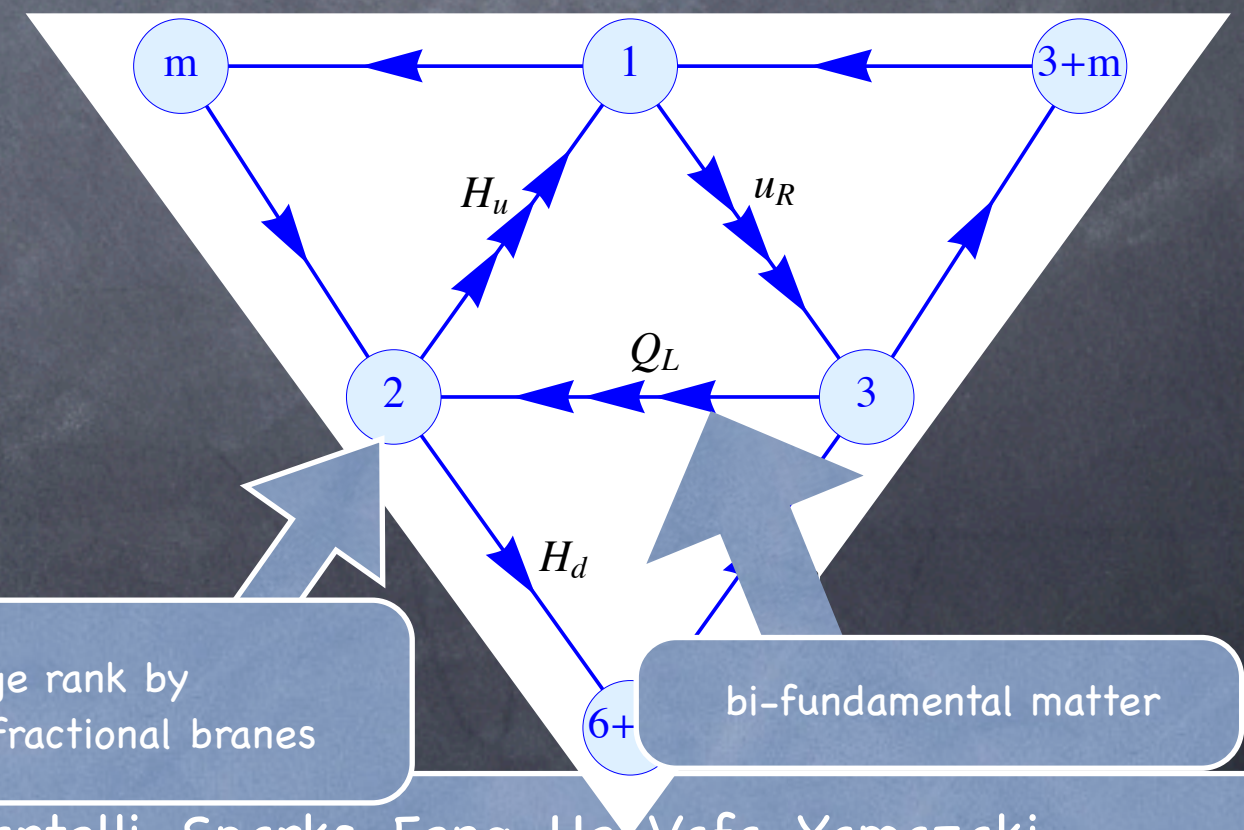
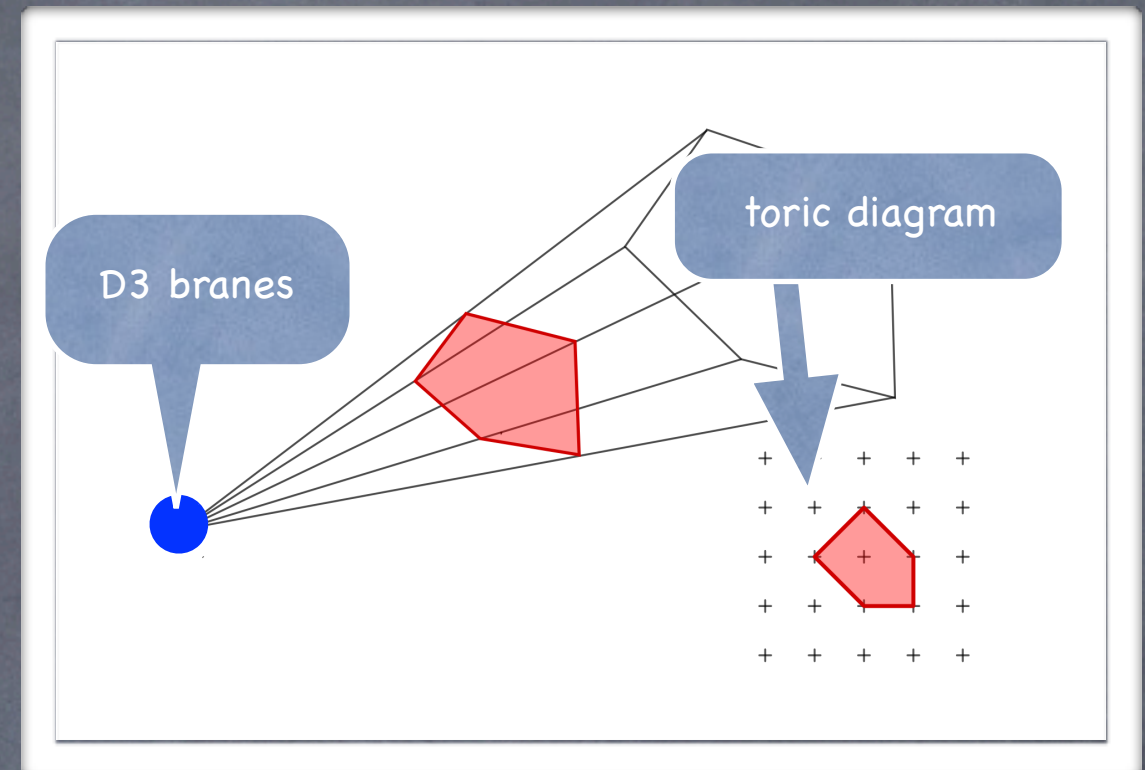
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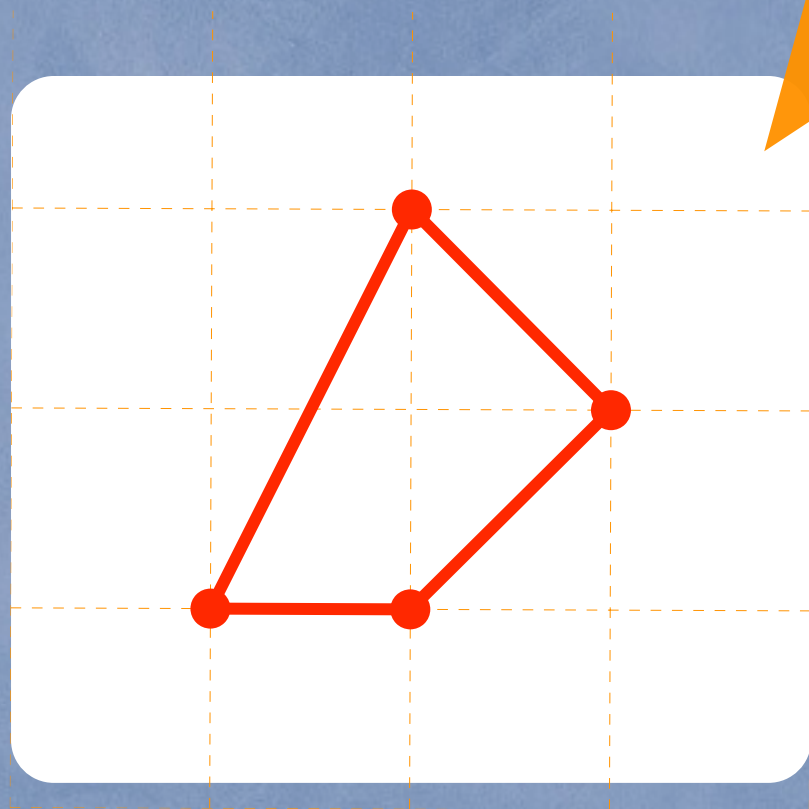
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The Dimer Language

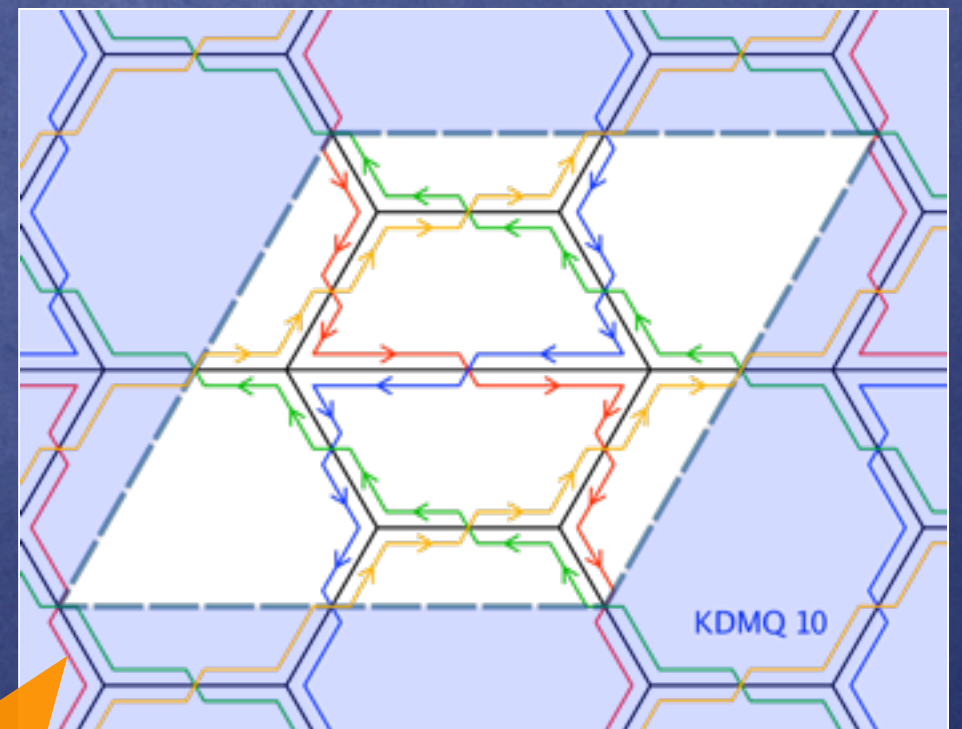
Dimers visualise the gauge theory of toric singularities.

Geometry: Toric Diagram

inverse slopes in toric diagram



Gauge Theory: Dimer



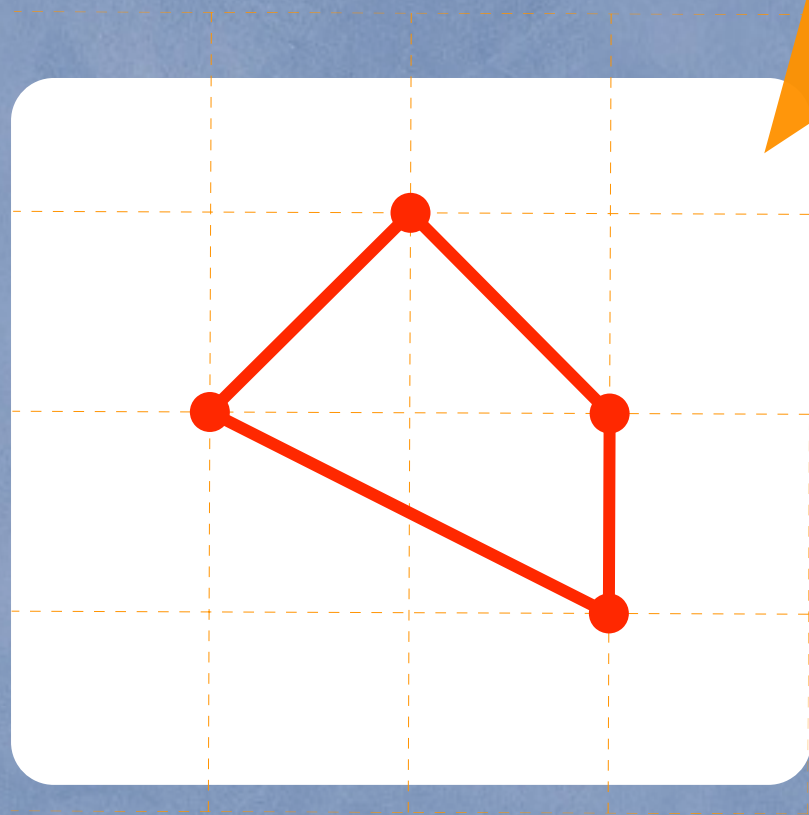
winding numbers of toric diagram

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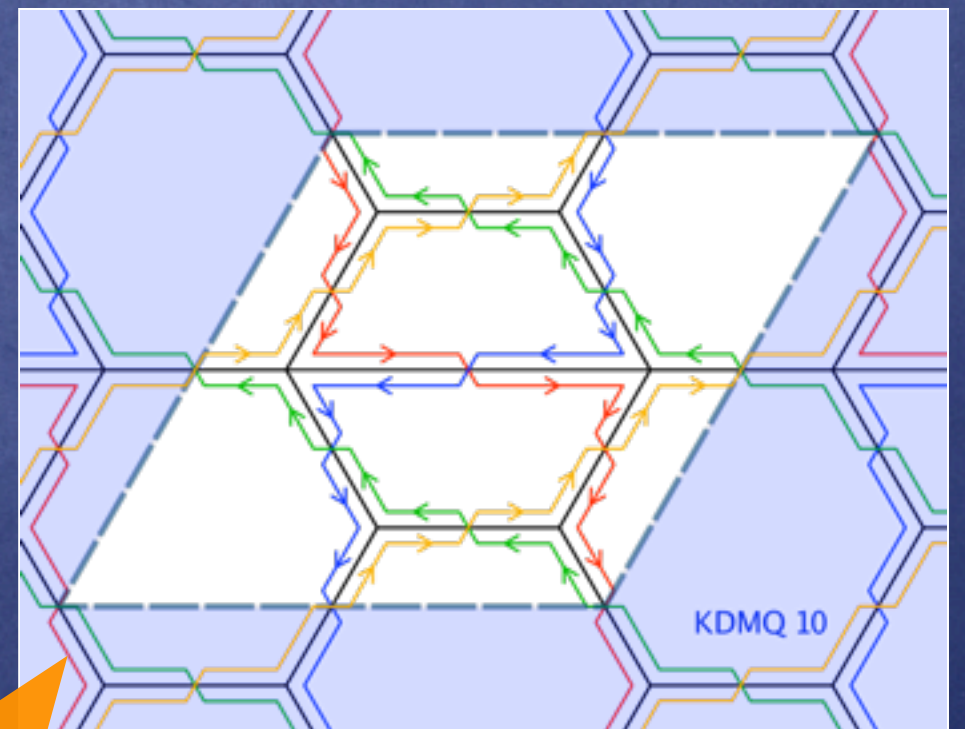
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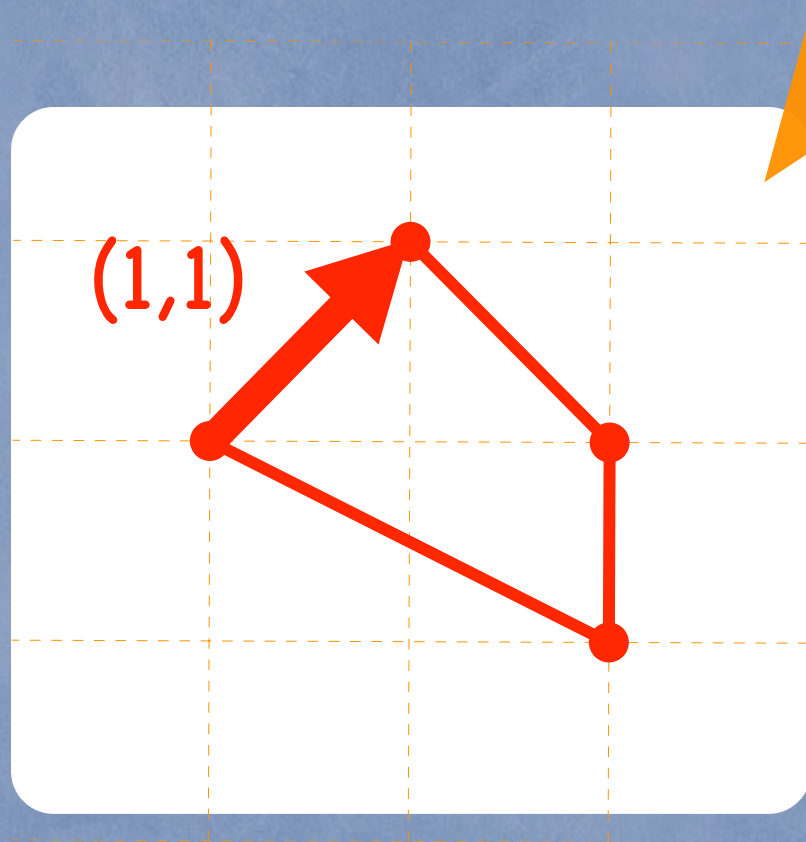
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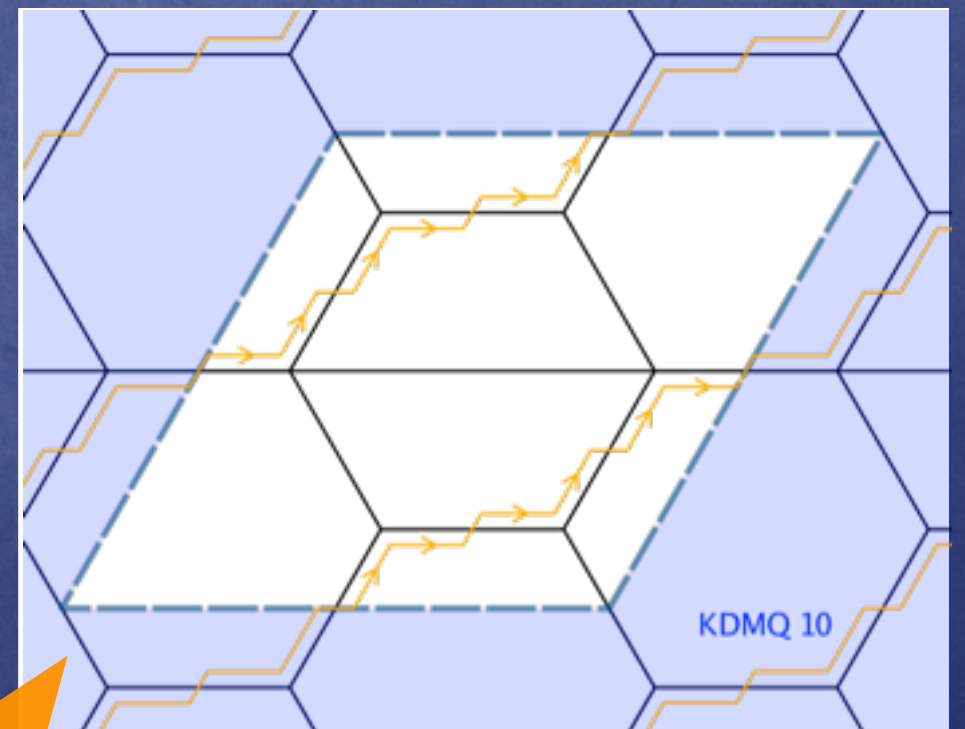
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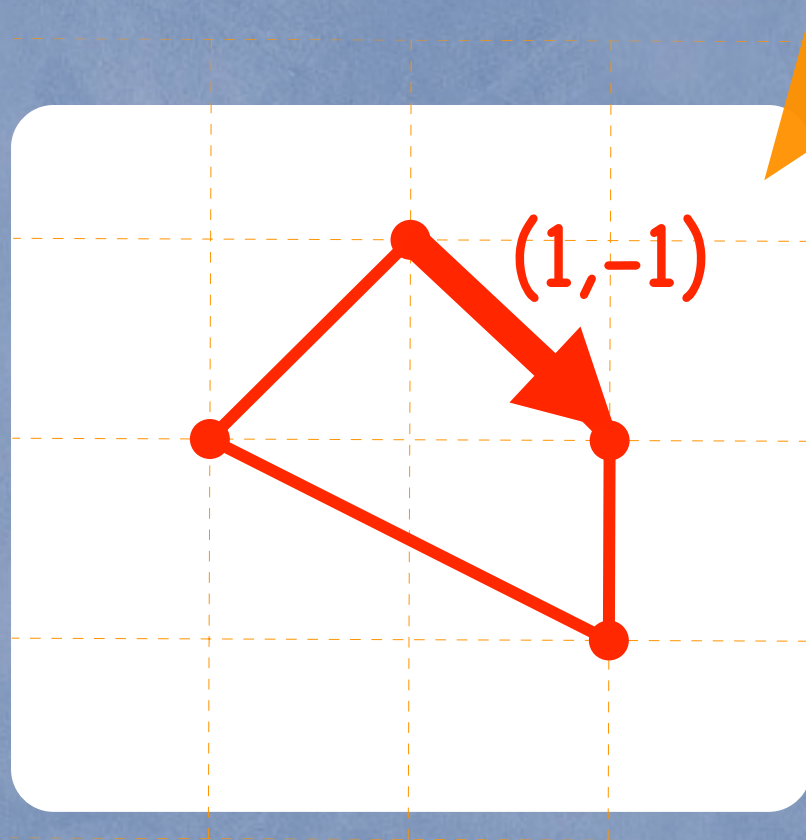
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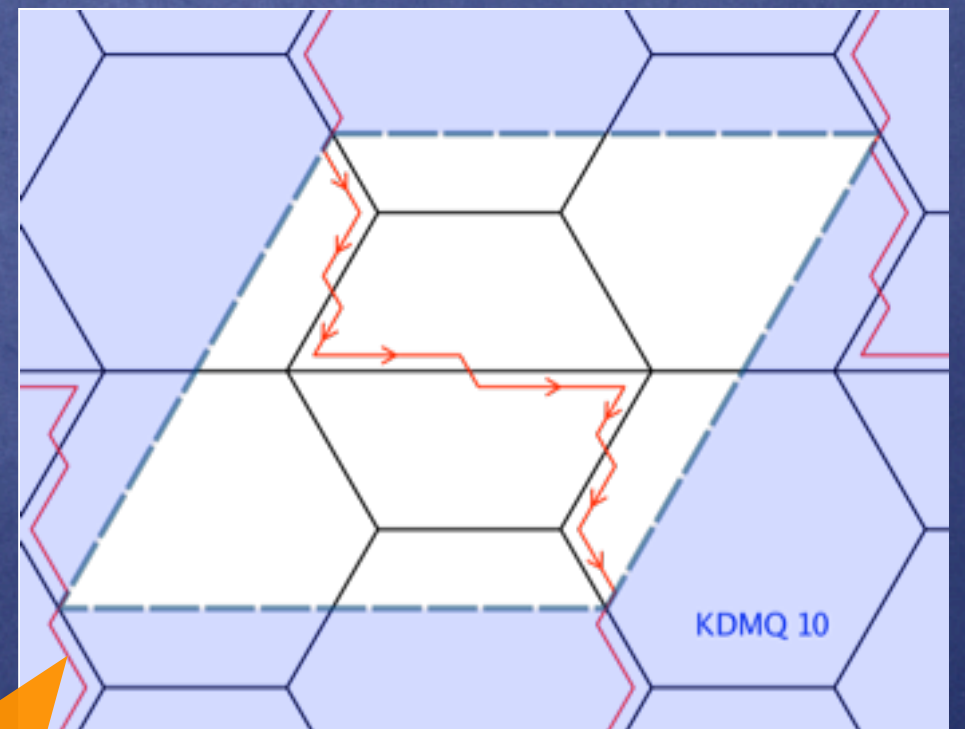
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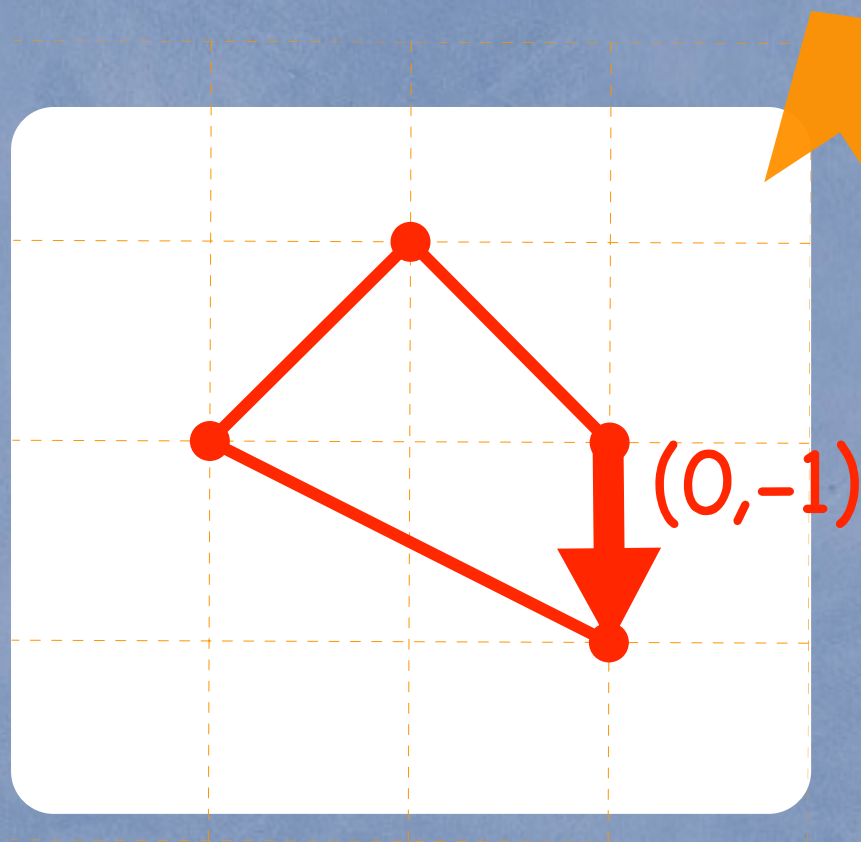
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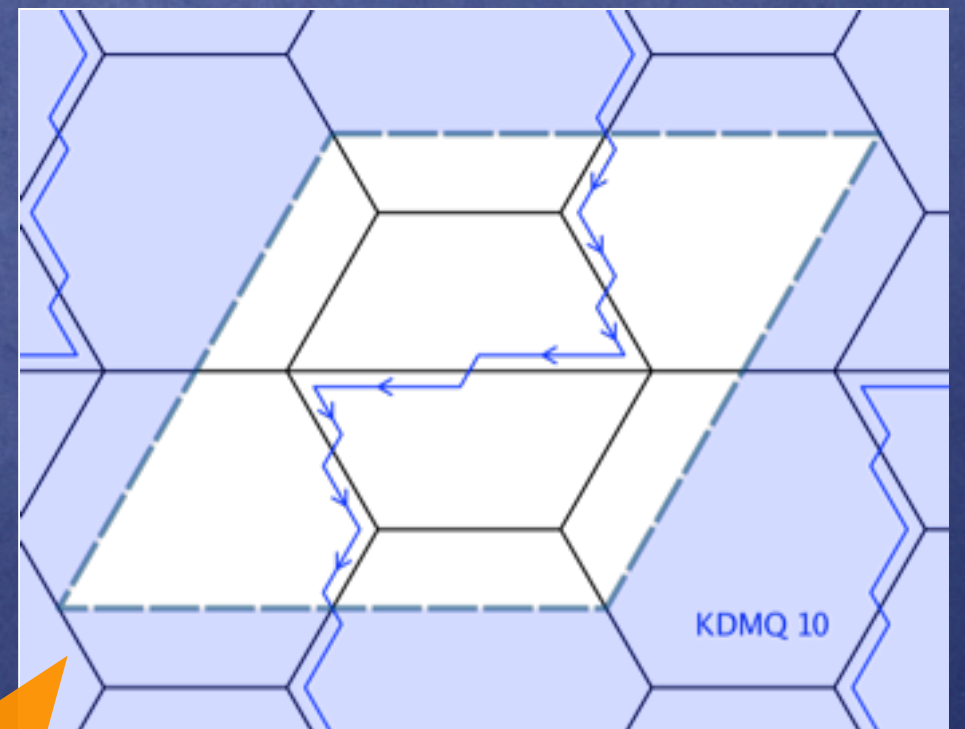
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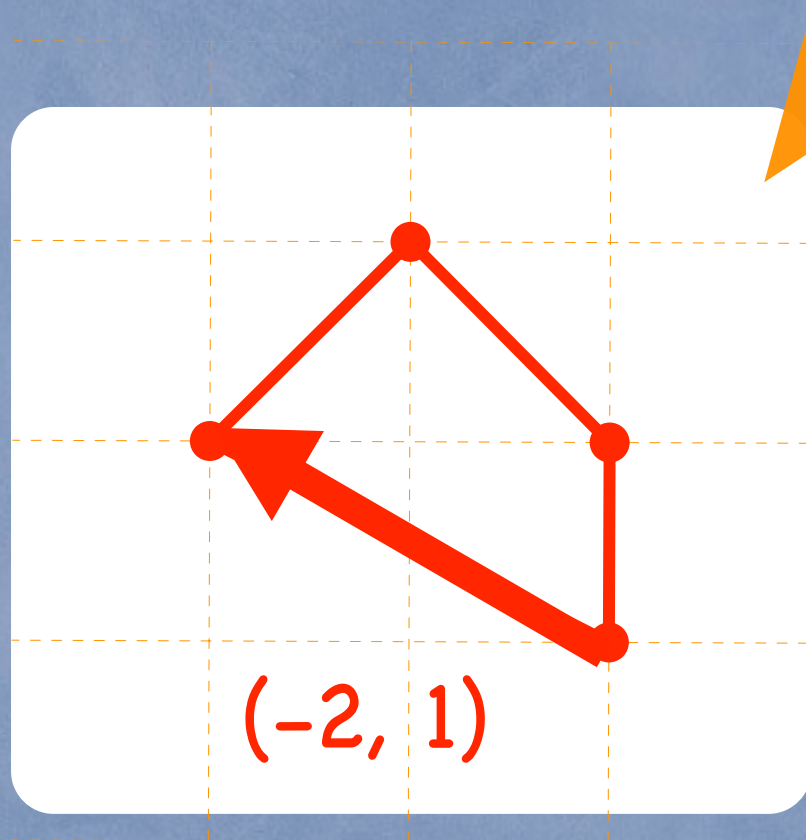
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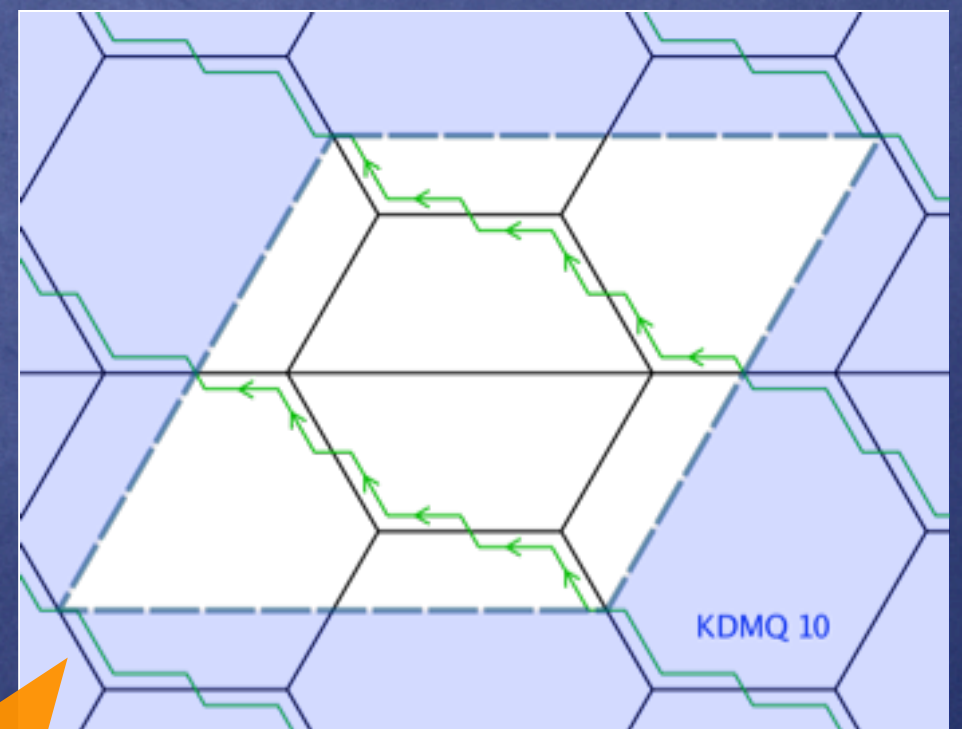
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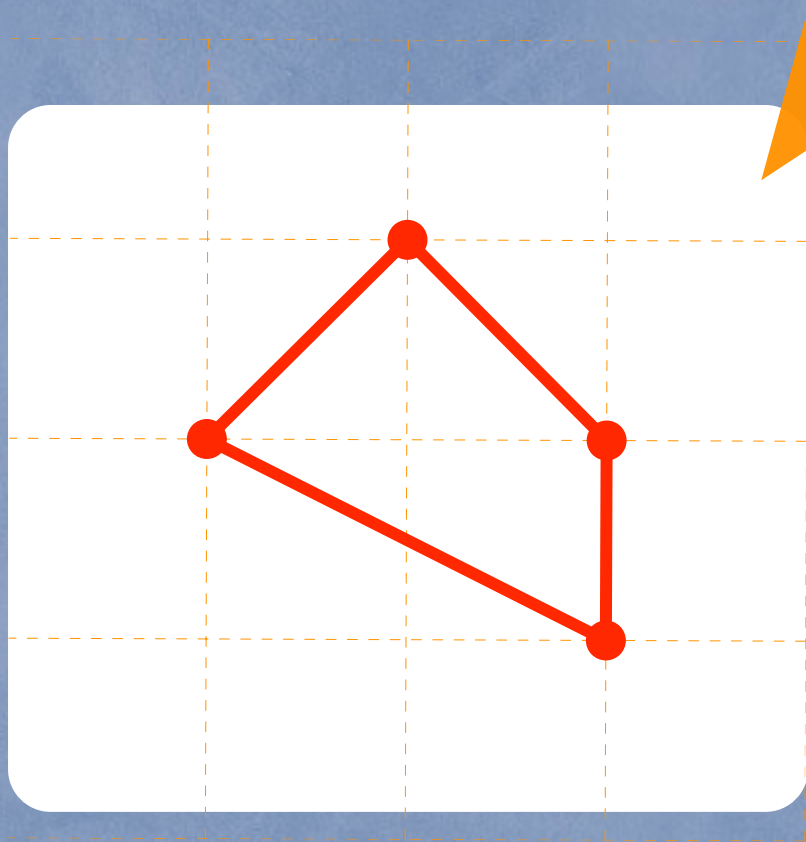
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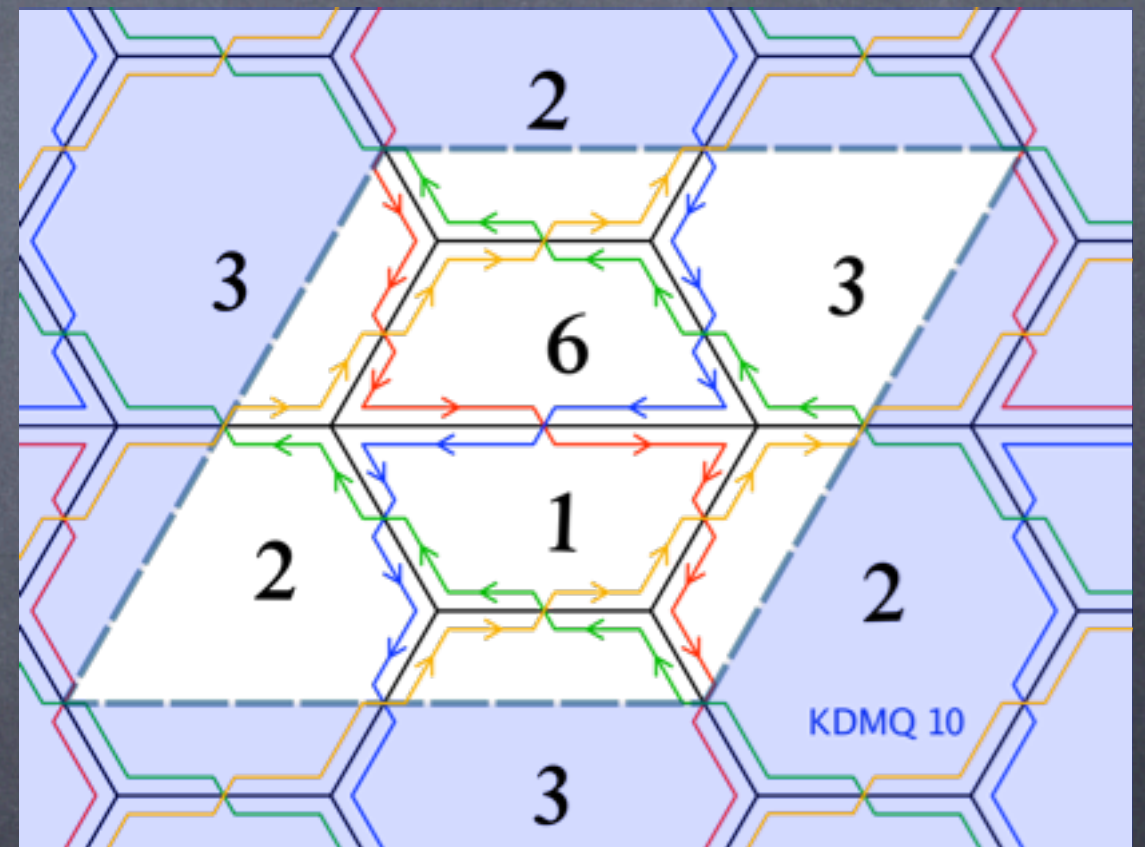


winding numbers of toric diagram

Dimer Language II

Reading off the gauge theory

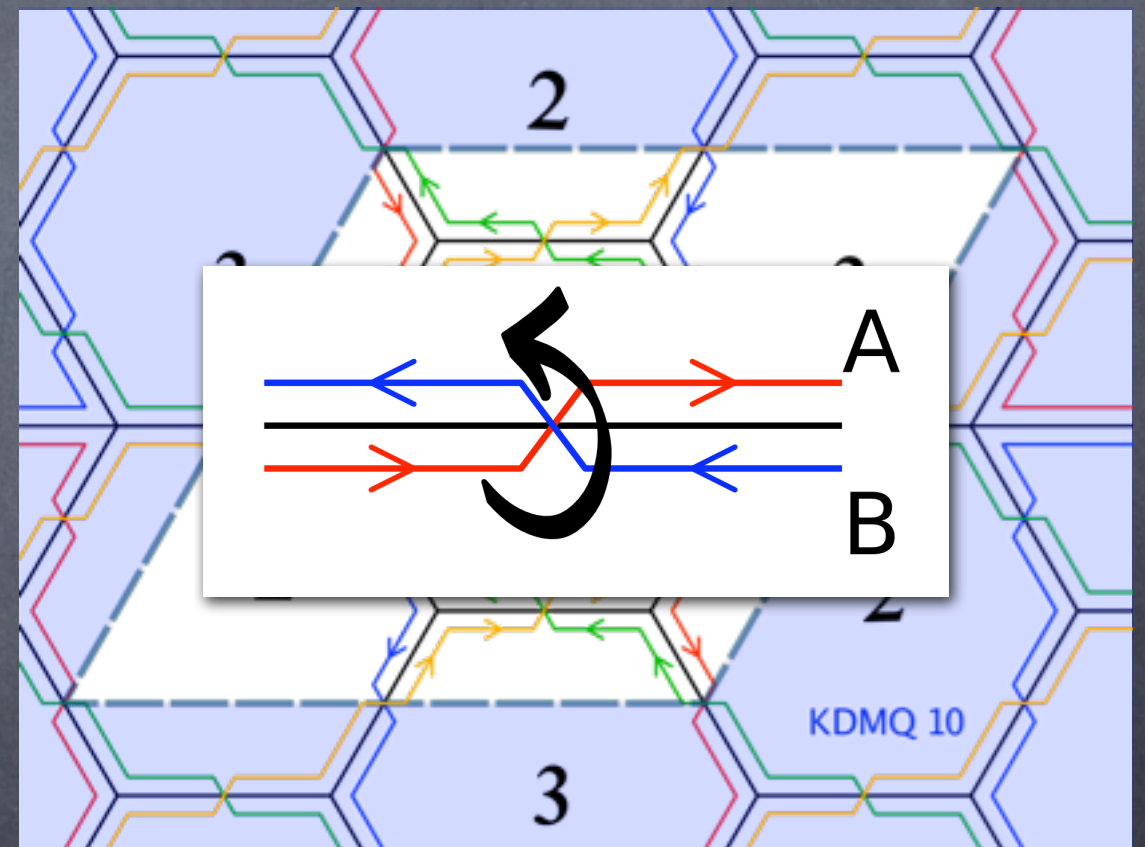
- Faces
= gauge groups
- Intersection of zigzag paths
= bi-fundamental matter
- Vertices (faces orbited by zigzag paths)
= superpotential terms



Dimer Language II

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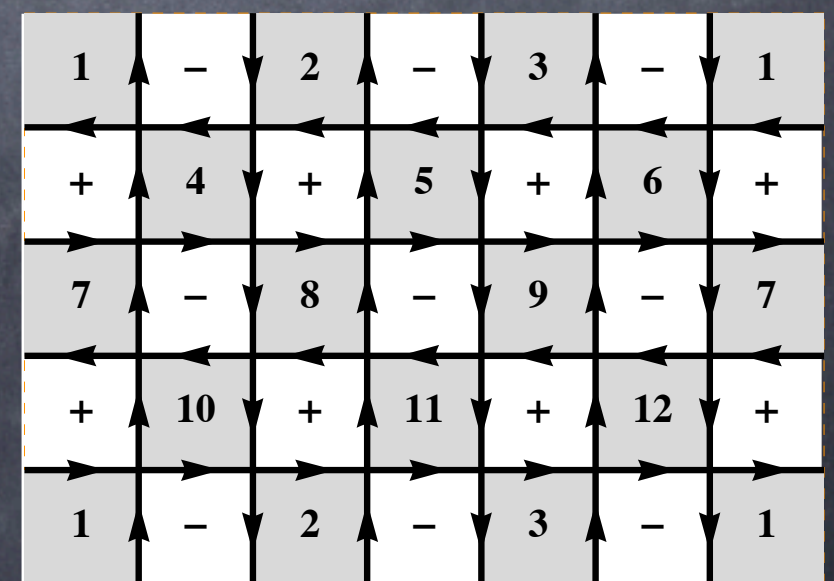
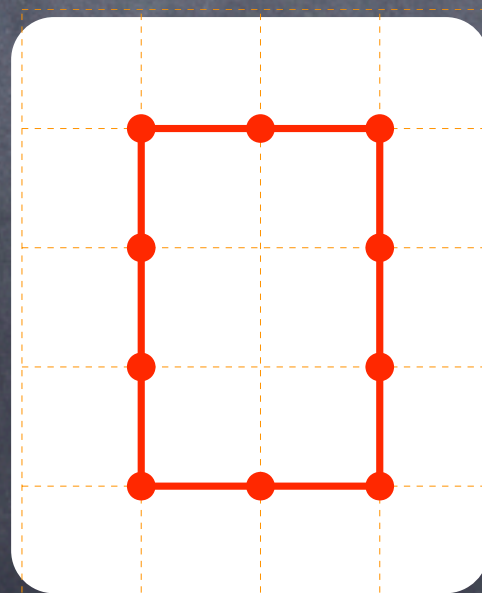
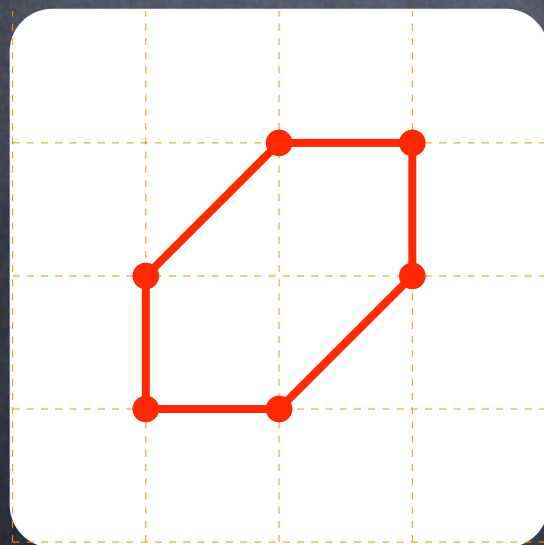
$$W = X_{13}X_{32}X_{21} - X_{14}X_{43}X_{32}X_{21}$$

Dimer Language III: How do I get a dimer?

- this is the inverse problem (following Gulotta)
- embed toric singularity in orbifold of conifold whose dimer is known (chess-board).
- collapse cycles in singularity (= cutting toric diagram)
- merge zigzag paths according to cutting of toric diagram
- higher slopes: merge zigzag path of given winding with further e.g. $[0,1]$ paths (Farey tree)
- create first all positive slopes then negative slopes

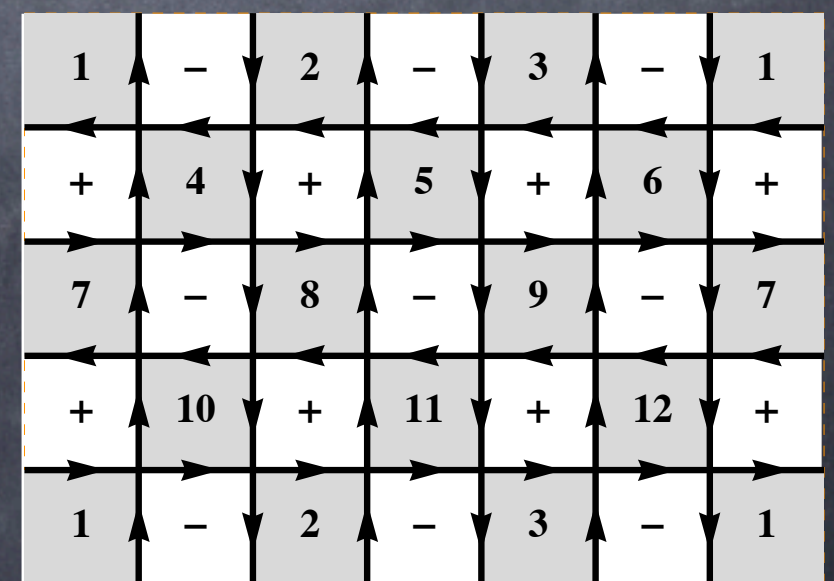
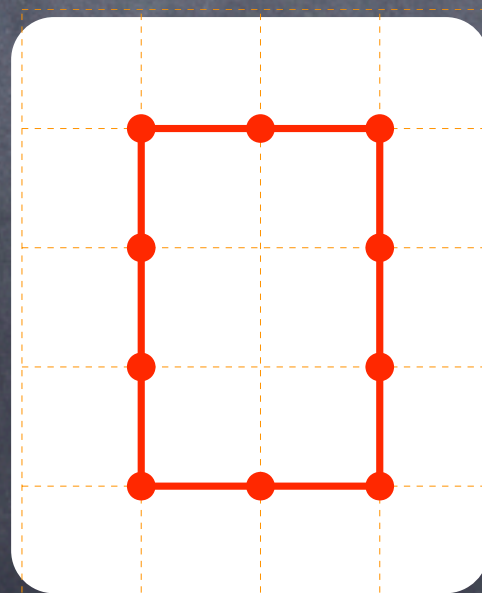
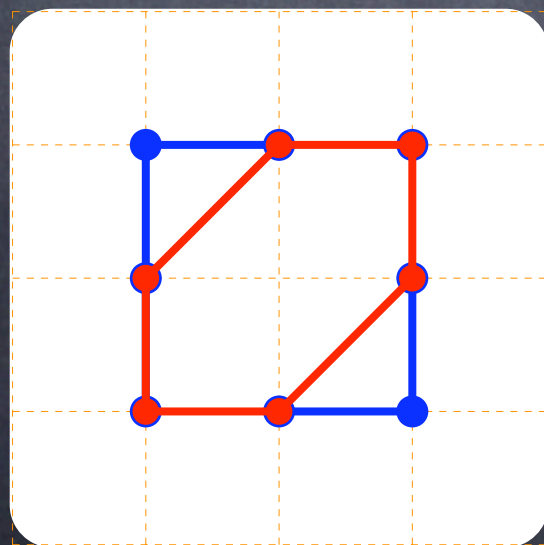
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- create first all positive slopes then negative slopes



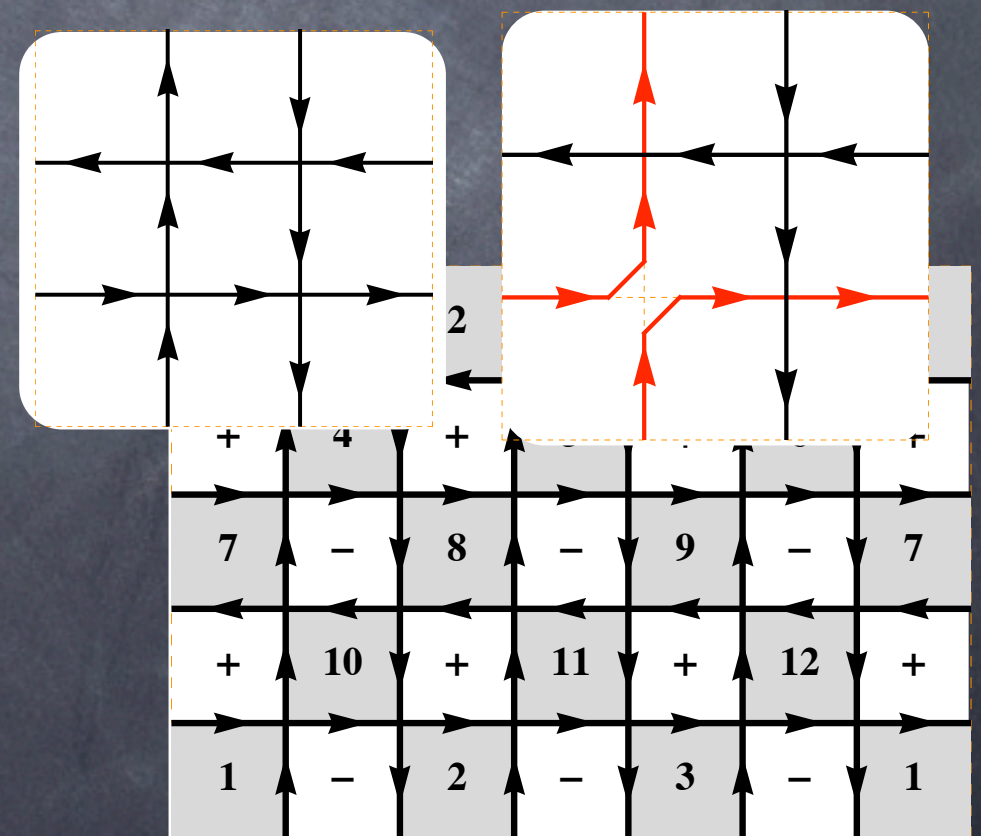
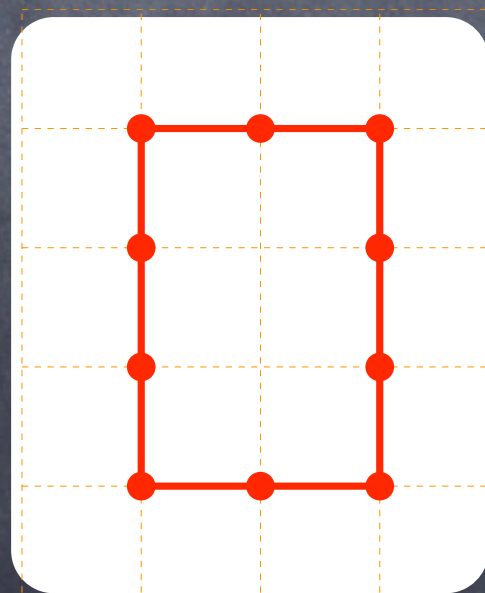
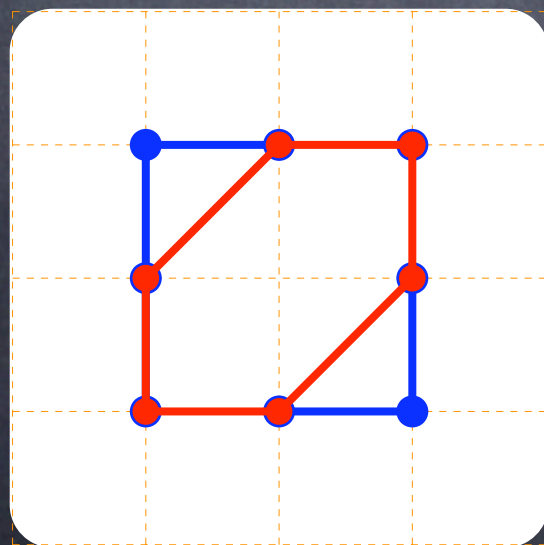
Dimer Language III: How do I get a dimer?

- this is the inverse problem (following Gulotta)
- embed toric singularity in orbifold of conifold whose dimer is known (chess-board).
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- The diagrams illustrate the merging of zigzag paths. The left diagram shows a grid with a horizontal path of arrows pointing left and a vertical path of arrows pointing up and down. The right diagram shows the same grid with a red vertical path of arrows pointing up and down, and a horizontal path of arrows pointing left.



Operations on the dimer

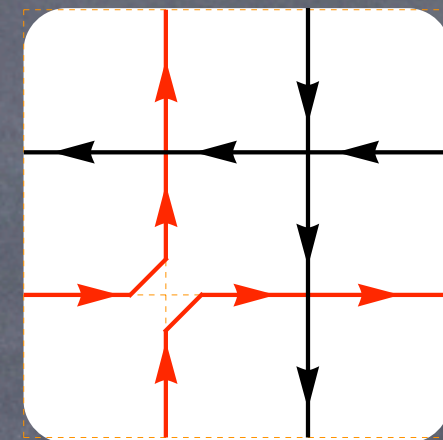
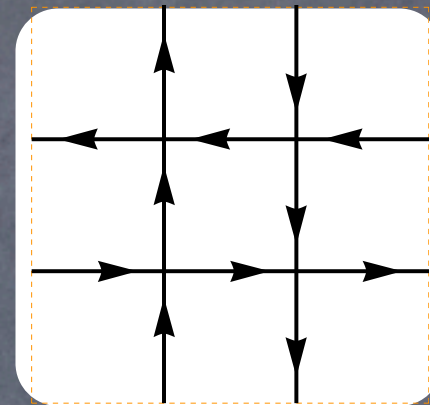
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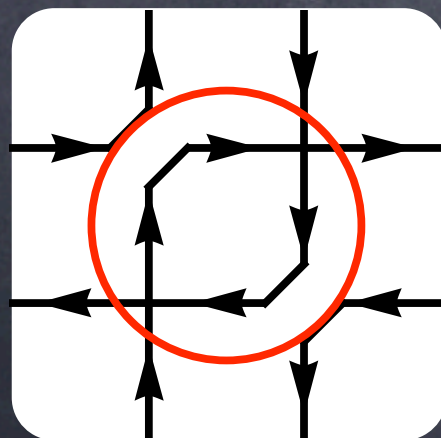
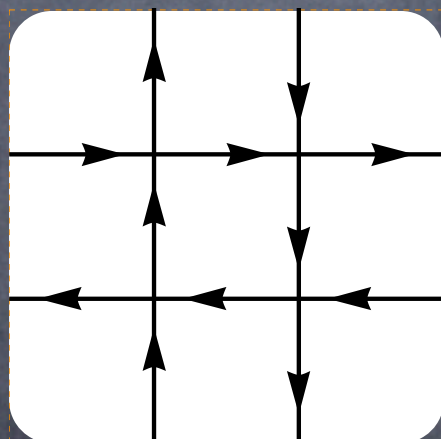
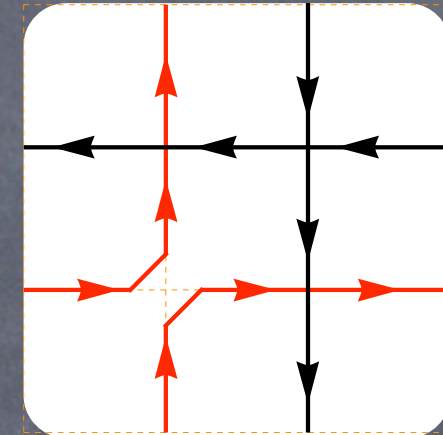
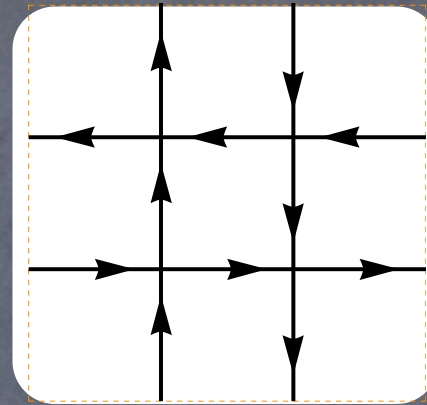
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Operations on the dimer

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Operation 2:

$$(1,0) + (0,1) \text{ \& } (-1,0) + (0,-1) \rightarrow (1,1) + (-1,-1)$$

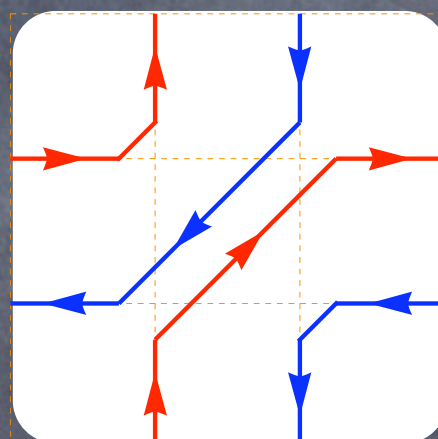
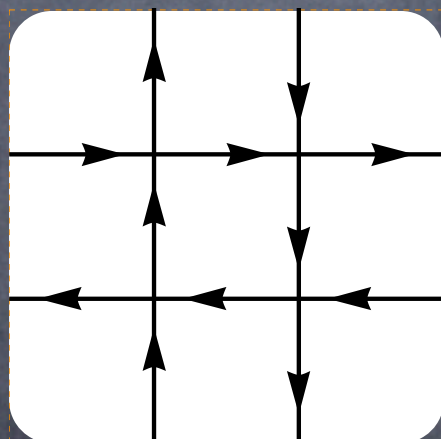
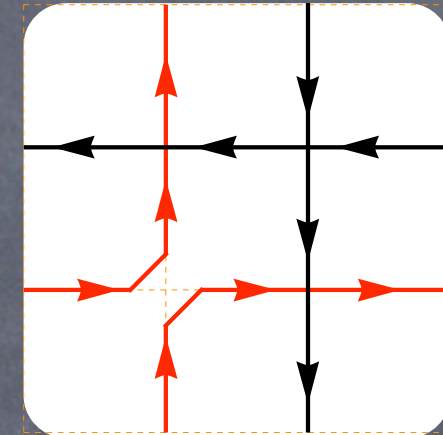
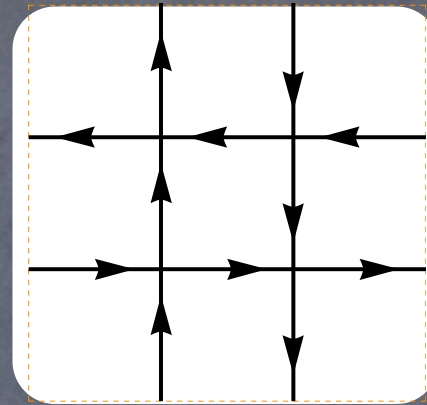
$$I_{(1,1),(-1,-1)} = 0$$

$$I_{ab} = n_a m_b - m_a n_b$$

Operations on the dimer

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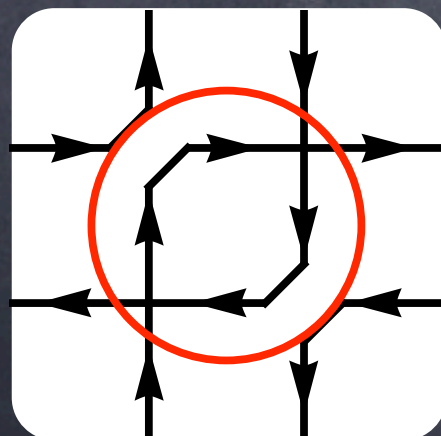


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Off-topic: General Results for toric singularities using dimer techniques

1002.1790

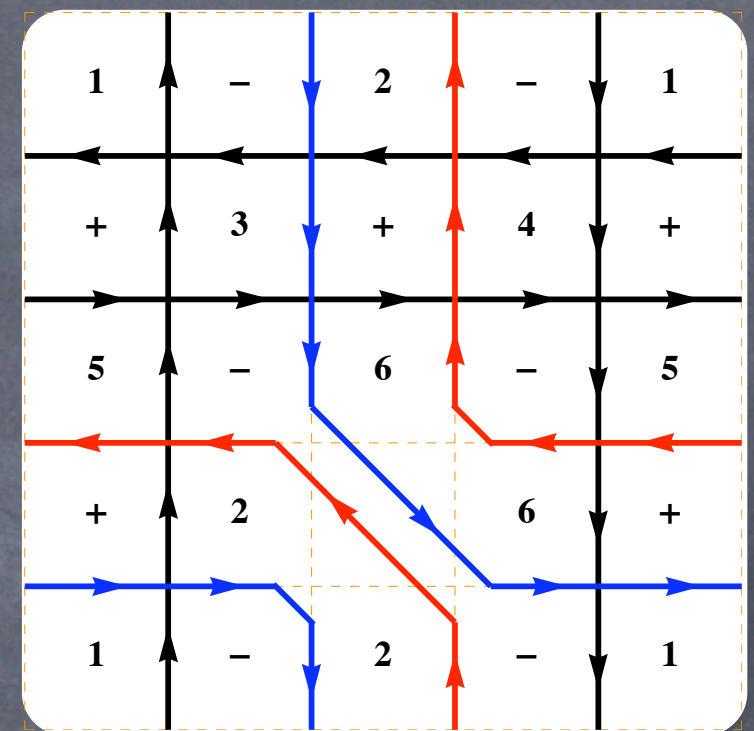
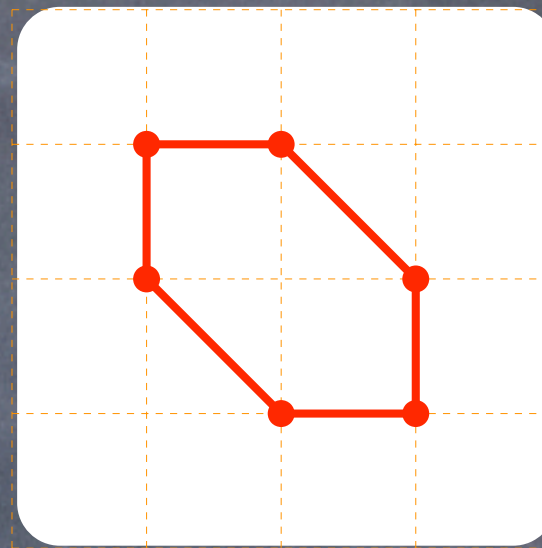
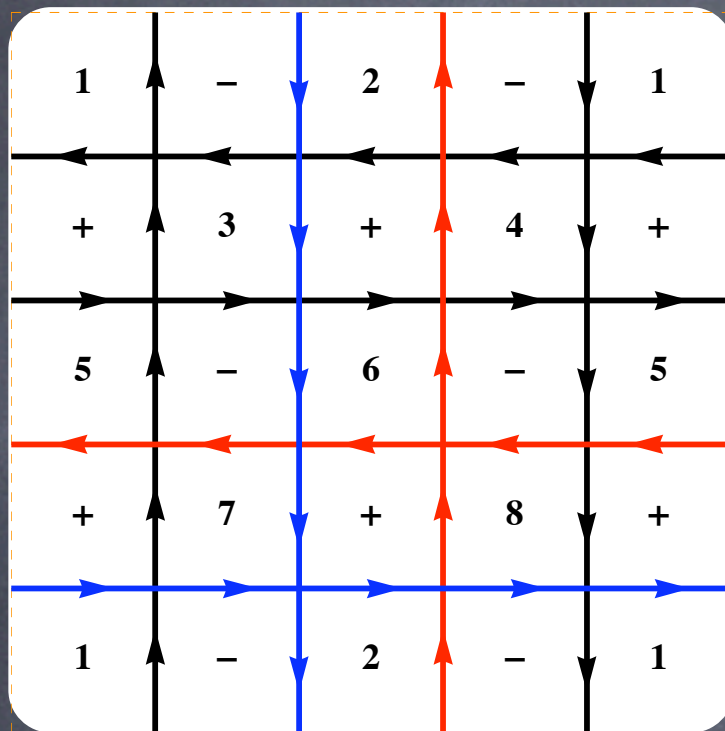
- Maximum number of families = 3 (one exception zeroth Hirzebruch surface with 4 families)
- Mass Hierarchy $(0, m, M)$

Off-topic: General Results for toric singularities using dimer techniques

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- Maximum number of families = 3 (one exception zeroth Hirzebruch surface with 4 families)
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e.g. del Pezzo 3



$$W_{dP_3} = -X_{12}Y_{31}Z_{23} - X_{45}Y_{64}Z_{56} + X_{45}Y_{31}Z_{14}\rho_{53} + X_{12}Y_{25}Z_{56}\Phi_{61} \\ + X_{36}Y_{64}Z_{23}\Psi_{42} - X_{36}Y_{25}Z_{14}\rho_{53}\Phi_{61}\Psi_{42}$$

$$= \begin{pmatrix} X_{45} \\ Y_{23} \\ Z_{25} \end{pmatrix} \begin{pmatrix} 0 & Z_{14}\rho_{53} & -Y_{64} \\ -Z_{14}\rho_{53}\Phi_{61}\Psi_{42} & 0 & X_{12}\Phi_{61} \\ Y_{64}\Psi_{42} & -X_{12} & 0 \end{pmatrix} \begin{pmatrix} X_{36} \\ Y_{31} \\ Z_{56} \end{pmatrix}.$$

note: no complex structure moduli dependence for toric del Pezzos in the superpotential

dP3 superpotential from global symmetries

$$W_{dP_3} = -X_{12}Y_{31}Z_{23} - X_{45}Y_{64}Z_{56} + X_{45}Y_{31}Z_{14}\rho_{53} + X_{12}Y_{25}Z_{56}\Phi_{61} \\ + X_{36}Y_{64}Z_{23}\Psi_{42} - X_{36}Y_{25}Z_{14}\rho_{53}\Phi_{61}\Psi_{42}$$

Fields	$SU(2) \times SU(3)$	$U(1)_R$
$(X_{36}, Y_{25}, Z_{14}, \Psi_{42}, \rho_{53}, \Phi_{61})$	$(2, 3)$	$1/3$
(X_{45}, Z_{56}, Y_{64})	$(1, \bar{3})$	$2/3$
(Z_{23}, Y_{31}, X_{12})	$(1, \bar{3})$	$2/3$

E_n global symmetry in dP_n : intersection of 2-cycles is Cartan matrix of E_n

dP3 superpotential from global symmetries

Franco, Hanany, Kazakopoulos 0404065

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E_n global symmetry in dP_n : intersection of 2-cycles is Cartan matrix of E_n

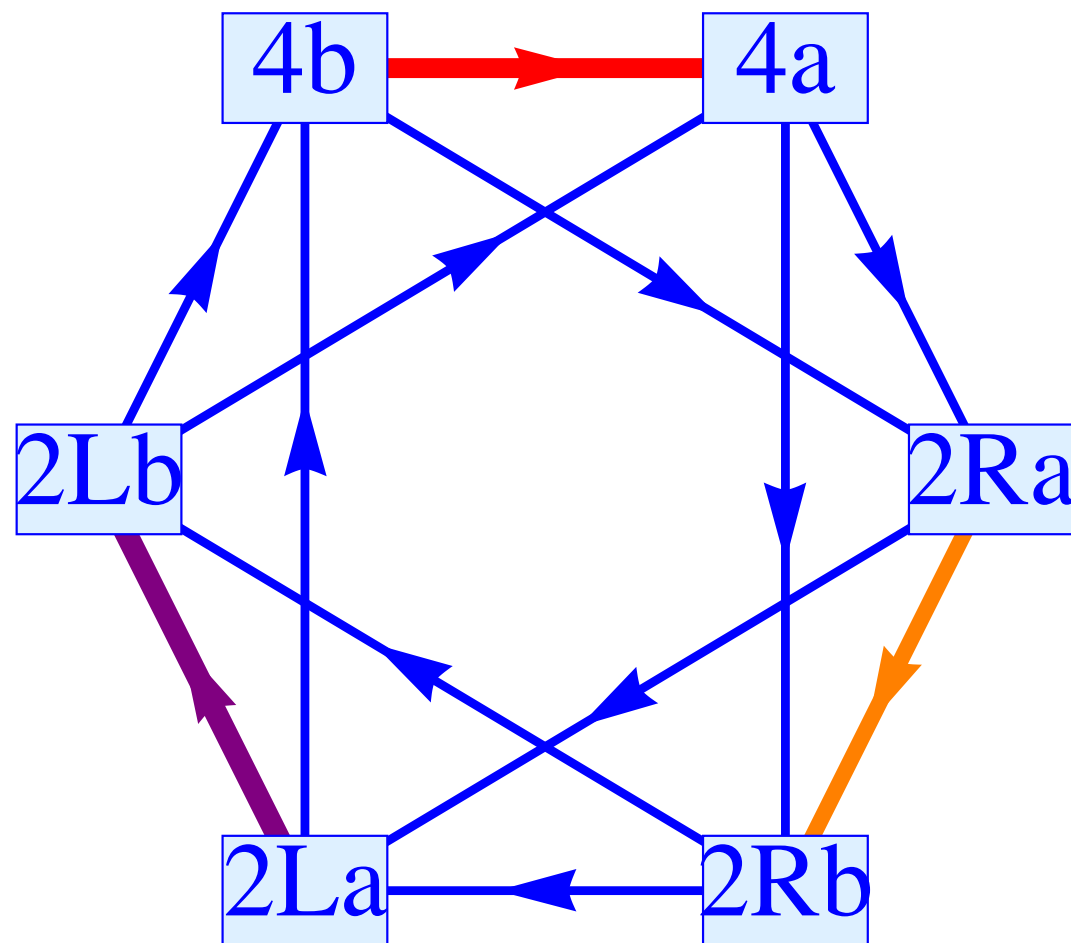
How to search for realistic Models?

- No simple GUT groups ($SU(5)$, $SO(10)$)
- Hypercharge massless (\rightarrow hypercharge should originate from non-abelian gauge group)
- All SM gauge groups/matter from D3s
 - ▶ This leads to Pati-Salam gauge groups

Minimal Del Pezzo?

- dP0 (no mass hierarchy unless non-commutative)
- dP1, dP2 no correct flavour structure (no control over kinetic terms)
- dP3 flavour diagonal kinetic terms and sufficient structure in mixing

the model of choice on dP3



$SU(4a) \times SU(4b)$
 $SU(2La) \times SU(2Lb)$
 $SU(2Ra) \times SU(2Rb)$

arrows correspond to
bi-fundamental matter

Properties of dP3 model

- Kähler potential flavour diagonal due to anomalous U(1) symmetries:

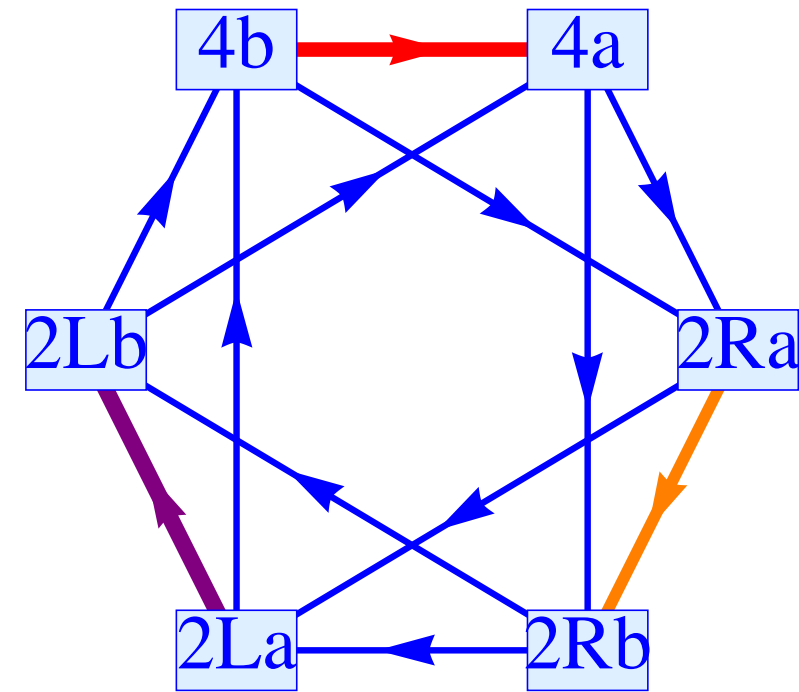
$$K_{\text{matter}} \supset \frac{a + f(\tau_s, \tau_b)}{\mathcal{V}^{2/3}} \left(Q_{L,R}^i \bar{Q}_{L,R}^i + H_i \bar{H}_i + \Phi_{61} \bar{\Phi}_{61} + \Psi_{42} \bar{\Psi}_{42} + \rho_{53} \bar{\rho}_{53} \right),$$

- All SM fields included + add. Higgses, needed for breaking

breaking fields
 $(\Phi_{61}, \Psi_{42}, \rho_{53})$

- The superpotential

$$W = \begin{pmatrix} Q_1^L \\ Q_2^L \\ Q_3^L \end{pmatrix} \begin{pmatrix} 0 & H_3 \frac{\rho_{53}}{\Lambda} & -H_2 \\ -H_3 \frac{\rho_{53} \Phi_{61} \Psi_{42}}{\Lambda^3} & 0 & H_1 \frac{\Phi_{61}}{\Lambda} \\ H_2 \frac{\Psi_{42}}{\Lambda} & -H_1 & 0 \end{pmatrix} \begin{pmatrix} Q_1^R \\ Q_2^R \\ Q_3^R \end{pmatrix},$$

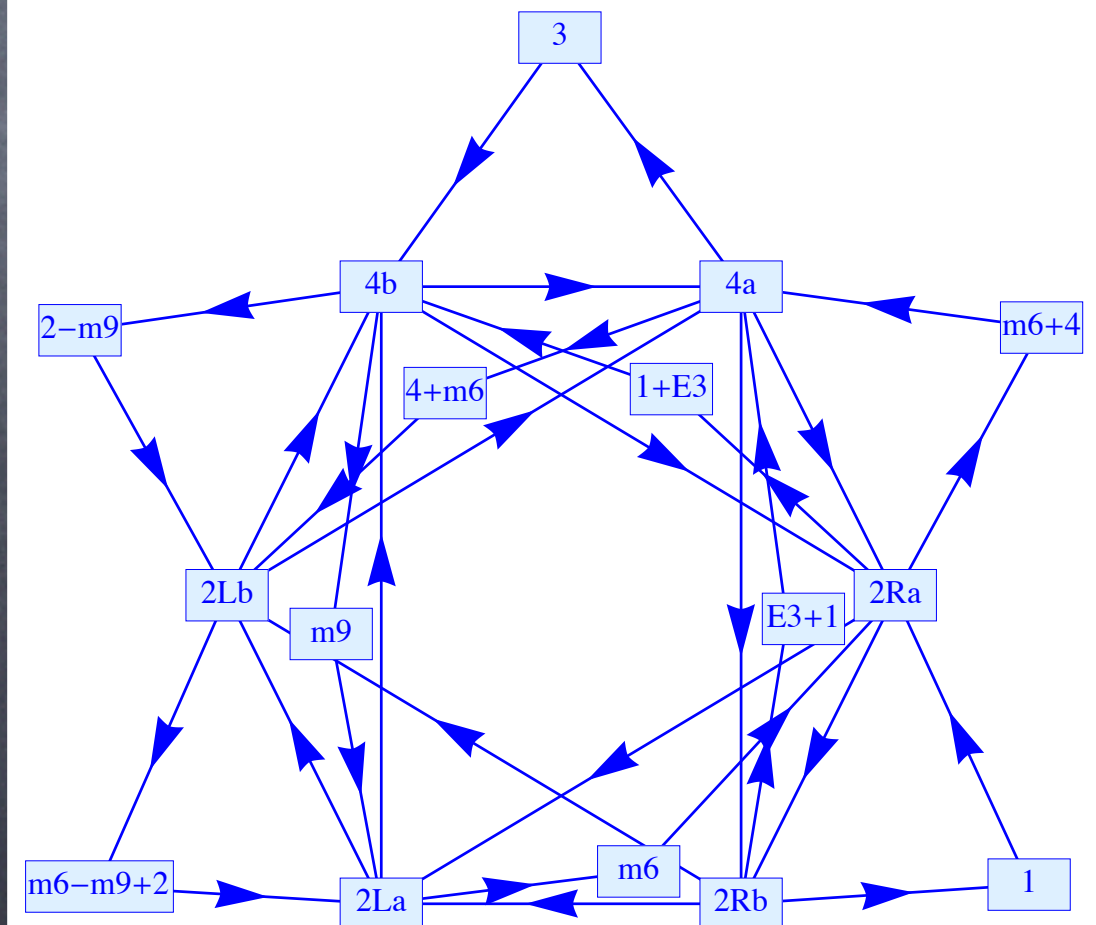
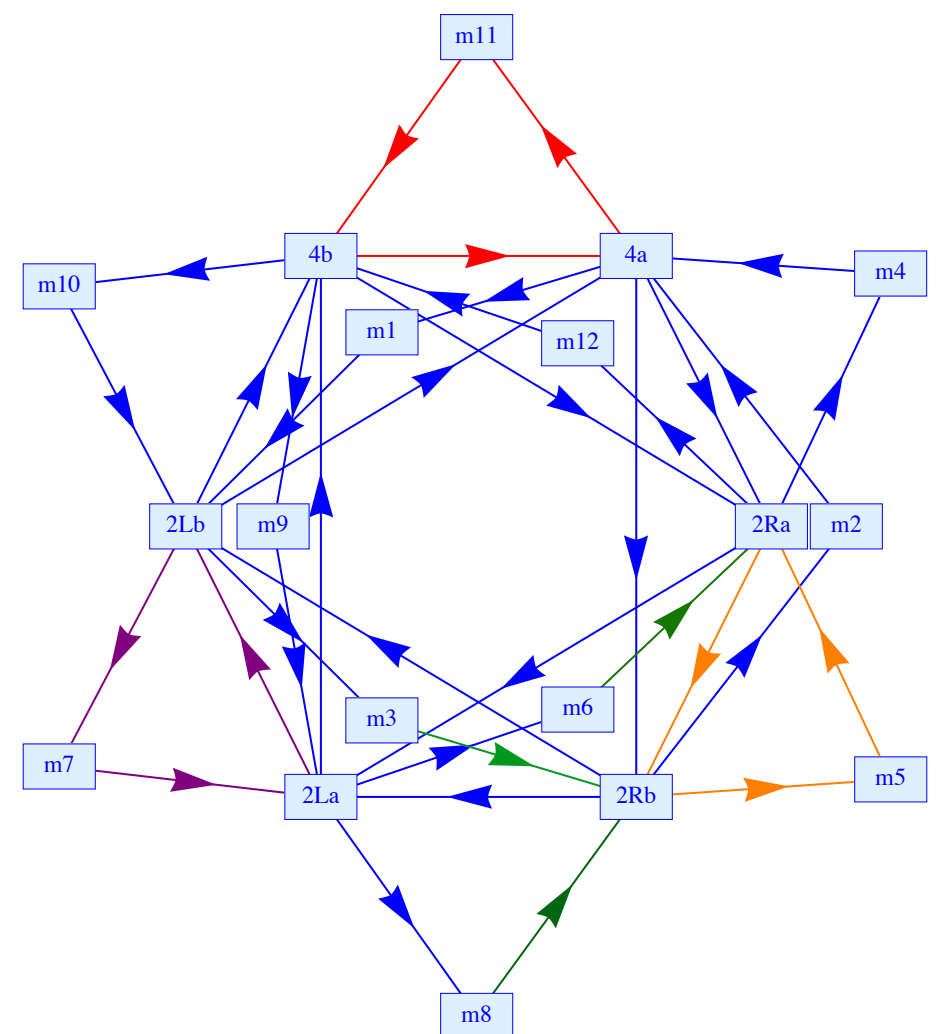


Check-List for dP3 Model

- Anomaly Cancellation
- μ -term
- Lepton Yukawas
- How to break to SM?
- Mass hierarchies
- Flavour Mixing
- Proton Stability
- Gauge coupling unification
- SUSY Breaking
- Radiative Corrections
- ...

Anomaly cancellation

- need to introduce D7 branes
- vast set of solutions is narrowed down by requirement of μ -term and Majorana neutrino masses
- 2 free D7 brane gauge groups
- assumption: large (string scale) masses for D7 branes \rightarrow absent in spectrum and running below string scale



Non-perturbative couplings

- Euclidean 3-branes intersecting with singularity (Ibanez, Uranga)

- ✓ Can generate μ -term

$$W_{\text{np}} = A e^{-a T_s} H.H$$

- ✗ Neutrino masses: heavily suppressed and break hypercharge

Breaking to the Standard Model

- how do the Standard Model gauge symmetries sit in this model?

$$SU(2L_a) \times SU(2L_b) \rightarrow SU(2)$$

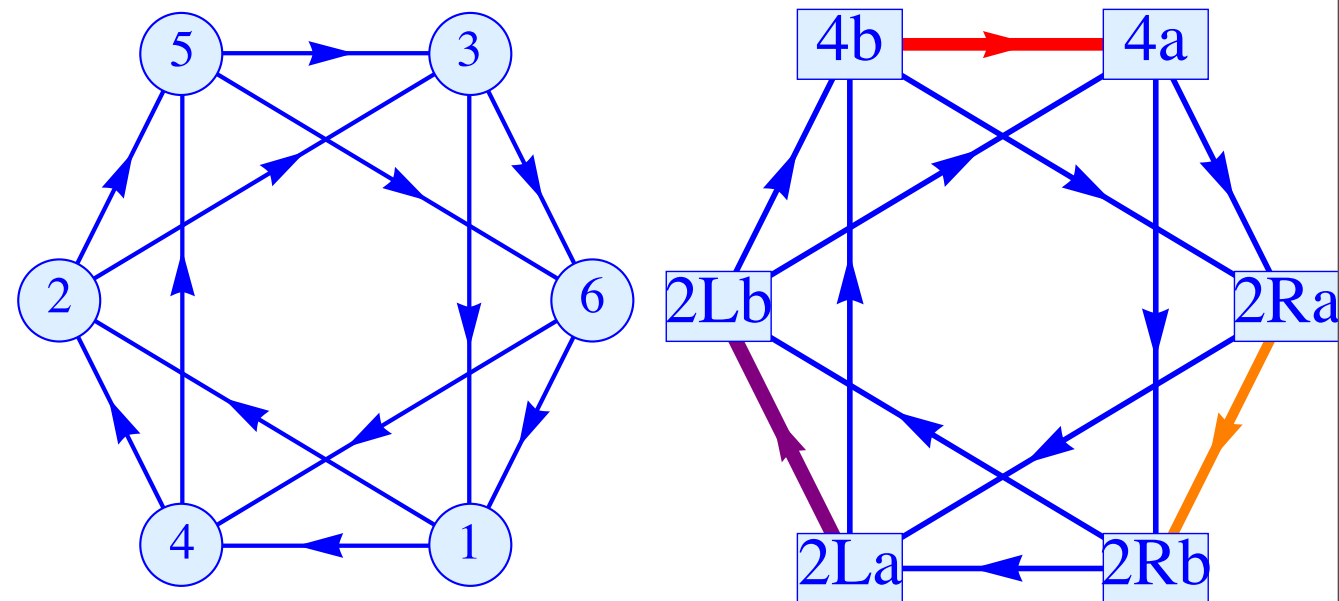
$$SU(4a) \times SU(4b) \rightarrow SU(3) \times U(1)_{B-L}$$

$$SU(2R_a) \times SU(2R_b) \rightarrow U(1)_2$$

- which fields can be used for breaking?

$$\langle \rho_{53} \rangle = \begin{pmatrix} v_1 & 0 & 0 & 0 \\ 0 & v_1 & 0 & 0 \\ 0 & 0 & v_1 & 0 \\ 0 & 0 & 0 & v_2 \end{pmatrix}.$$

$$\langle \Phi_{61} \rangle = \begin{pmatrix} \phi & 0 \\ 0 & \tilde{\phi} \end{pmatrix}, \quad \langle \Psi_{42} \rangle = \begin{pmatrix} \psi & 0 \\ 0 & \psi \end{pmatrix}.$$



total #	Fields	$SU(3)$	$SU(2)$	$U(1)_Y$
3	Q_1^L, Q_2^L, Q_3^L	$\bar{3}$	$\bar{2}$	a
3	u_1, u_2, u_3	$\bar{3}$	1	$-a + k$
3	d_1, d_2, d_3	$\bar{3}$	1	$-a - k$
3	L_1, L_2, L_3	1	$\bar{2}$	$-3a$
3	ν_1, ν_2, ν_3	1	1	$3a + k$
3	e_1, e_2, e_3	1	1	$3a - k$
3	H_1^u, H_2^u, H_3^u	1	2	$-k$
3	H_1^d, H_2^d, H_3^d	1	2	k

vev for right-handed sneutrino breaks completely to SM gauge symmetries + neutrino see-saw mechanism

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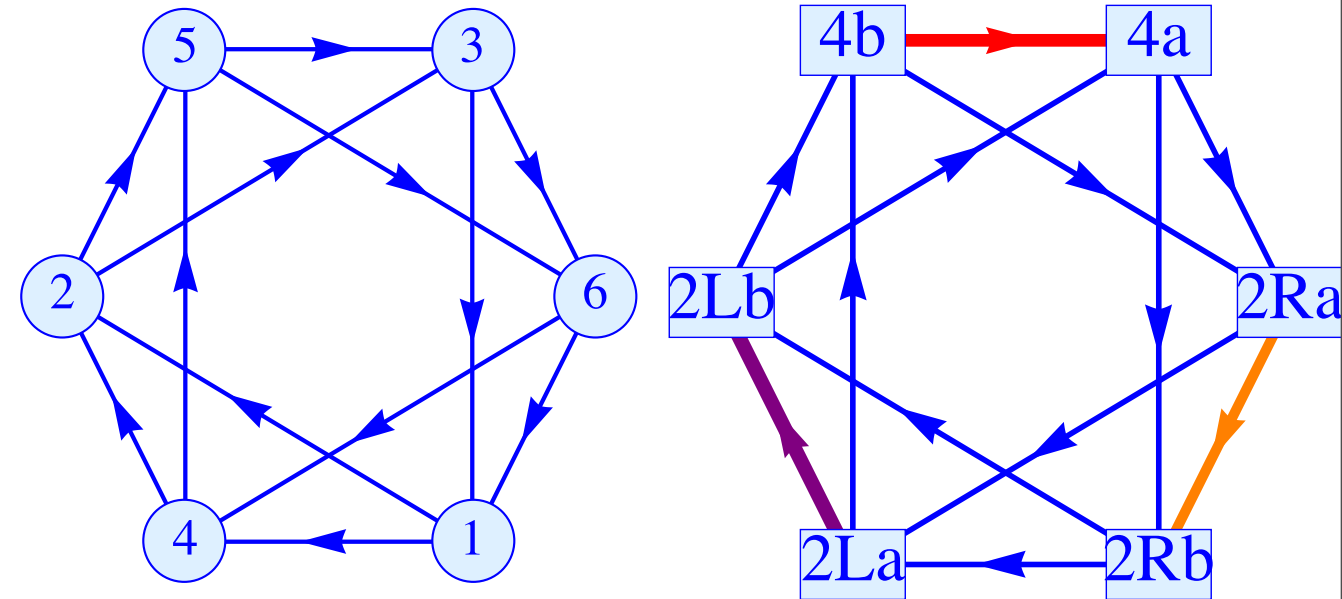
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3	L_1, L_2, L_3	1	$\bar{2}$	$-3a$
3	ν_1, ν_2, ν_3	1	1	$3a + k$
3	e_1, e_2, e_3	1	1	$3a - k$
3	H_1^u, H_2^u, H_3^u	1	2	$-k$
3	H_1^d, H_2^d, H_3^d	1	2	k

vev for right-handed sneutrino breaks completely to SM gauge symmetries + neutrino see-saw mechanism

Khalil, Masiero; Ambroso, Ovrut;
Fileviez-Perez, Spinner, ...

Induced R-parity violating operators
sufficiently suppressed by tuning $\frac{v_2}{\Lambda}$

$$W = W_{\text{MSSM}} - \frac{v_2 \langle \nu_2 \rangle}{\mu_z \Lambda} \frac{v_2 \tilde{\phi} \psi}{\Lambda^3} L_2 L_1 e_1 - \frac{v_2 \langle \nu_2 \rangle}{\mu_z \Lambda} \frac{v_1 \tilde{\phi} \psi}{\Lambda^3} Q_2^L L_1 d_1$$

Yukawa couplings 1

- To achieve hierarchy between up-quarks and neutrinos (same Yukawas), we need a see-saw mechanism.
- See-saw mechanism via vev for sneutrino

$$\begin{aligned}
 W = & \begin{pmatrix} Q_1^L \\ Q_2^L \\ Q_3^L \end{pmatrix} \begin{pmatrix} 0 & H_3^u \frac{v_1}{\Lambda} & -H_2^u \\ -H_3^u \frac{v_1 \phi \psi}{\Lambda^3} & 0 & H_1^u \frac{\phi}{\Lambda} \\ H_2^u \frac{\psi}{\Lambda} & -H_1^u & 0 \end{pmatrix} \begin{pmatrix} u_1^R \\ u_2^R \\ u_3^R \end{pmatrix} \\
 & + \begin{pmatrix} Q_1^L \\ Q_2^L \\ Q_3^L \end{pmatrix} \begin{pmatrix} 0 & H_3^d \frac{v_1}{\Lambda} & -H_2^d \\ -H_3^d \frac{v_1 \tilde{\phi} \tilde{\psi}}{\Lambda^3} & 0 & H_1^d \frac{\tilde{\phi}}{\Lambda} \\ H_2^d \frac{\tilde{\psi}}{\Lambda} & -H_1^d & 0 \end{pmatrix} \begin{pmatrix} d_1^R \\ d_2^R \\ d_3^R \end{pmatrix} \\
 & + \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} \begin{pmatrix} 0 & H_3^u \frac{v_2}{\Lambda} & -H_2^u \\ -H_3^u \frac{v_2 \phi \psi}{\Lambda^3} & 0 & H_1^u \frac{\phi}{\Lambda} \\ H_2^u \frac{\psi}{\Lambda} & -H_1^u & 0 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \\
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 & A_1 \nu_1^1 \bar{\nu}_1 + A_2 \nu_2 \bar{\nu}_2 + A_3 \nu_3 \bar{\nu}_3 + W_{D3D7}
 \end{aligned}$$

Hierarchical & distinct Masses

$$(m_u^i)^2 = \left(0, |H_1^u|^2 + |H_2^u|^2 \frac{|\Psi_{42}|^2}{\Lambda^2}, |H_2^u|^2 + |H_3^u|^2 \frac{|\rho_{53}^{(1)}|^2}{\Lambda^2} \right),$$

$$(m_d^i)^2 = \left(0, |H_2^d|^2 \frac{|\Psi_{42}|^2}{\Lambda^2}, |H_2^d|^2 + |H_3^d|^2 \frac{|\rho_{53}^{(1)}|^2}{\Lambda^2} \right),$$

$$m_{e_L}^2 = \left(0, |H_2^d|^2 \frac{|\Psi_{42}|^2}{\Lambda^2}, |H_2^d|^2 + |H_3^d|^2 \frac{|\rho_{53}^2|^2}{\Lambda^2} \right).$$

$$m_{\nu_R}^2 = (4|A_1|^2, 4|A_2|^2, 4|A_3|^2).$$

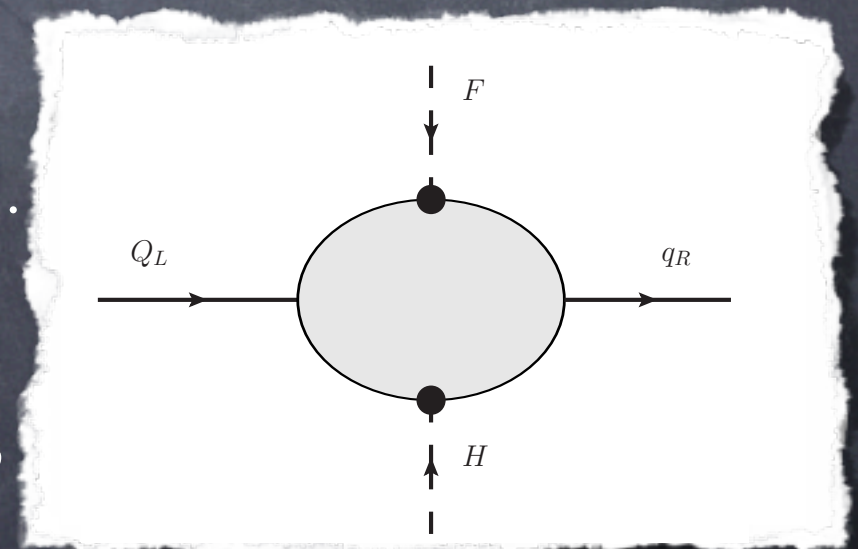
$$m_{\nu_L}^2 = \left(0, \left(\frac{2|H_1^u|^2}{|A_2|} + \frac{2|H_2^u|^2|\Psi_{42}|^2}{|A_1|\Lambda^2} \right)^2, \left(\frac{2|H_2^u|^2}{|A_3|} + \frac{2|H_3^u|^2|\rho_{53}^2|^2}{|A_2|\Lambda^2} \right)^2 + \frac{8|H_1^u|^2|H_3^u|^2|\rho_{53}^2|^2}{|A_2|\Lambda^2} \right),$$

Radiative Fermion Masses (1102.1973)

- How can we lift the zero mass eigenvalue for the lightest generation?
- As in field theory, can be traced back to global symmetry in local model
- Local isometries broken upon compactification (no global symmetries in theory of quantum gravity)
- Here breaking can arise from add. Higgses or flavour-violating soft-masses (in particular scalar masses contribution not suppressed (unlike Ibanez 1982 estimate) due to SUSY breaking structure in Large Volume Scenario)

$$\frac{1}{M_{\text{UV}}^2} CC^\dagger MM^\dagger \Big|_{\theta\theta\bar{\theta}\bar{\theta}} \sim \frac{1}{M_{\text{UV}}^2} CC^\dagger F_M \bar{F}_M \sim m_{3/2}^2 CC^\dagger .$$

$$\frac{1}{\tilde{M}^{2+n}} C_L C_R H M f(\Phi_{\text{hidden}}) \Big|_{\theta\theta\bar{\theta}\bar{\theta}} \sim M_{\text{weak}} C_L C_R \frac{F_M}{\tilde{M}^2} ,$$



Yukawa couplings 2

- CKM with the following vevs:

$$|V_{\text{CKM}}| = \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}.$$

$$\frac{H_1^u}{H_2^u} \sim \epsilon, \quad \frac{H_3^u v_1}{H_2^u \Lambda} \sim \epsilon, \quad \frac{\Phi_{61}^u}{\Lambda} \sim \epsilon^2 \text{ and } \frac{\Phi_{61}^d v_1 H_3^d}{\Lambda^2 H_2^d} \sim \epsilon$$

- PNMS, e.g. large mixing between first and second generation:

$$\frac{H_3^d \rho_{53} \Phi_{61}^d}{\Lambda^2 H_2^d} \quad \frac{v_1}{v_2} \sim \epsilon^2$$

- Get 21 (masses+angles) from 14 parameters:

$\langle \rho_{35} \rangle$	$\langle \Phi_{61} \rangle$	$\langle \Psi_{42} \rangle$	$\langle H_1^{u,d} \rangle$	$\langle H_2^{u,d} \rangle$	$\langle H_3^{u,d} \rangle$	A_i
2	2	1	2	2	2	3

Proton stability

- At the UV scale all perturbative operators are fixed by global symmetries $SU(3) \times SU(2) \times U(1)_R$ and anomalous $U(1)$ symmetries. Only these operators could lead perturbatively to proton decay. \rightarrow no dangerous operators, e.g. $Q_R Q_R Q_R Q_R$ forbidden by $U(1)$ s
- After symmetry breaking the operators breaking the symmetries were operators allowed by the UV theory.
- Anomalous $U(1)$ s can be broken non-perturbatively by D-instantons. However forbidding them is much easier than generating them (e.g. by placing some D7 branes on that cycle). Only a global model can give a definite answer. In this example higher suppression compared to μ -term:

$$\frac{f(\phi_{\text{hidden}})}{\Lambda} Q_R Q_R Q_R Q_R e^{-aT}$$

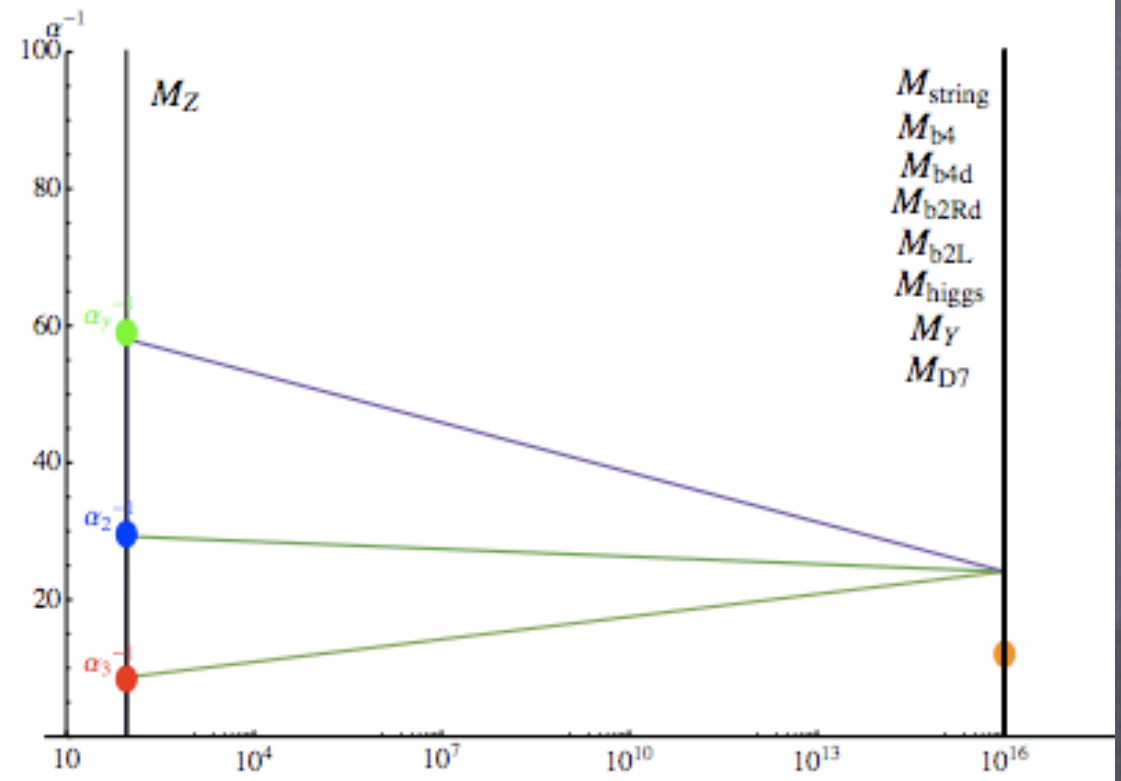
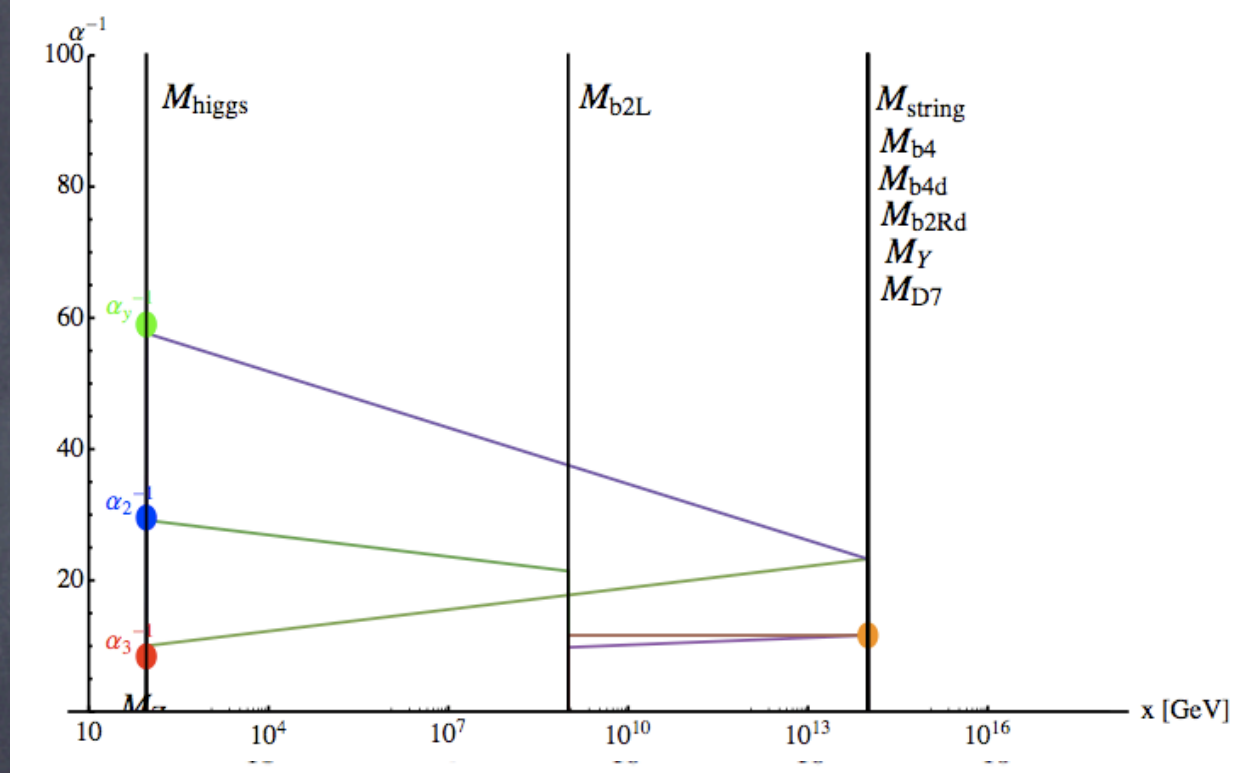
Typical Questions

- D-flatness, F-flatness (using D7-D3 states)
- FCNC from add. Higgses (bounds not very severe) but the Higgs sector needs to be studied more whether this can be achieved
- add. Higgses and unification

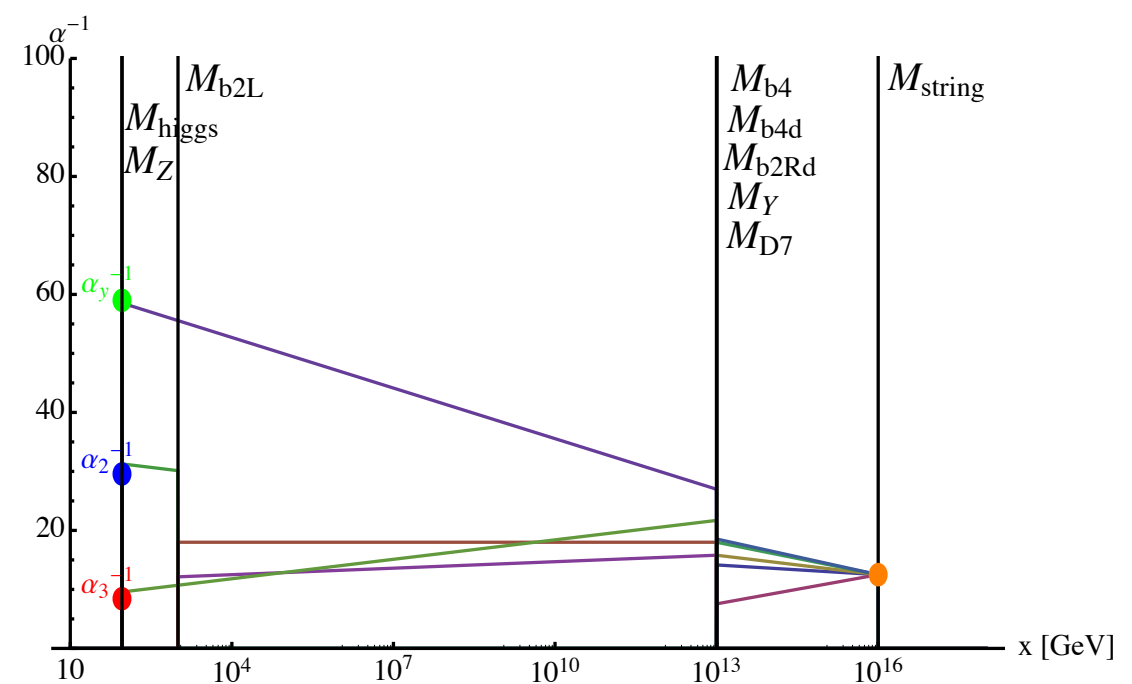
Gauge coupling unification

- Gauge coupling at UV scale given by dilaton.
- 1-loop running no threshold corrections, just interested in order of magnitude estimate.
- Hypercharge normalisation fixed to standard $5/3$. \rightarrow Standard GUT scale unification or $M_s > 10^{12}$ GeV (possible due to additional matter content)
- Q: Since we have gauge coupling unification are we consistent with low-energy gauge coupling data?
- Q: Which breaking/unification scales are possible?

Gauge Coupling Unification



Q: What fixes the scales? Running of soft-masses given some soft-masses should give you the answer.



Check-List for dP3 Model

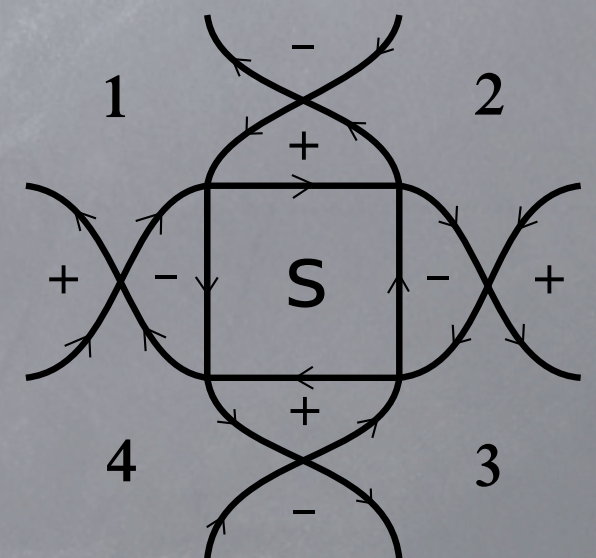
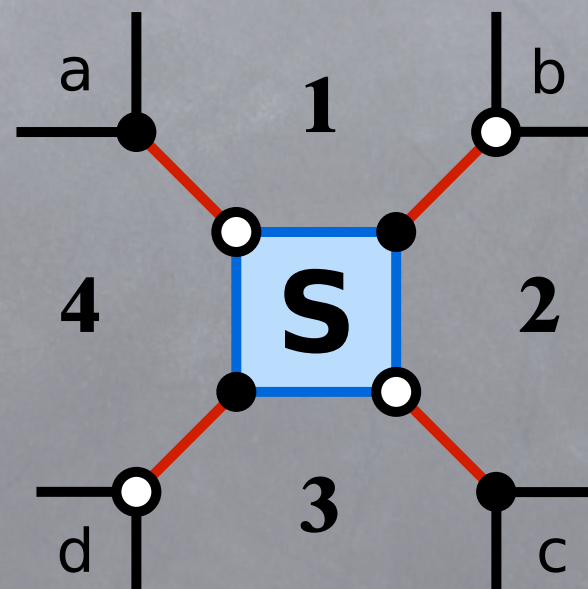
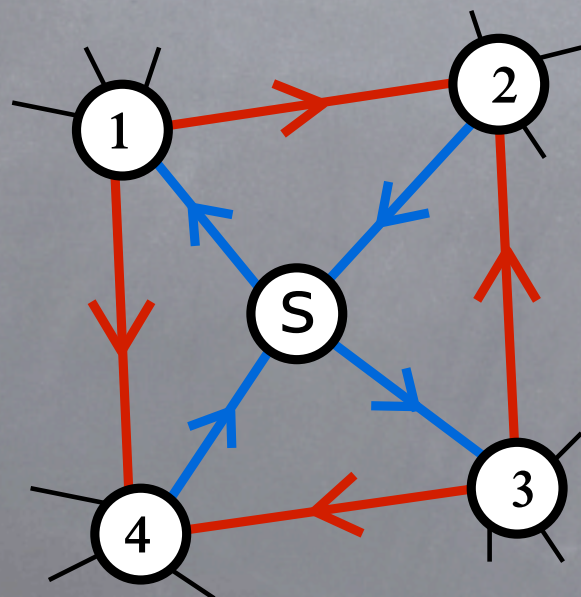
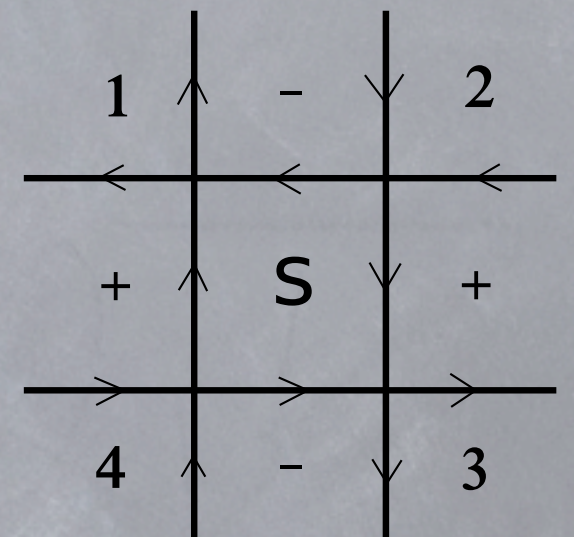
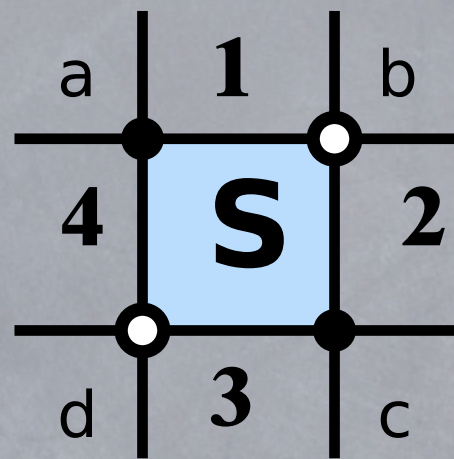
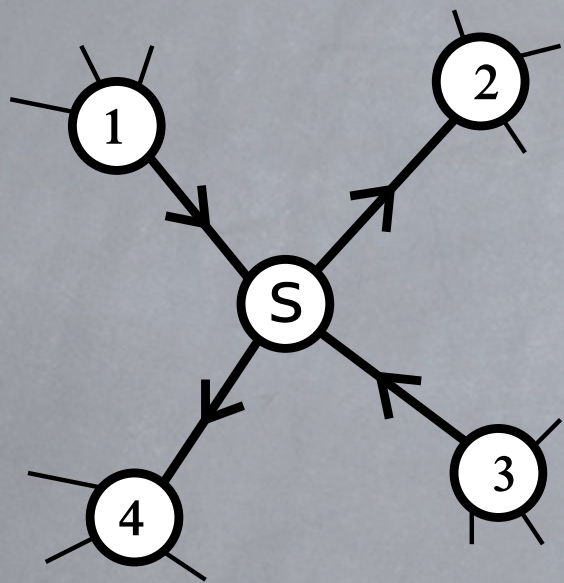
- ✓ Anomaly Cancellation
- ✓ μ -term
- ✓ Lepton Yukawas
- ✓ How to break to SM?
- ✓ Mass hierarchies
- ✓ Flavour Mixing
- ✓ Proton Stability
- ✓ Gauge coupling unification
- SUSY Breaking
- Radiative Corrections
- ...

Conclusions

- Progress in local model building with D-branes at toric (del Pezzo) singularities
[bound on families, mass hierarchies, CKM, PMNS mixing, proton stability, unification]
- Interesting phenomenological extensions of MSSM (Higgses, additional gauge symmetries)
- open questions: global embedding, dynamical assignment of vevs, sub-leading effects in SUSY breaking sector (\rightarrow flavour physics)

Thank you!

Seiberg duality in quivers and dimers



The zeroth Hirzebruch surface

