Discrete symmetries in GUTs and in the MSSM

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Bonn, November 7, 2011

Based on:

- H.M. Lee, S. Raby, G. Ross, M.R., R. Schieren & P. Vaudrevange, Phys. Lett. **B** 694, 491-495 (2011) & Nucl. Phys. **B** 850, 1-30 (2011)
- R. Kappl, B. Petersen, S. Raby, M.R., R. Schieren & P. Vaudrevange, Nucl. Phys. **B** 847, 325-349 (2011)
- M. Fallbacher, M.R. & P. Vaudrevange, Phys. Lett. **B** 705, 503-506 (2011)

MSSM: good features and open questions

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➥ Supersymmetry alone seems not to be enough

Disclaimer & apologies

The topic

> “Discrete symmetries in GUTs and in the MSSM”

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- ☹ CP and flavor problems

This ignores many interesting developments in model building:

- flavor symmetries:
  - discrete vs. continuous
  - VEV alignment
- CP violation
  - spontaneous? or geometric?
  - relation to baryon asymmetry?
  - . . .

Proton decay operators

- ☞ Gauge invariant superpotential terms up to order 4 include

$$\begin{aligned}\mathscr{W} &= \mu\, H_d H_u + \kappa_i\, L_i H_u \\ &\quad + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\ &\quad + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\ &\quad + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell\end{aligned}$$

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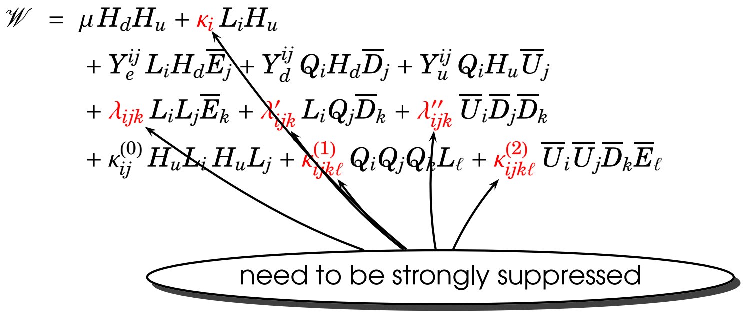

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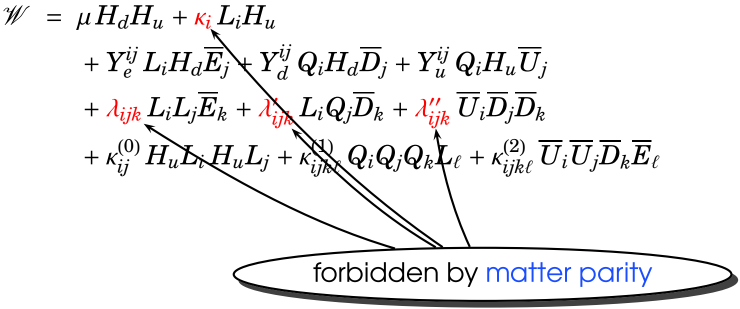

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forbidden by matter parity

Farrar & Fayet (1978); Dimopoulos, Raby & Wilczek (1981)

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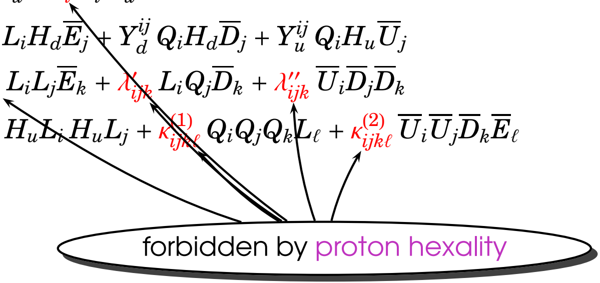

forbidden by proton hexality

Babu, Gogoladze & Wang (2002); Dreiner, Luhn & Thormeier (2006)

- Proton hexality = matter parity + baryon triality

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Proton hexality

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☞ Proton hexality P_6 = matter parity $\mathbb{Z}_2^{\mathcal{M}}$ × baryon triality B_3

	Q	U	D	L	E	H_u	H_d	$\overline{\nu}$
$\mathbb{Z}_2^{\mathcal{M}}$	1	1	1	1	1	0	0	1
B_3	0	-1	1	-1	2	1	-1	0
P_6	0	1	-1	-2	1	-1	1	3

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  - forbids dimension four and five proton decay operators
  - allows Yukawa couplings & Weinberg operator $\kappa_{ij}^{(0)} H_u L_i H_u L_j$
  - unique anomaly–free symmetry with the above features
    . . . with the common notion of anomaly freedom

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☞ However:

- ☹ not consistent with unification for matter (i.e. inconsistent with universal discrete charges for all matter fields)

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- ☹ embedding into string theory not yet fully convincing

Förste, Nilles, Ramos-Sánchez, Vaudrevange (2010)

Outline

1. Introduction & Motivation ✓
2. Anomaly–free discrete symmetries & grand unification
3. String theory completion
4. Summary

Anomaly–free

discrete symmetries

and

grand unification

- anomaly cancellation
- consistency with unification

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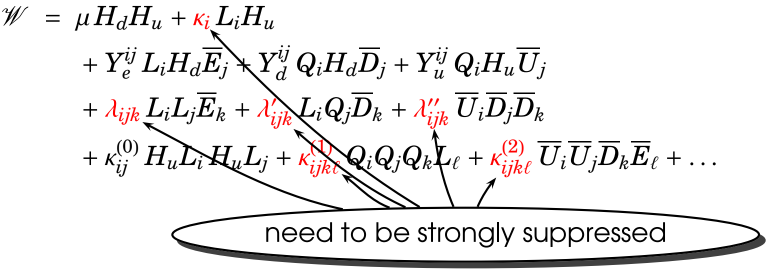

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need to be strongly suppressed

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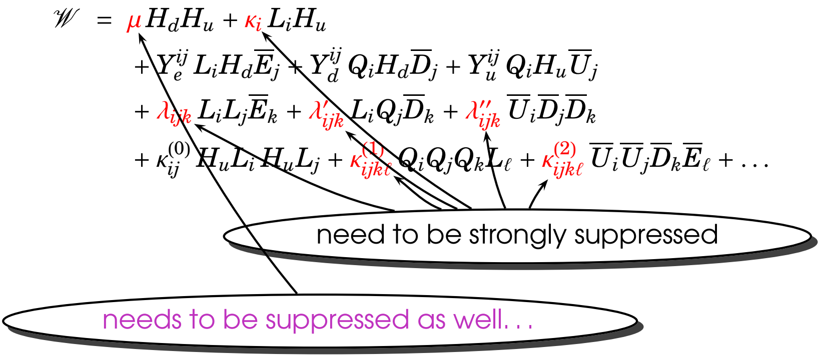

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Discrete anomaly cancellation

- ☞ Example: anomaly coefficients for $\mathbb{Z}_N$ symmetries

Ibáñez & Ross (1990)

$$A_{G-G-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

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sum over all representations of G

sum over all fermions

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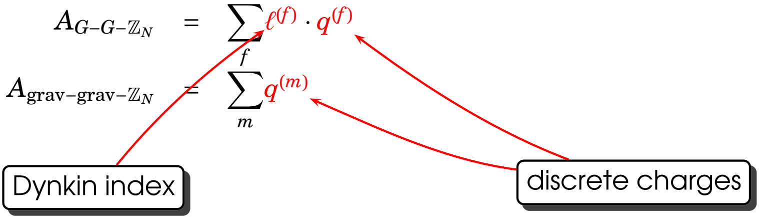

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Dynkin index

discrete charges

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traditional anomaly constraints:

all A coefficients vanish (mod η)

Discrete anomaly cancellation revisited

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main message:

▸ details

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☞ **Note**: discrete GS anomaly cancellation at work in many explicit string models

Anomaly–free symmetries, μ and unification

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(i) anomaly freedom (allow for GS anomaly cancellation)

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1. assuming (i) & $\mathrm{SU}(5)$ relations:
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Anomaly–free symmetries, μ and unification

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3. assuming (i)–(iii) & $\mathrm{SU}(5)$ relations:
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   only five discrete symmetries possible
4. R symmetries are not available in 4D GUTs

Claim 1: Non–R symmetries cannot forbid μ

H.M. Lee, S. Raby, M.R., G. Ross, R. Schieren, K. Schmidt–Hoberg, P. Vaudrevange (2011)

☞ Anomaly coefficients for non-R symmetry with SU(5) relations for matter charges

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \sum_{g=1}^{3}\left[\frac{3}{2}q^g_{\mathbf{10}}+\frac{1}{2}q^g_{\overline{\mathbf{5}}}\right]$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \sum_{g=1}^{3}\left[\frac{3}{2}q^g_{\mathbf{10}}+\frac{1}{2}q^g_{\overline{\mathbf{5}}}\right]+\frac{1}{2}\left(q_{H_u}+q_{H_d}\right)$$

sum over matter charges

Claim 1: Non–R symmetries cannot forbid μ

H.M. Lee, S. Raby, M.R., G. Ross, R. Schieren, K. Schmidt–Hoberg, P. Vaudrevange (2011)

- Anomaly coefficients for non-R symmetry with SU(5) relations for matter charges

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \sum_{g=1}^{3}\left[\frac{3}{2}q_{\mathbf{10}}^{g}+\frac{1}{2}q_{\overline{\mathbf{5}}}^{g}\right]$$

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- Anomaly universality: $A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = 0 \mod \eta$

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bottom-line:

non–R $\mathbb{Z}_N$ symmetry cannot forbid μ term

Claim 2: SO(10) implies unique symmetry

H.M. Lee, S. Raby, M.R., G. Ross, R. Schieren, K. Schmidt–Hoberg, P. Vaudrevange (2011)

☞ Assumption: quarks and leptons have universal charge q

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- Assumption: quarks and leptons have universal charge q
- u- and d-type Yukawas allowed requires that

$$2q + q_{H_u} = 2 \mod N \quad \text{and} \quad 2q + q_{H_d} = 2 \mod N$$

superpotential has R charge 2

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- first conclusion:

$$q_{H_u} = q_{H_d} = 0 \mod N$$

Claim 2: SO(10) implies unique symmetry (cont'd)

- ☞ Anomaly coefficients for Abelian discrete R symmetry

$$\begin{aligned} A_{\mathrm{SU}(3)^2-\mathbb{Z}_N^R} &= 6(q-1)+3 \;=\; 6q-3 \\ A_{\mathrm{SU}(2)^2-\mathbb{Z}_N^R} &= 6q+\frac{1}{2}\left(q_{H_u}+q_{H_d}\right)-5 \end{aligned}$$

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$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N^R} - A$$

$$\Leftrightarrow \quad q_{H_u} + q_{H_d} = 4 \mod \Bigg\{ \; N \quad \text{for } N \text{ even}$$

however: there is no meaningful $\mathbb{Z}_2^R$ symmetry

cf. e.g. Dine & Kehayias (2009)

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$N = 4$ unique

Unique $\mathbb{Z}_4^R$ symmetry

☞ We know:

- it is a $\mathbb{Z}_4^R$ symmetry
- Higgs fields have charge $q_{H_u} = q_{H_d} = 0 \mod 4$

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A_{\mathrm{SU}(3)^2-\mathbb{Z}_N^R} &= 6(q-1)+3 = 6q-3 = 1 \mod 4/2 \\
A_{\mathrm{SU}(2)^2-\mathbb{Z}_N^R} &= 6q + \frac{1}{2}\left(q_{H_u}+q_{H_d}\right) - 5 = 1 \mod 4/2 \\
A_{\mathrm{U}(1)_Y^2-\mathbb{Z}_N^R} &= 6q + \frac{3}{5}\cdot\frac{1}{2}\cdot\left(q_{H_u}+q_{H_d}-2\right)
\end{aligned}$$

$$\boxed{\text{e.g. } q_{H_u} = q_{H_d} = 16}$$

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gravitino contribution

gaugino contributions

$$A_{\mathrm{U}(1)_Y^2-\mathbb{Z}_N^R} = 6q + \frac{}{5}\cdot\frac{}{2}(q_{H_u} + q_{H_d} - 2) = 1 \mod 4/2$$

$$\frac{1}{24}A_{\mathrm{grav}^2-\mathbb{Z}_N^R} = \frac{1}{24}\left[-21 + 8 + 3 + 1 + 48(q-1) + 2(q_{H_u} + q_{H_d} - 2) - 1\right]$$

only defined $\mod 4$

axino contribution

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A $4/2$

$\frac{1}{24}A$

bottom-line:

- $\mathbb{Z}_4^R$ is anomaly free via GS mechanism
- GS axino contribution important for gravitational anomaly

Claim 3: only 5 symmetries obey SU(5) relations

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☞ Demanding SU(5) rather than SO(10) relations we find that the order N of possible $\mathbb{Z}_N^R$ symmetries has to divide 24

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- Demanding SU(5) rather than SO(10) relations we find that the order N of possible $\mathbb{Z}_N^R$ symmetries has to divide 24
- There are only five viable charge assignments

N	$q_{\mathbf{10}}$	$q_{\overline{\mathbf{5}}}$	q_{H_u}	q_{H_d}	ρ	$A_0^R(\mathrm{MSSM})$
4	1	1	0	0	1	1
6	5	3	4	0	0	1
8	1	5	0	4	1	3
12	5	9	4	0	3	1
24	5	9	16	12	9	7

gravitational anomaly

Recall

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} q^{(f)} \stackrel{!}{=} \rho \mod \eta$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum q^{(m)} \stackrel{!}{=} \rho \mod \eta$$

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- All $\mathbb{Z}_N^R$ symmetries can be obtained from $\mathbb{Z}_N^{R\,\prime} \times \mathrm{SO}(10)$ by spontaneous breaking

- N divides 24: hint at realization of $\mathbb{Z}_N^R$ as discrete rotational symmetry in orbifolds
(The geometry of orbifolds with $\mathcal{N} = 1$ SUSY is constrained that the order of discrete R symmetries also divides 24)

No–Go for R symmetries in 4D

M. Fallbacher, M.R., P. Vaudrevange (2011)

☞ Assumptions:

(i) GUT model in four dimensions based on $G \supset \mathrm{SU}(5)$

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☞ Will prove that it is impossible to get low–energy effective theory with both:

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1. just the MSSM field content
2. residual R symmetries

The basic argument

☞ Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6}$$

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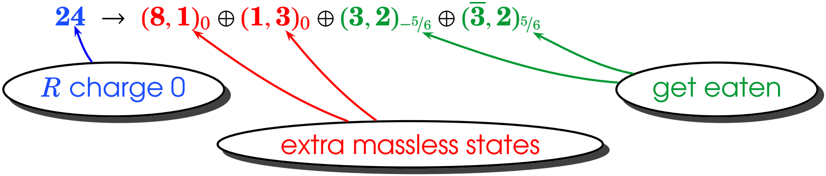

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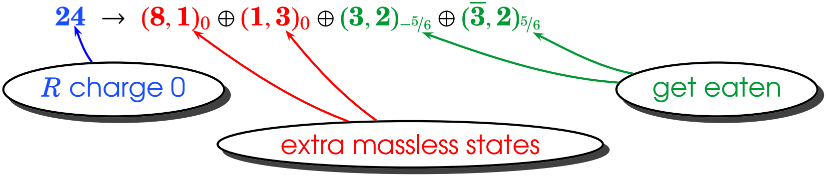

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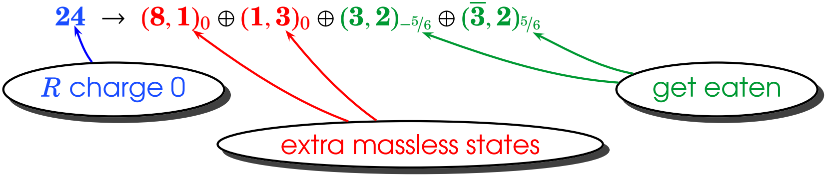

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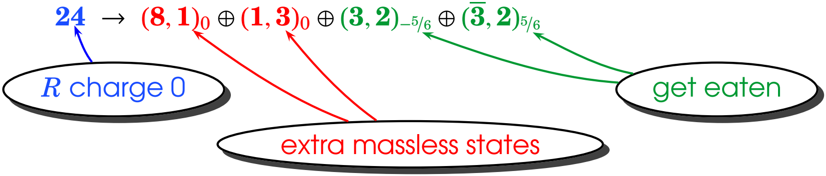

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- Loophole for infinitely many $\mathbf{24}$–plets

cf. Goodman & Witten (1986)

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bottom–line:

'Natural' solutions to the
μ and/or doublet–triplet splitting problems
are not available in four dimensions!

Higher–dimensional GUTs

and

string realization

- evading the no–go theorem
- origin of $\mathbb{Z}_4^R$
- higher–dimensional operators (effective μ term etc.)

Grand unification in higher dimensions

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Witten (1985); Breit, Ovrut & Segre (1985) . . . Kawamura (1999) . . .

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- ☞ **Remainder of this talk**: explicit string–derived example

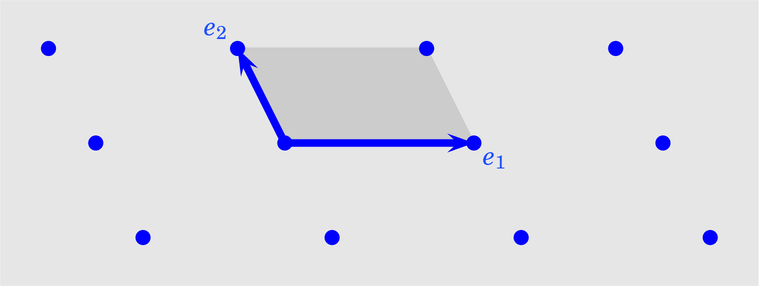
based on: M. Blaszczyk, S. Groot Nibbelink, M.R., F. Ruehle, M. Trapletti, P. Vaudrevange (2009)

R. Kappl, B. Petersen, S. Raby, M.R., R. Schieren & P. Vaudrevange (2011)

The $\mathbb{Z}_2$ orbifold plane

2D space with SO(2) rotational symmetry

The $\mathbb{Z}_2$ orbifold plane



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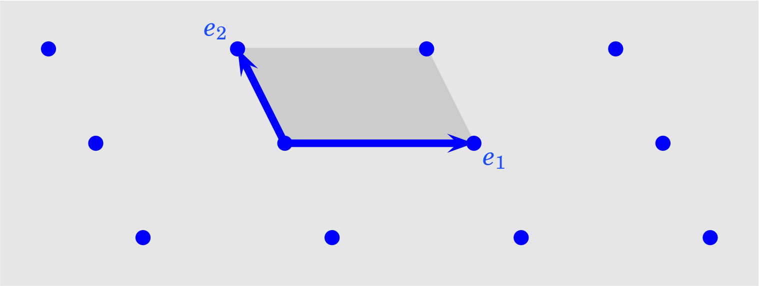

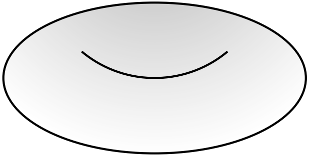

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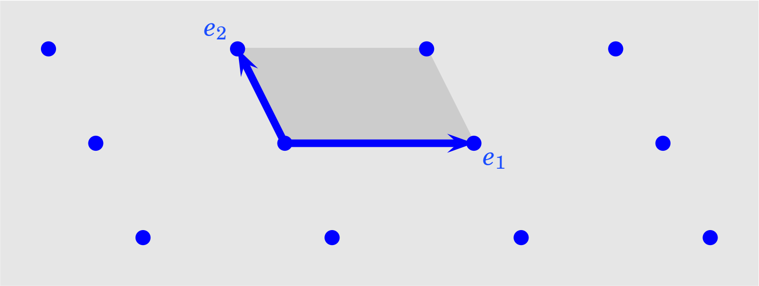

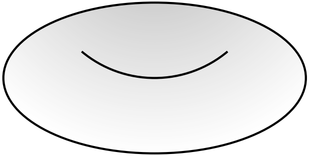

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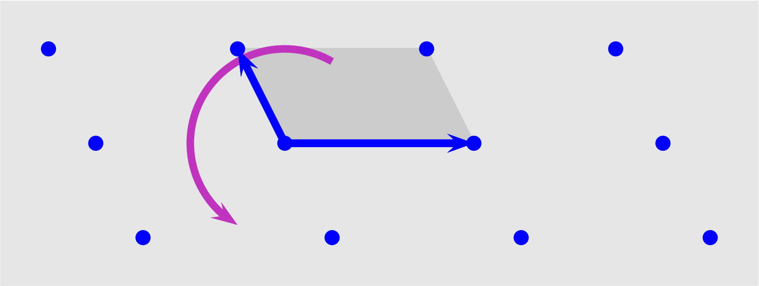

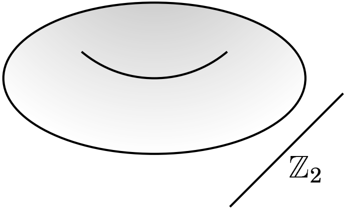

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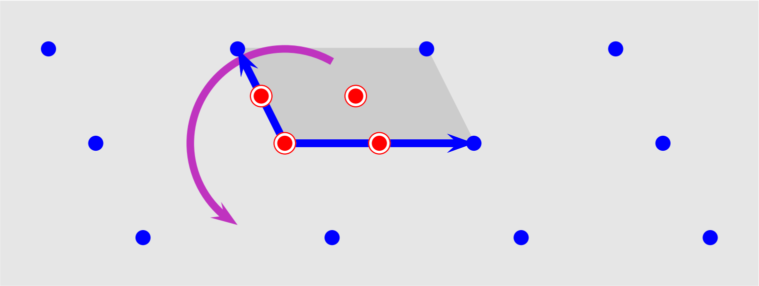

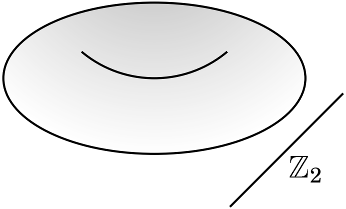

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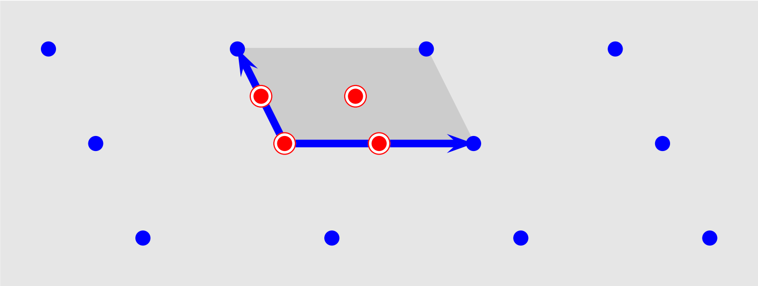
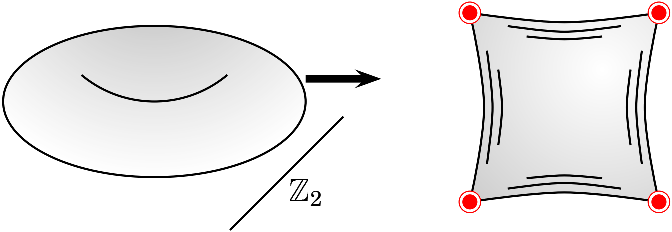
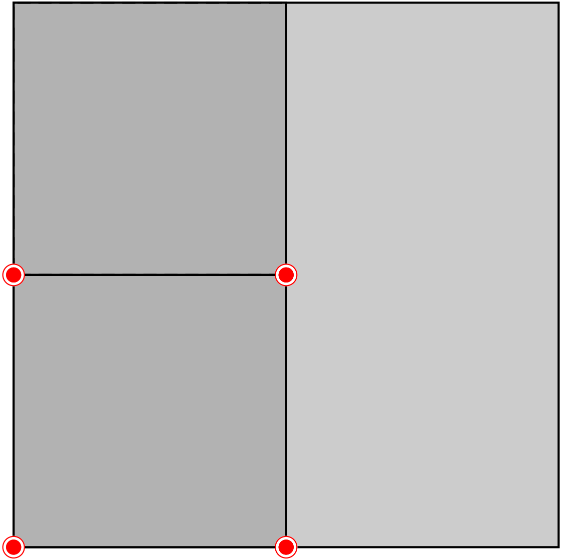




$\mathbb{Z}_2$ orbifold pillow



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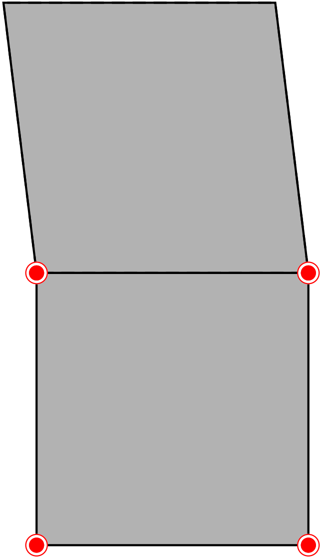

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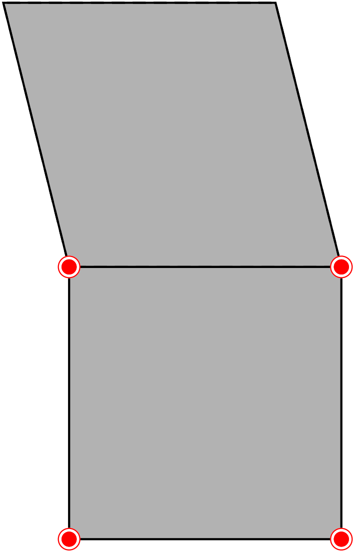

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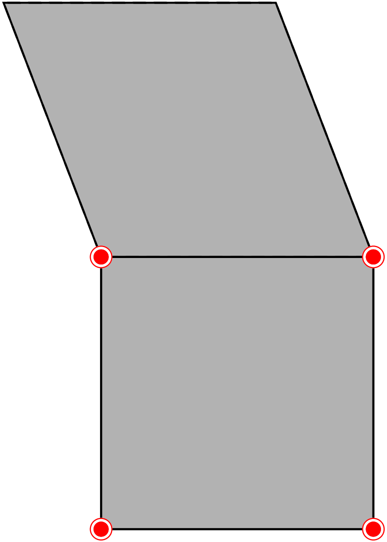

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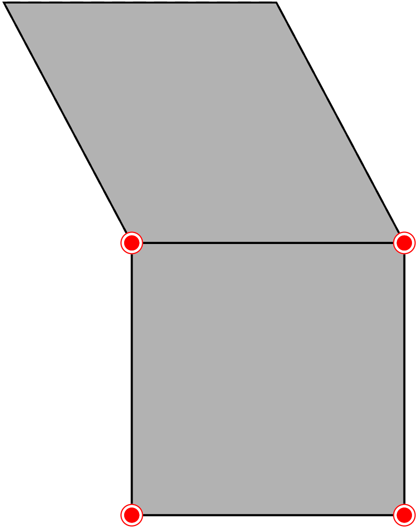

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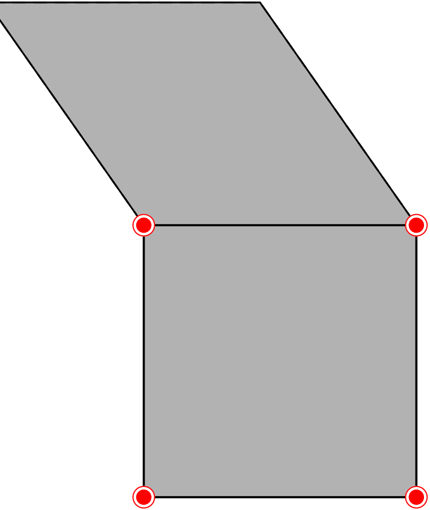

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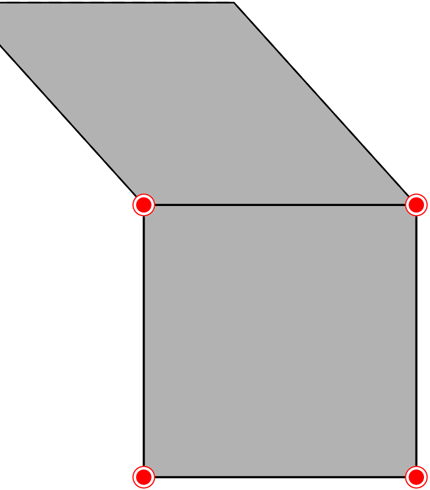

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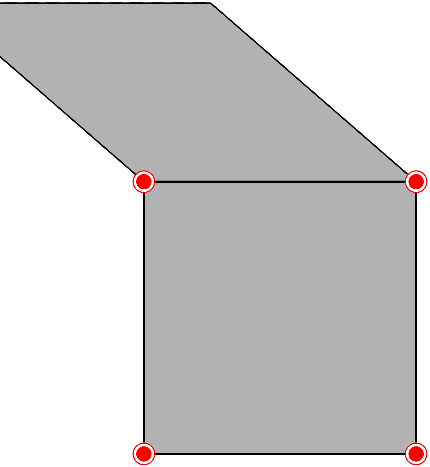

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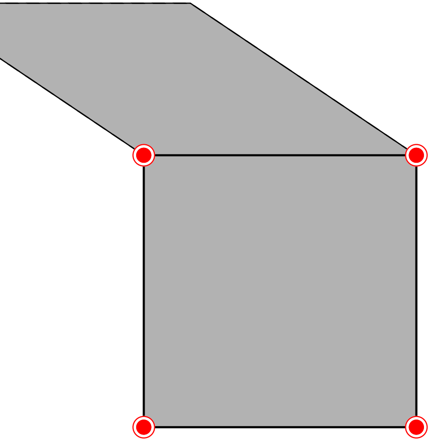

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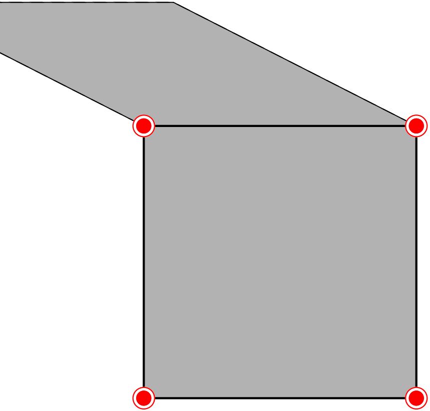

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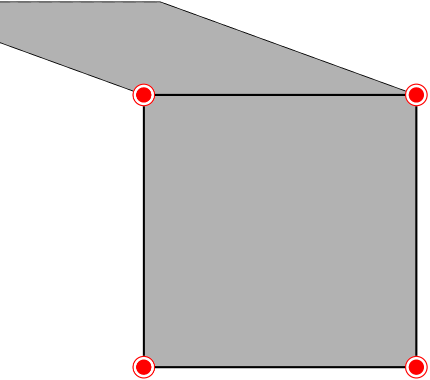

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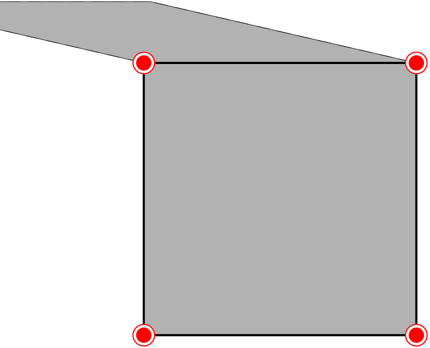

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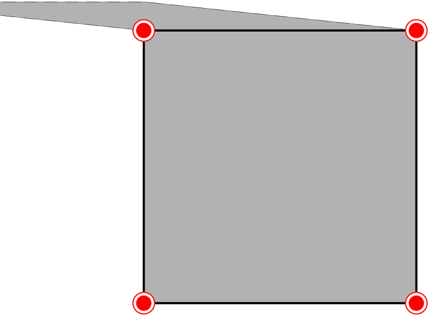

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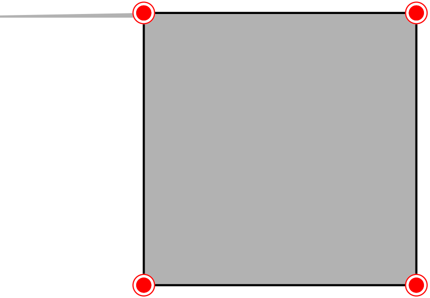

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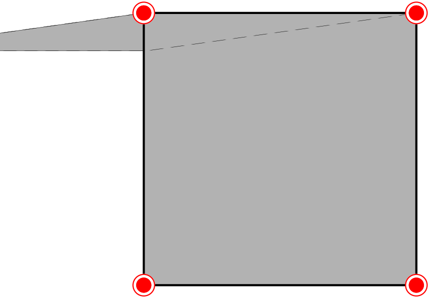

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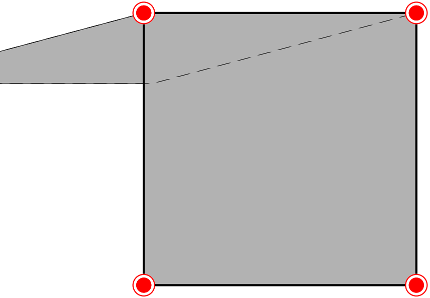

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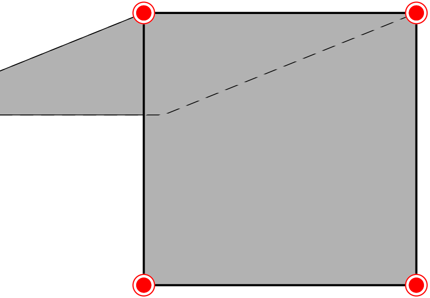

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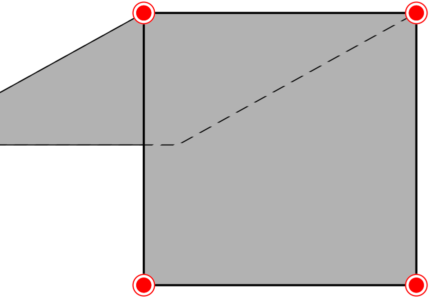

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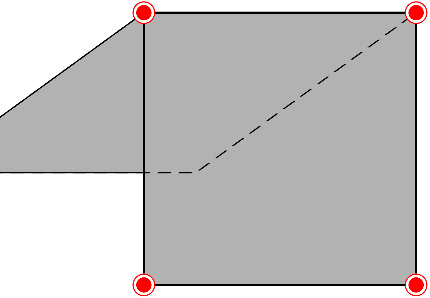

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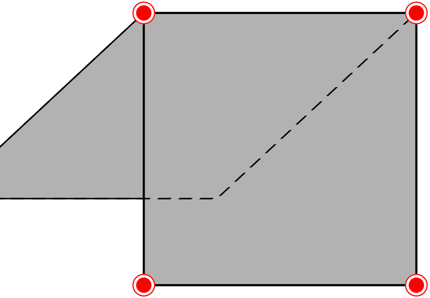

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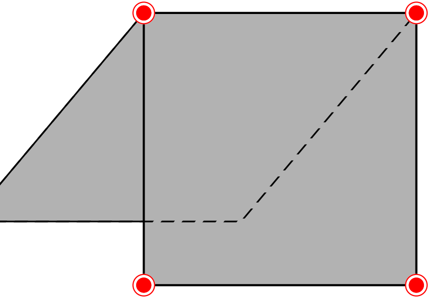

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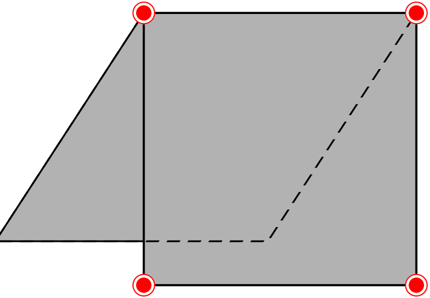

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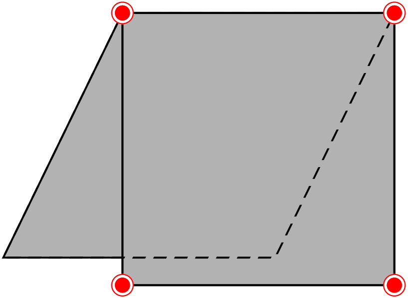

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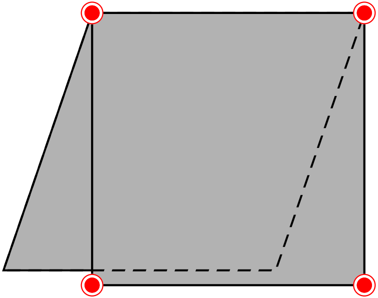

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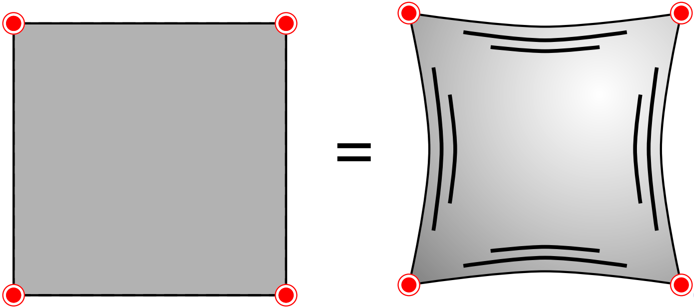
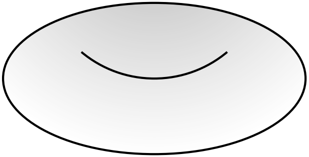


▸ back

$\mathbb{Z}_2$ orbifold plane & R symmetries

☞ **Crucial**: $\mathbb{Z}_4^R$ symmetry arises as a remnant of the Lorentz group in compact dimensions



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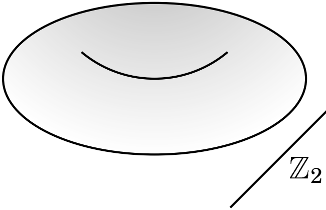

$\mathbb{Z}_2$ orbifold plane & R symmetries

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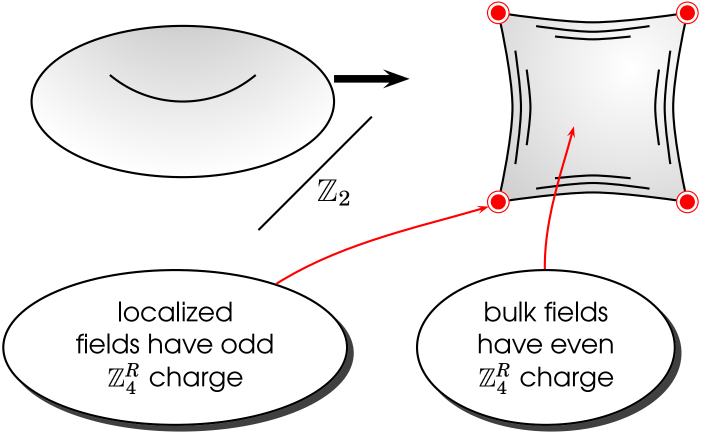


localized fields have odd $\mathbb{Z}_4^R$ charge

bulk fields have even $\mathbb{Z}_4^R$ charge

$\mathbb{Z}_2$ orbifold plane & R symmetries

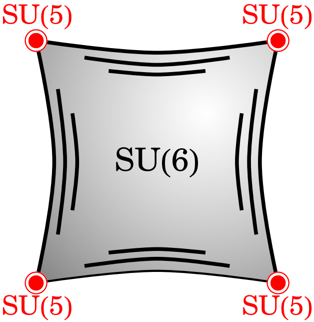
☞ **Crucial**: $\mathbb{Z}_4^R$ symmetry arises as a remnant of the Lorentz group in compact dimensions

➥ **Remainder of this talk**: discuss globally consistent string model with these features

more details on heterotic orbifolds will be provided in tomorrows talk by P. Vaudrevange

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold example

M. Blaszczyk, S. Groot Nibbelink, M.R., F. Ruehle, M. Trapletti, P. Vaudrevange (2009)



1. step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry

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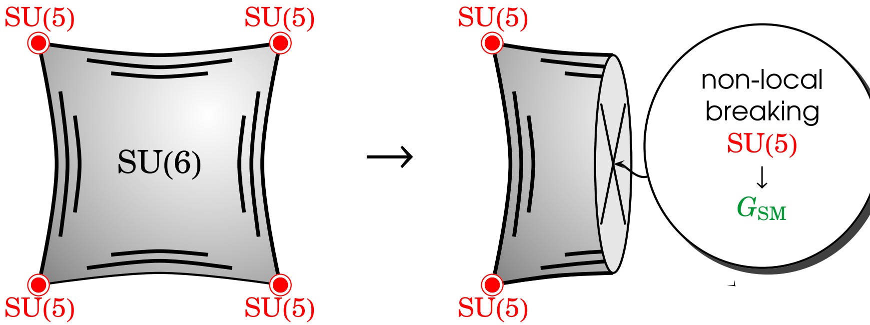

❶ step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry

❷ step: mod out a freely acting $\mathbb{Z}_2$ symmetry which:

- breaks $\mathrm{SU}(5) \rightarrow \mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$
- reduces the number of generations to 3

analogous mechanism in CY MSSMs Bouchard & Donagi (2005)
Braun, He, Ovrut, Pantev (2005)

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold example

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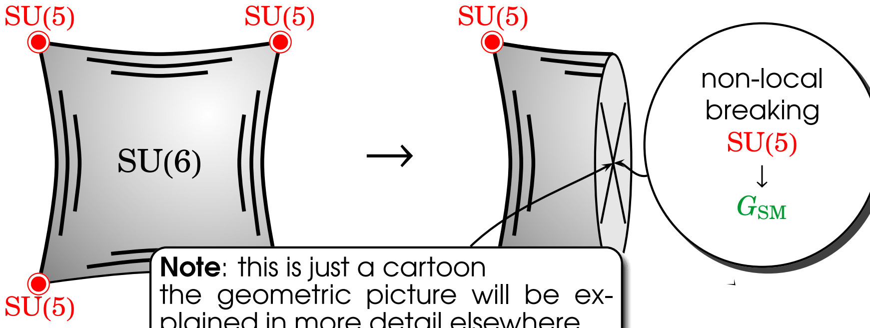

Note: this is just a cartoon
the geometric picture will be explained in more detail elsewhere

M. Fischer, M.R., P. Vaudrevange (to appear)

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Main features

❶ GUT symmetry breaking non-local
↷ no ‘logarithmic running above the GUT scale’

Hebecker, Trapletti (2004)

↷ precision gauge unification
with distinctive pattern of soft masses

Raby, M.R., Schmidt-Hoberg (2009)

Main features

1. GUT symmetry breaking non-local
2. No localized flux in hypercharge direction
   $\curvearrowright$ complete blow-up without breaking SM gauge symmetry in principle possible

Main features

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2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{B-L} \times [\mathrm{SU}(3) \times \mathrm{SU}(2)^2 \times \mathrm{U}(1)^7]$

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❹ massless spectrum

$$\text{spectrum} = 3 \times \text{generation} + \text{vector-like}$$

❺ Various appealing features:

- vacua where exotics decouple at the linear level in SM singlets
- non-trivial Yukawa couplings
- gauge-top unification
- SU(5) relation $y_\tau \simeq y_b$ (but also for light generations)

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ model

☞ We succeeded in finding vacua with the $\mathbb{Z}_4^R$

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- ✓ F- and D-flatness explicitly verified
- ✓ exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$
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➥ Successful string embedding of $\mathbb{Z}_4^R$ possible!

SUSY vacua with $\mathbb{Z}_4^R$

- ☞ Recall: situation for gauge theories with generic superpotential

e.g. Luty & Taylor (1995)

solutions of D-equations $\cap$ solutions of F-equations $=$ non-trivial

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- ☞ Example: consider one field ϕ_0 with R-charge 0 and one field ϕ_2 with R-charge 2

$$\mathscr{W} \;=\; \phi_2 \cdot f(\phi_0) + \mathcal{O}({\phi_2}^2) \qquad \text{with} \quad \langle \mathscr{W} \rangle \;=\; 0 \text{ automatic}$$

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$$F_{\phi_0} = \frac{\partial \mathscr{W}}{\partial \phi_0} = \phi_2 \cdot f'(\phi_0) + \mathcal{O}({\phi_2}^2) = 0 \quad @\ \phi_2 = 0$$

$$F_{\phi_2} = \frac{\partial \mathscr{W}}{\partial \phi_2} = f(\phi_0) \overset{!}{=} 0 \quad \text{fixes } \phi_0$$

SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

☞ Generalization: consider N fields $\phi_0^{(i)}$ with R-charge 0 and M fields $\phi_2^{(j)}$ with R-charge 2

SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

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- ☞ Have identified configurations with $N \geq M$ in our $\mathbb{Z}_2 \times \mathbb{Z}_2$ model(s)

$\mathbb{Z}_4^R$ phenomenology

- ☞ Gauge invariant superpotential terms up to order 4

$$\begin{aligned}\mathscr{W} &= \mu\, H_d H_u + \kappa_i\, L_i H_u \\ &\quad + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\ &\quad + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\ &\quad + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \ldots\end{aligned}$$

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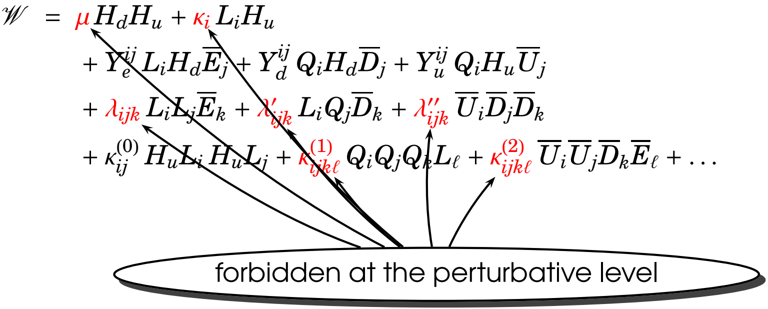

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forbidden at the perturbative level

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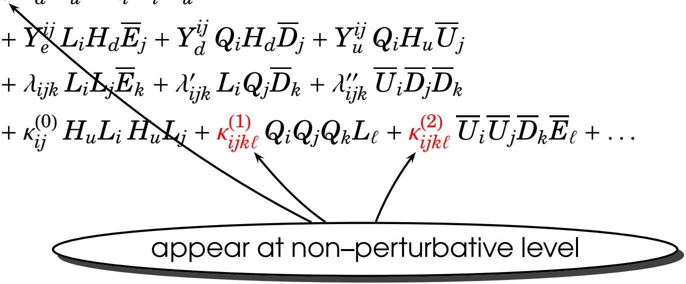

appear at non–perturbative level

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also forbidden at
non-perturbative level by
non-anomalous $\mathbb{Z}_2$ subgroup
which is equivalent
to matter parity

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non-perturbative generation of μ solves the μ problem

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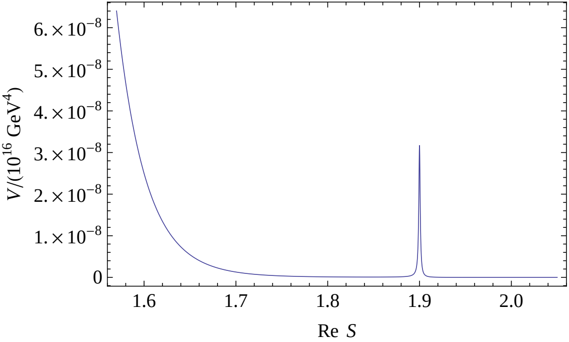
non-perturbatively generated terms harmless

Minimal realization of $\mathbb{Z}_4^R$

☞ MSSM + Kähler stabilized dilaton



- non–perturbative corrections to the Kähler potential lead to a bump in the potential of $\mathrm{Re}\,S$
- $\mathrm{Im}\,S$ has a flat potential ↷ GS axion remains light

Minimal realization of $\mathbb{Z}_4^R$

- ☞ MSSM + Kähler stabilized dilaton
- ☞ Non–perturbative superpotential

$$\mathscr{W}_{\mathrm{np}} \supset M_{\mathrm{P}}^3\,\mathrm{e}^{-b\,S}$$

is $\mathbb{Z}_4^R$ covariant (i.e. has R charge 2) as $S \to S + \frac{\mathrm{i}}{2}\Delta_{\mathrm{GS}}$

- ☞ Comments:
  - Of course $\mathscr{W}_{\mathrm{np}}$ is just the effective description of some hidden sector strong dynamics
  - $\mathbb{Z}_4^R$ anomaly universality leads to non–trivial constraints on the (β-function) coefficient b
  - discrete shift of dilaton not uniquely fixed:

$$4\,\pi\,\Delta_{\mathrm{GS}} \equiv \frac{1}{24}A_{\text{grav}-\text{grav}-\mathbb{Z}_4^R} = A_{G-G-\mathbb{Z}_4^R} \mod 2$$

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- ☞ Effective μ term and $QQQL$ coefficients

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  are also $\mathbb{Z}_4^R$ covariant

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$$\mathscr{W}_{\text{eff}} \supset \underbrace{\mathcal{O}(m_{3/2})}_{\overset{!}{\sim}\, m_{\text{soft}}} H_d H_u + \underbrace{\frac{\mathcal{O}(m_{3/2})}{M_{\text{P}}^2}}_{\overset{!}{\lesssim}\, \frac{10^{-8}}{M_{\text{P}}}} Q_i Q_j Q_k L_\ell + \ldots$$

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- ➥ General singlet extension of the MSSM w/ $m_N \sim m_{3/2}$ (no domain wall/tadpole problems)

Summary

&

outlook

Summary

☞ Assumptions:

(i) anomaly freedom (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

(iv) $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT relations for quarks and leptons

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☞ Have shown:

1. assuming (i) & $\mathrm{SU}(5)$ relations:
   $\curvearrowright$ only R symmetries can forbid the μ term
2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
   $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry
3. assuming (i)–(iii) & $\mathrm{SU}(5)$ relations:
   $\curvearrowright$ only five discrete symmetries possible
4. R symmetries are not available in 4D GUTs
   $\curvearrowright$ no 'natural' solution to doublet–triplet splitting in 4D!

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☞ A simple ‘anomalous’ $\mathbb{Z}_4^R$ symmetry can

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$$\mathbb{Z}_4^R \curvearrowright \left\{\begin{array}{l}\text{dim. 4 proton decay operators completely forbidden}\\ \text{dim. 5 proton decay operators highly suppressed}\\ \mu \text{ appears non-perturbatively}\end{array}\right.$$

Summary

☞ Embedding into string theory allows us to understand where the $\mathbb{Z}_4^R$ symmetry comes from: it may arise as a discrete remnant of Lorentz symmetry in extra dimensions

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Summary & outlook

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- ☞ (Unique) $\mathbb{Z}_4^R$ will also be available in other constructions (F-theory, D-branes, . . .)

Vielen Dank!

Green–Schwarz anomaly cancellation

- ☞ Under ‘anomalous’ $\mathrm{U}(1)$ symmetry the path integral measure exhibits non-trivial transformation Fujikawa (1979)

$\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi} \;\rightarrow\; J(\alpha)\,\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}$ with non-trivial $J(\alpha)$

Green–Schwarz anomaly cancellation

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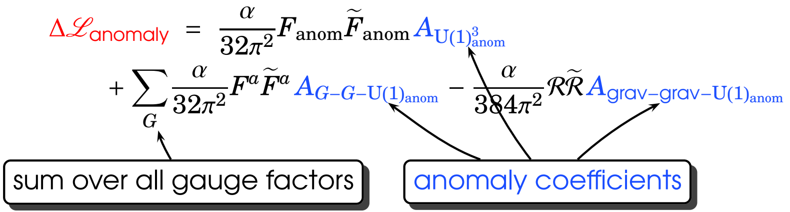
Fujikawa (1979)

- One can absorb the change of the path integral measure in a change of Lagrangean



$$\Delta\mathscr{L}_{\text{anomaly}} = \frac{\alpha}{32\pi^2} F_{\text{anom}} \widetilde{F}_{\text{anom}}\, A_{\mathrm{U}(1)^3_{\text{anom}}} + \sum_G \frac{\alpha}{32\pi^2} F^a \widetilde{F}^a\, A_{G-G-\mathrm{U}(1)_{\text{anom}}} - \frac{\alpha}{384\pi^2} \mathcal{R}\widetilde{\mathcal{R}}\, A_{\text{grav}-\text{grav}-\mathrm{U}(1)_{\text{anom}}}$$

sum over all gauge factors

anomaly coefficients

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- ☞ Provided the Lagrangean also includes axion couplings

$$\mathscr{L} \supset -\frac{a}{8} F_{\mathrm{anom}} \widetilde{F}_{\mathrm{anom}} - \frac{a}{8} F^a \widetilde{F}^a + \frac{a}{4} \mathcal{R}\widetilde{\mathcal{R}}$$

$\Delta\mathscr{L}_{\text{anomaly}}$ can be compensated by a shift of the axion a if the anomaly coefficients are universal

Green & Schwarz (1984)

Discrete GS anomaly cancellation

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Discrete GS anomaly cancellation

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- ☞ Specifically for a $\mathbb{Z}_N$ transformation

$$\Phi^{(f)} \;\to\; \mathrm{e}^{-\mathrm{i}\,\frac{2\pi}{N}\,q^{(f)}}\,\Phi^{(f)}$$

the dilaton (containing the axion) has to transform as

$$S \;\to\; S + \frac{\mathrm{i}}{2}\Delta_{\mathrm{GS}}$$

where

$$\pi N\,\Delta_{\mathrm{GS}} \;\equiv\; \frac{1}{24}A_{\mathrm{grav-grav-}\mathbb{Z}_N} \;=\; A_{G-G-\mathbb{Z}_N} \mod \eta \quad \forall\, G$$

back

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  - no discussion of mixed hypercharge nor gravitational anomalies