Whither† SUSY?

G. Ross, Bethe Forum, Bonn, November 2011

† **whither** *Archaic or poetic*
adv
1. to what place?
2. to what end or purpose?
conj
to whatever place, purpose, etc.
[Old English *hwider, hwæder;* related to Gothic *hvadrē;* modern English form influenced by HITHER]

Little hierarchy problem ⇒ definite SUSY structure

Fine Tuning measure:

$$\Delta(a_i) = \left| \frac{a_i}{M_Z} \frac{\partial M_Z}{\partial a_i} \right|,$$

$$\Delta_{\max} = Max_{a_i} \Delta(a_i)$$

Ellis, Enquist, Nanopoulos, Zwirner

Barbieri, Giudice

MSSM: 105 +(19) Parameters

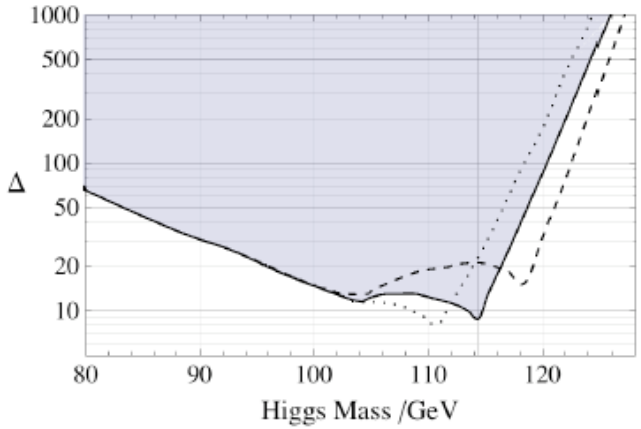
$$M_Z^2 = \sum_{\tilde{q},\tilde{l}} a_i \tilde{m}_i^2 + \sum_{\tilde{g},\tilde{W},\tilde{B}} \tilde{M}_i^2 + \ldots$$

$$m_{\tilde{q}} > 0.6 - 1 TeV \quad \Rightarrow \quad \Delta > a \frac{\tilde{m}^2}{M_Z^2} \sim 100$$

⇒ Correlations between SUSY breaking parameters and/or additional low-scale states

- The CMSSM $\mu_0, m_0, m_{1/2}, A_0, B_0$

$$M_Z^2 = a_0 m_0^2 + a_{1/2} m_{1/2}^2 + a_\mu \mu^2 + .. \ll \tilde{m}_{q_i}^2, M_i^2$$



Constraints

SUSY particle masses

$3.20 < 10^4 \ \mathrm{Br}(b \to s\gamma) < 3.84$

$\mathrm{Br}(b \to \mu\mu) < 1.8 \times 10^{-8}$

$\delta a_\mu < 292 \times 10^{-11}$

$-0.0007 < \delta\rho < 0.0012$

Radiative EW breaking

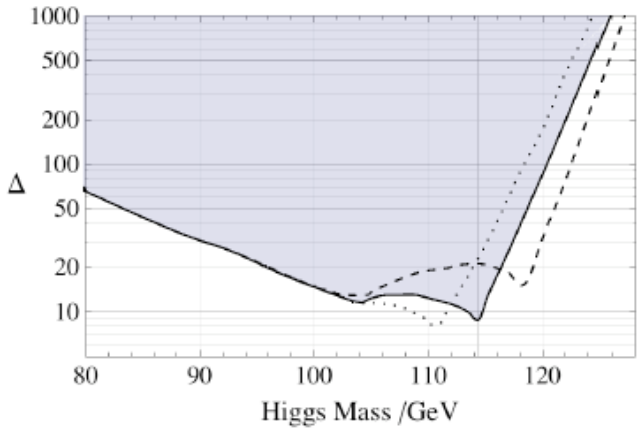
Relic density unrestricted

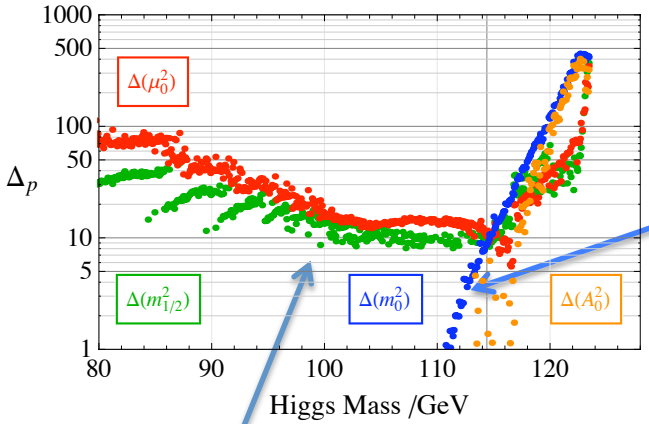
$$\Delta \equiv \max \left| \Delta_p \right|_{p=\{\mu_0^2, m_0^2, m_{1/2}^2, A_0^2, B_0^2\}}, \qquad \Delta_p \equiv \frac{\partial \ln v^2}{\partial \ln p}$$

$$\Delta_{Min} = 9, \quad m_h = 114 \pm 2 GeV$$

(No Higgs bound applied)

The CMSSM





Limit of focus point

λ decrease

Constraints

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Radiative EW breaking

Relic density unrestricted

$$v^2 = -\frac{m^2}{\lambda}$$

S.Cassel, D.Ghilencea, GGR

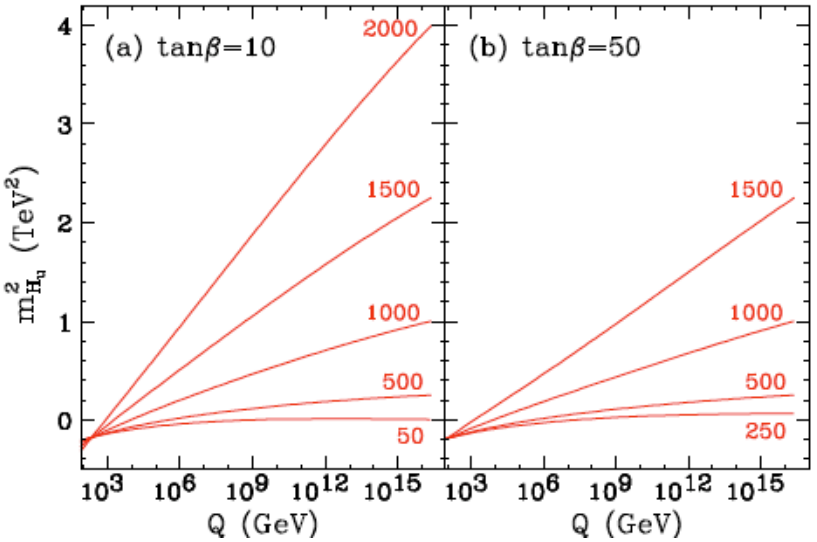
Focus Point

$2\,|\,y_t\,|^2\,(m^2_{H_u}+m^2_{Q_3}+m^2_{u_3})+2\,|\,a_t\,|^2$

$$16\pi^2\frac{d}{dt}m^2_{H_u}=3X_t-6g_2^2\,|\,M_2\,|^2-\frac{6}{5}g_1^2\,|\,M_1\,|^2$$

$$16\pi^2\frac{d}{dt}m^2_{Q_3}=X_t+X_b-\frac{32}{3}g_3^2\,|\,M_3\,|^2-6g_2^2\,|\,M_2\,|^2-\frac{2}{15}g_1^2\,|\,M_1\,|^2$$

$$16\pi^2\frac{d}{dt}m^2_{u_3}=2X_t-\frac{32}{3}g_3^2\,|\,M_3\,|^2-\frac{32}{15}g_1^2\,|\,M_1\,|^2$$



$$m^2_{H_u}\left(Q^2\right)=m^2_{H_u}\left(M_P^2\right)+\frac{1}{2}\left(m^2_{H_u}\left(M_P^2\right)+m^2_{Q_3}\left(M_P^2\right)+m^2_{u_3}\left(M_P^2\right)\right)\left[\left(\frac{Q^2}{M_P^2}\right)^{\frac{3y_t^2}{4\pi^2}}-1\right]$$

m_0^2 $\qquad$ $3m_0^2$ $\qquad$ $\simeq -\frac{2}{3},\ Q^2\simeq M_Z^2$

"Focus point": $m^2_{H_u}\left(0\right)=m^2_{Q_3}\left(0\right)=m^2_{u_3}\left(0\right)\equiv m^2$

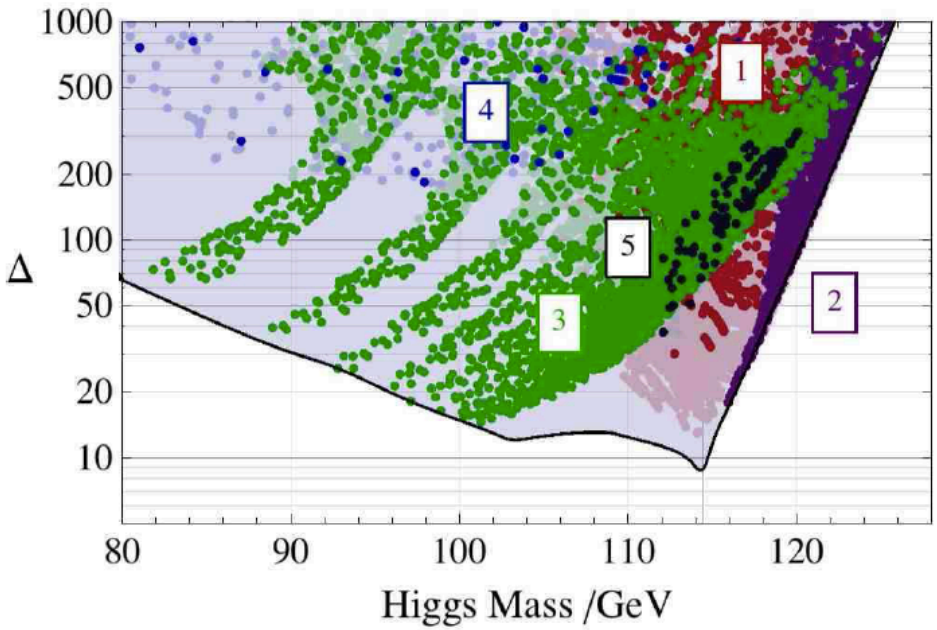
$$m^2_{H_u}\left(t_0\right)=a_0\,m^2+..,\ a_0\le 0.1$$

Natural choice

$i.e.\ m^3_{Q_3}, m^2_{u_3}\gg M_Z^2$ possible

Feng, Matchev, Moroi
Chan, Chattopadhay, Nath
Barbieri, Giudice

Dark Matter structure



Relic density restricted

1 h^0 resonant annihilation

2 $\tilde{h}$ t-channel exchange

3 $\tilde{\tau}$ co-annihilation

4 $\tilde{t}$ co-annihilation

• 5 A^0 / H^0 resonant annihilation

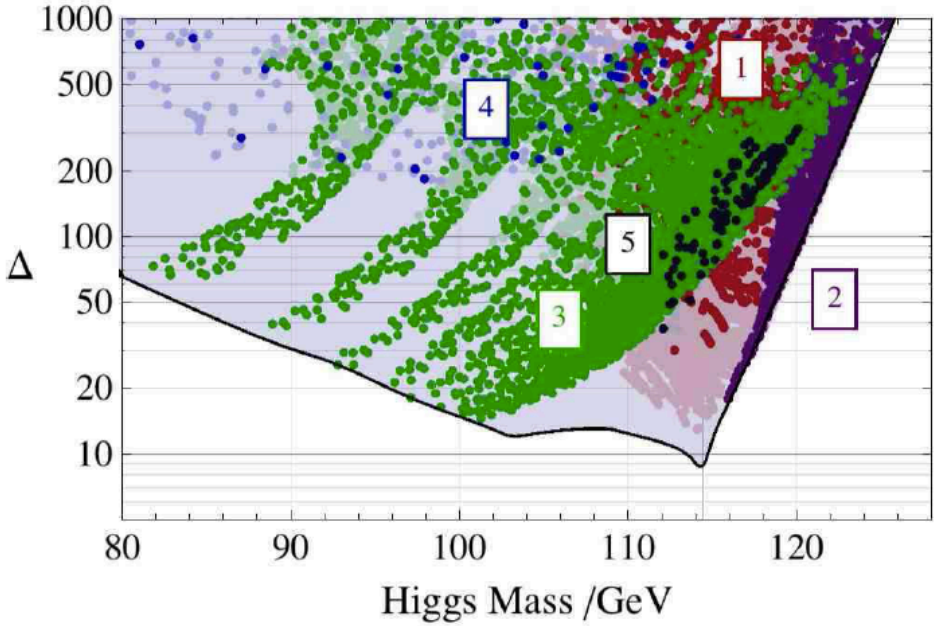
Within 3σ WMAP:

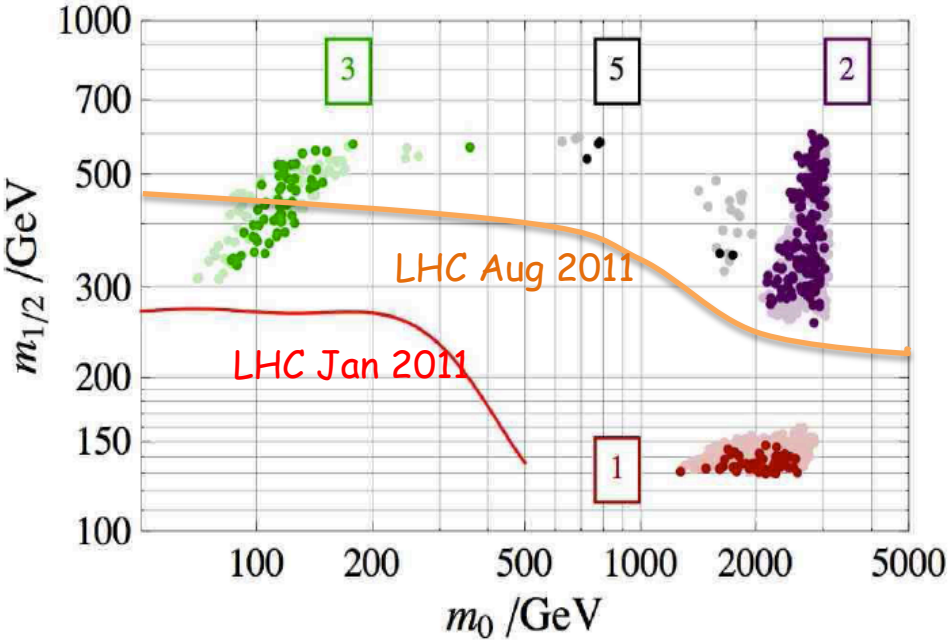
$\Delta_{Min} = 15, \quad m_h = 114.7 \pm 2GeV$

< 3σ WMAP:

$\Delta_{Min} = 18, \quad m_h = 115.9 \pm 2GeV$

Cassel, Ghilencea, GGR





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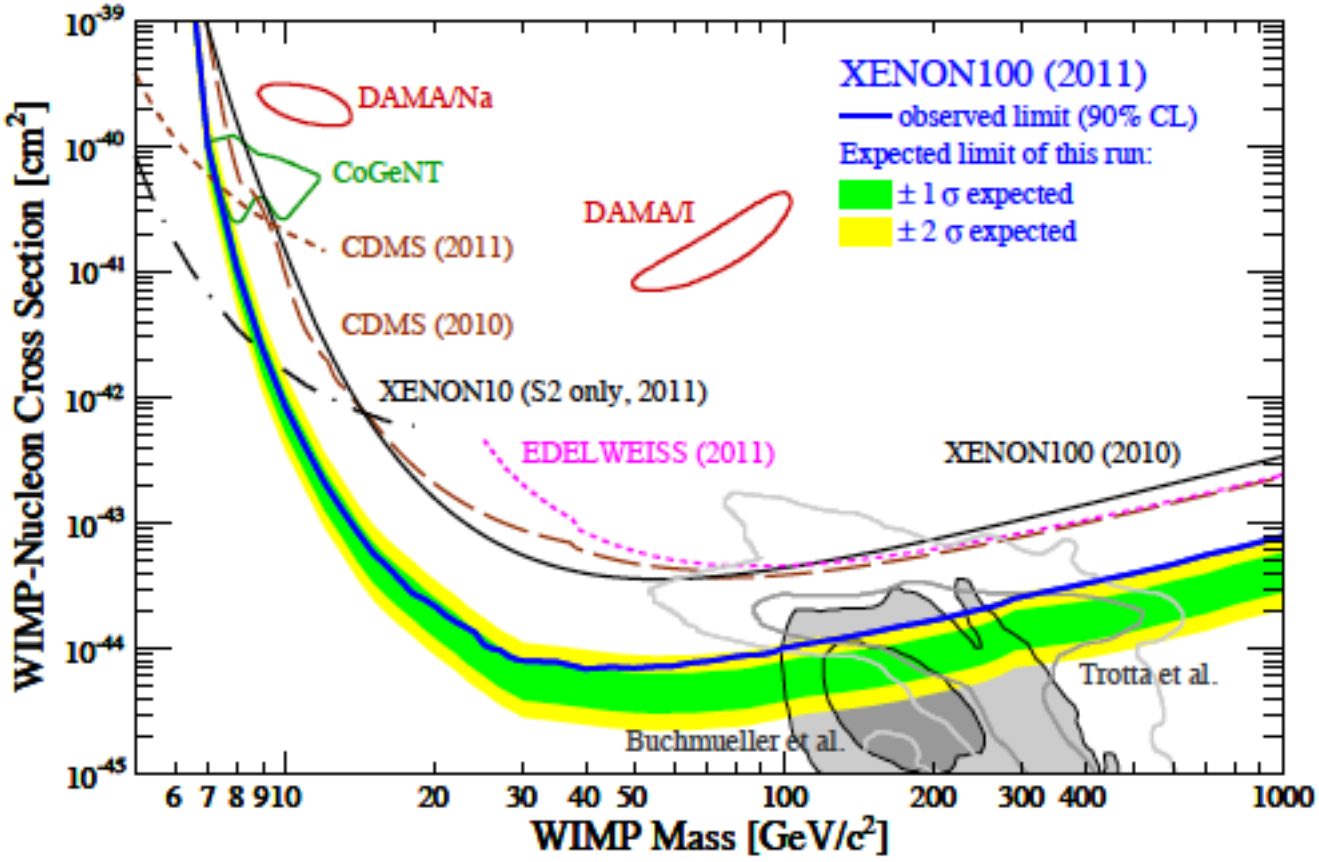
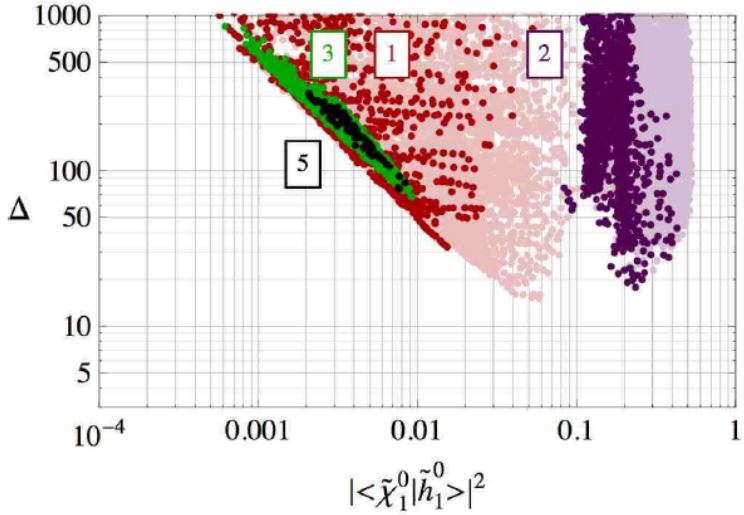
Cassel, Ghilencea, GGR

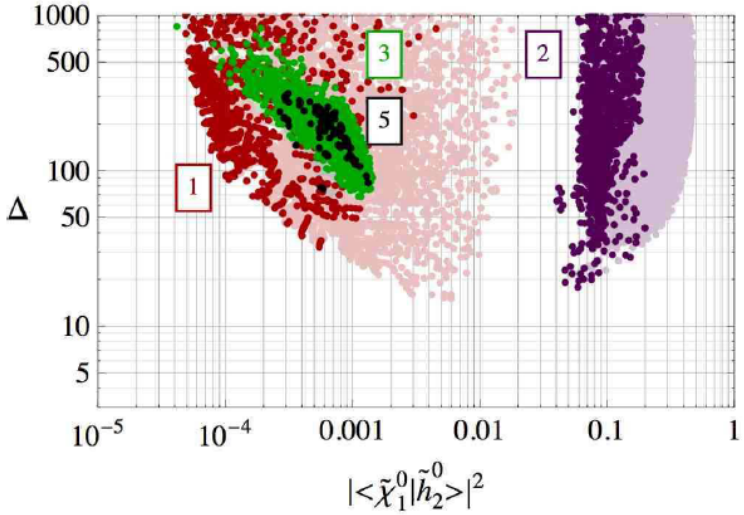
Direct dark matter searches: (spin independent)



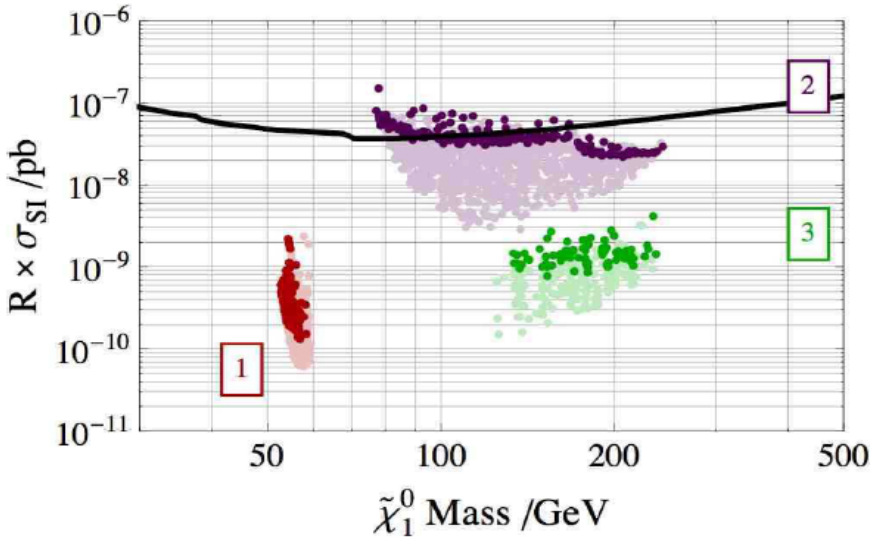
DM - Scaled spin independent cross section for LSP-proton scattering:



(a) LSP $\tilde{h}_1^0$ component

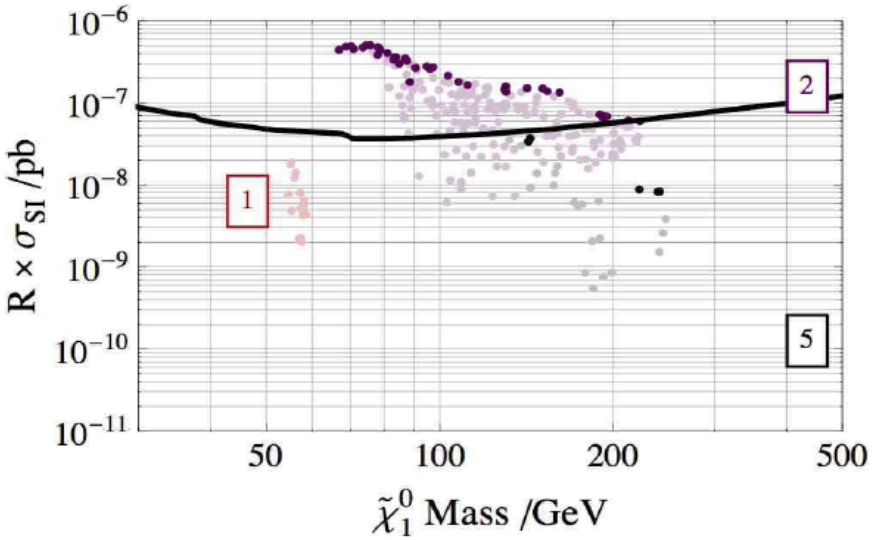


(b) LSP $\tilde{h}_2^0$ component



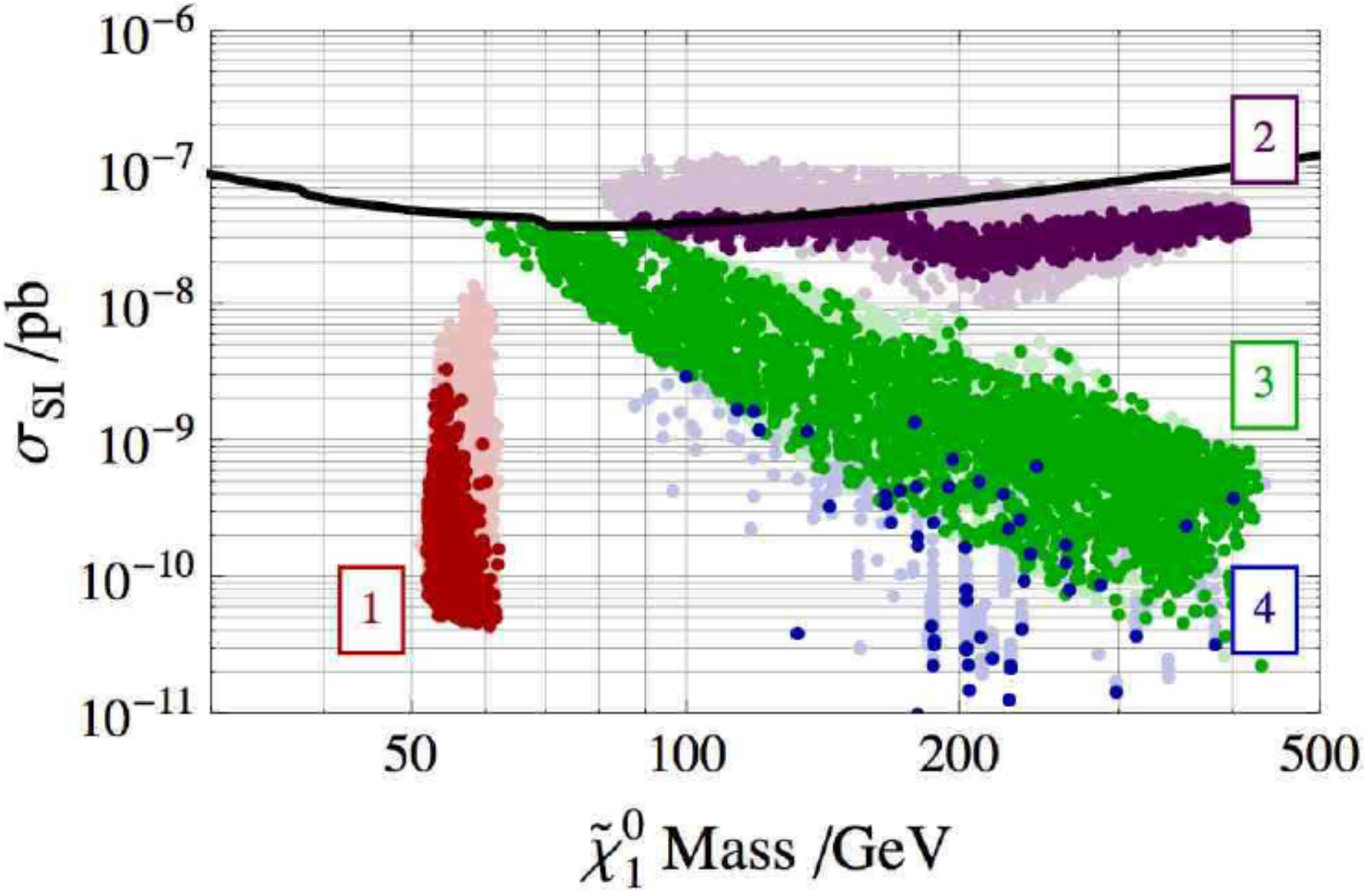
(a) $\tan\beta \leq 45$

$\Delta < 100$



(b) $50 \leq \tan\beta \leq 55$

$\Delta < 100$


10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
σ_{SI} /pb
50
100
200
500
$\tilde{\chi}_1^0$ Mass /GeV
1
2
3
4

Reduced fine tuning

- New degrees of freedom e.g. singlet extensions

- New focus points?

Gauginos: $M_{\tilde{g},\widetilde{W},\tilde{B}}$ Non-universal gaugino correlations

Kane,King
Lebedev, Nilles, Ratz…
Horton, GGR
…

Scalars: M_0, A_0 correlations?

Feldman, Kane, Kuflik, Lu

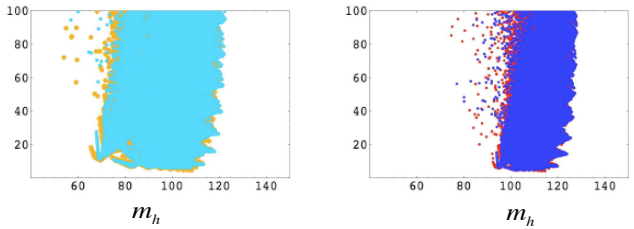
$$M_0, A_0, B_0 \gg \mu, m_a \qquad m_{h_u^2}^2(t) = f_{M_0} M_0^2 - f_{A_0} A_0^2$$

$$f_{M_0} \sim f_{A_0} \sim 0.1 \quad + M_0 \sim A_0 \quad \Rightarrow \qquad m_{h_u}^2 \sim 10^{-2} m_{3/2}^2$$

$$\Delta_{h_t} \sim 10^4, \quad M_0 \sim 10 TeV$$

Reduced fine tuning : singlet extensions

Δ



$$W = W_{\rm Yukawa} + (\mu + \lambda S) H_u H_d + \frac{\mu_S}{2} S^2 + \frac{\kappa}{3} S^3 + \xi S$$ GNMSSM

$\mu_S >> m_{3/2}$

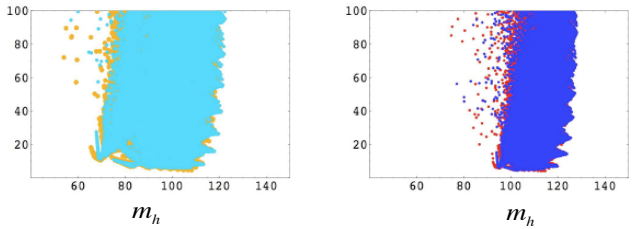
c.f. $$W = W_{\rm Yukawa} + \lambda S H_u H_d + \frac{\kappa}{3} S^3$$ NMSSM

$$W_{eff}^{SMSSM} = \left(H_u H_d\right)^2 / \mu_s + \mu H_u H_d$$

$$\varsigma_1 2 \frac{\mu_0}{M_*}\left(|h_1|^2 + |h_2|^2\right) h_1 h_2 \qquad v^2 = -\frac{m^2}{\lambda}$$

Cassel, Ghilencea, GGR
Casas, Espinosa, Hidalgo
Dine, Seiberg, Thomas
Batra, Delgardo, Tait
Kaplan,

Reduced fine tuning : singlet extensions



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Cassel, Ghilencea, GGR
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Kaplan,

GNMSSM (NMSSM) unnatural?.....discrete R-symmetry.....

$$\begin{aligned} \Delta \mathscr{W}_{\mathbb{Z}_4^R} &= Y + Y^2 N + Y N^2 + Y H_u H_d \\ &\sim m_{3/2} M_{\rm P}^2 + m_{3/2}^2 N + m_{3/2} N^2 + m_{3/2} H_u H_d \end{aligned}$$

M	$q_{\mathbf{10}}$	$q_{\overline{\mathbf{5}}}$	q_{H_u}	q_{H_d}	q_N
4	1	1	0	0	2

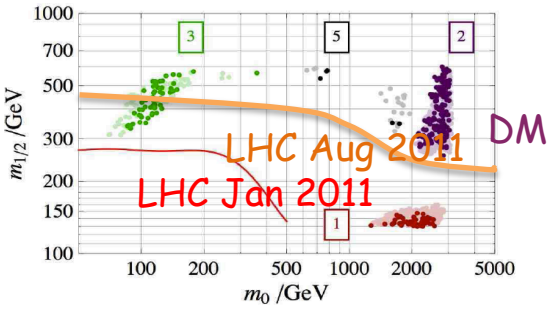
$$\begin{aligned} \Delta \mathscr{W}_{\mathbb{Z}_8^R} &= Y + Y^2 \left(N + Y N^2 + Y H_u H_d\right) \\ &\sim m_{3/2} M_{\rm P}^2 + m_{3/2}^2 N + \frac{m_{3/2}^3}{M_{\rm P}^2} N^2 + \frac{m_{3/2}^3}{M_{\rm P}^2} H_u H_d \end{aligned}$$

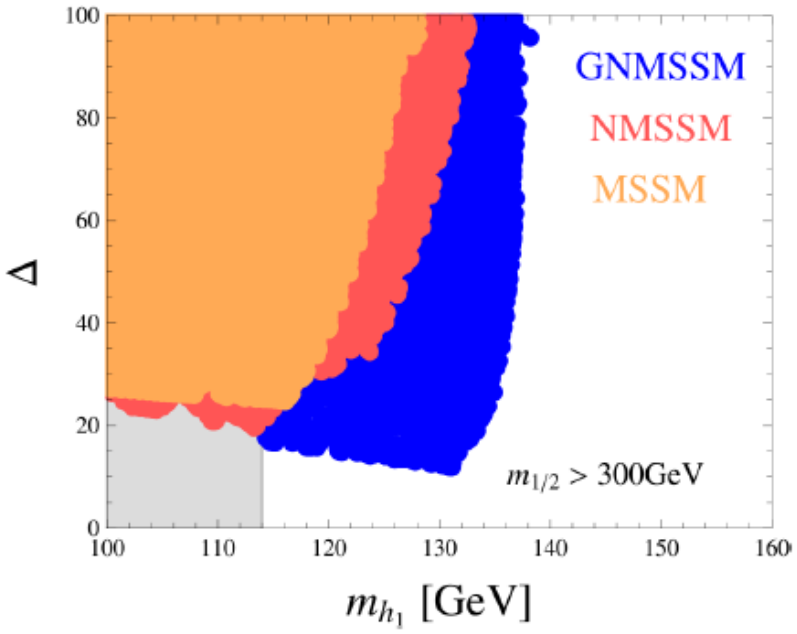
M	$q_{\mathbf{10}}$	$q_{\overline{\mathbf{5}}}$	q_{H_u}	q_{H_d}	q_N
8	1	5	0	4	6

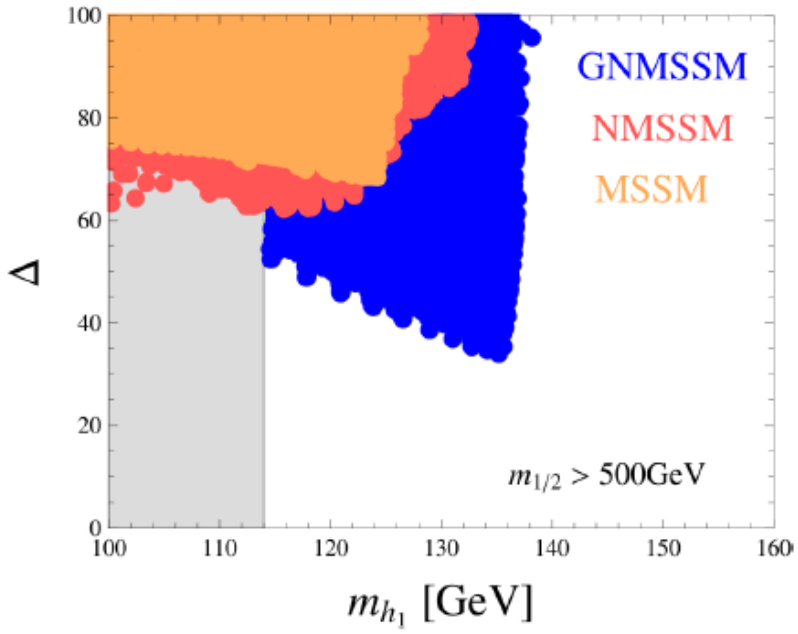
R-symmetry guarantees tadpoles under control

Abel

Fine tuning in the GNMSSM







GGR, Schmidt-Hoberg

$\left|\lambda H_u H_d\right|^2$ small increase in SUSY masses possible

GENERAL-NMSSM PHENOMENOLOGY

$$W = W_{\text{Yukawa}} + (\mu + \lambda S) H_u H_d + \frac{\mu_S}{2} S^2 + \frac{\kappa}{3} S^3 + \xi S$$ GNMSSM

c.f. $$W = W_{\text{Yukawa}} + \lambda S H_u H_d + \frac{\kappa}{3} S^3$$ NMSSM

- Higgs structure (h_u, h_d, s)

$\mu_s \gg \mu$ MSSM SUSY structure

$\mu_s, m_s, b_s \sim \mu$ $h_1 \simeq H_u + \varepsilon S, \quad h_2 = S - \varepsilon H_u$ $M_{h_1} \ll M_{h_2}, \quad \varepsilon \ll 1$

$$BR\left(\frac{h_1 \to \tilde{S}\tilde{S}}{h_1 \to \gamma\gamma, b\bar{b}}\right) \gg 1$$

Invisible Higgs decay $45\ GeV < m_{\tilde{S}} < 70\ GeV$

(applies for a wide range of parameters)

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2 \mid y_t \mid^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + 2 \mid a_t \mid^2\right) - 6 g_2^2 \mid M_2 \mid^2 - \frac{6}{5} g_1^2 \mid M_1 \mid^2$$

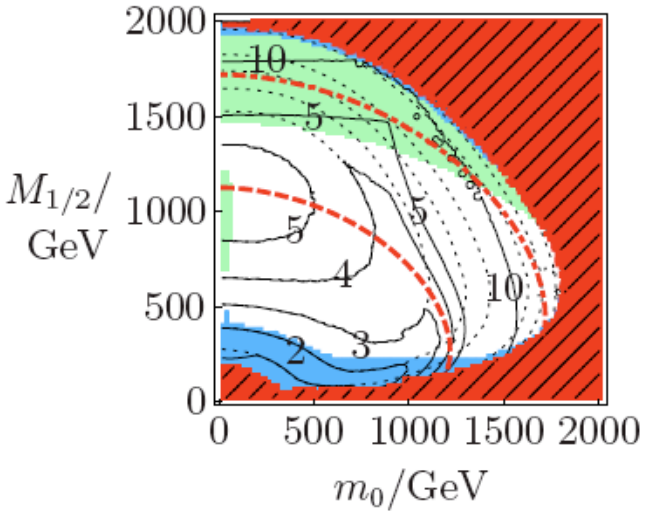
New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

Natural ratios? e.g.:

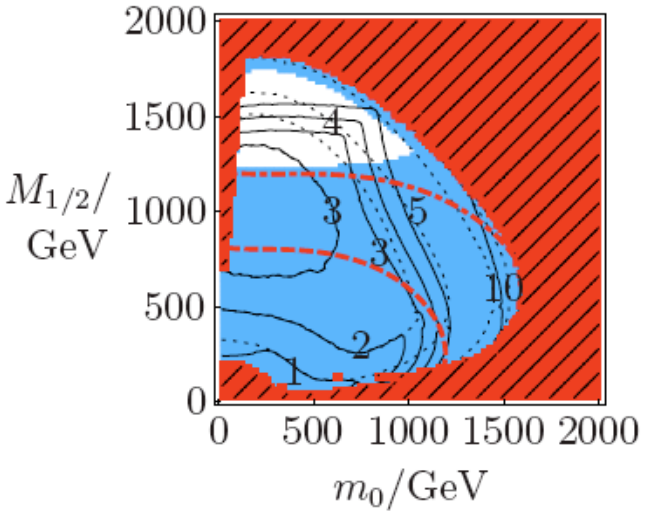
GUT: $SU(5): \ \Phi^N \subset (24 \times 24)_{symm} = 1 + 24 + 75 + 200; \quad SO(10): \ (45 \times 45)_{symm} = 1 + 54 + 210 + 770$

	$\eta_3 : 1 : \eta_1$	$2.7\eta_3 : 1 : 0.5\eta_1$
Representation	$M_3 : M_2 : M_1$ at M_{GUT}	$M_3 : M_2 : M_1$ at M_{EWSB}
1	1:1:1	6:2:1
24	2:(-3):(-1)	12:(-6):(-1)
75	1:3:(-5)	6:6:(-5)
200	1:2:10	6:4:10

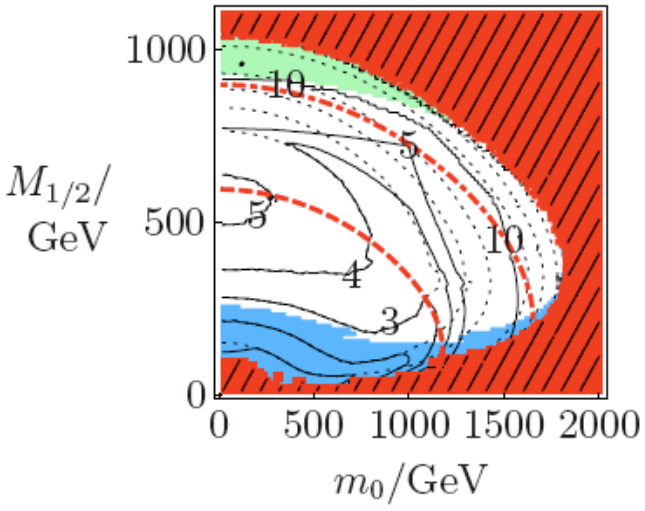
String: $(3 + \delta_{GS}) : (-1 + \delta_{GS}) : \left(-\frac{33}{5} + \delta_{GS}\right)$ (OII, also mixed moduli anomaly)



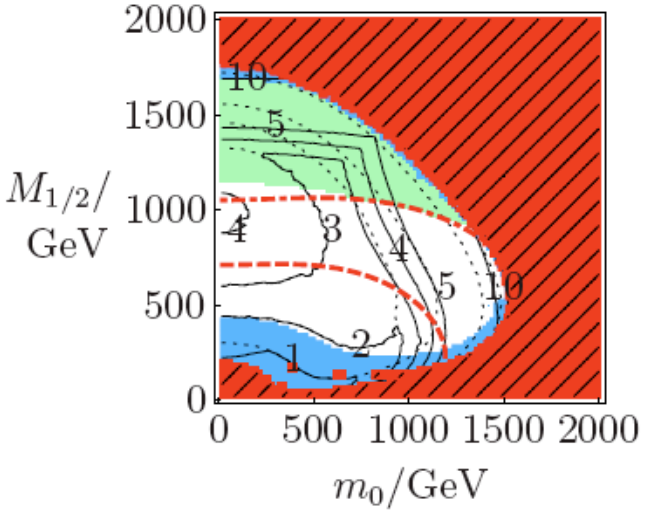
(a) 54



(b) 210



(c) 770



(d) O-II

Phenomenology

- Gaugino mass ratios

$$\frac{M_i(Q)}{M_{1/2}} = \eta_i \frac{\alpha_i(Q)}{\alpha_i(M_X)} \quad \Rightarrow \quad \begin{aligned} \frac{M_1(Q)}{M_2(Q)} &\approx 0.5\eta_1 \\ M_2(Q) &\approx 0.8 M_{1/2} \\ \frac{M_3(Q)}{M_2(Q)} &\approx 2.7\eta_3 \end{aligned}$$

…. gauginos can be very heavy

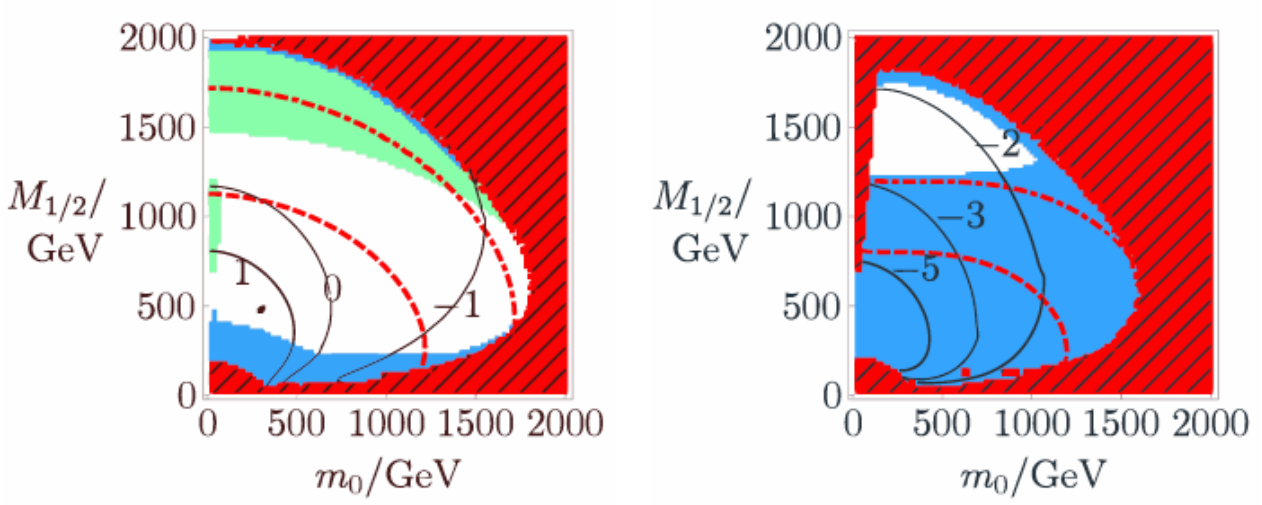
- Light neutralino and 2 charginos nearly degenerate

$$m_{\chi_2^0} - m_{\chi_1^0} = M_Z^2 \left(\frac{s_W^2}{M_1} + \frac{c_W^2}{M_2} \right) + \mathcal{O}(\frac{M_Z^3}{M_2^2})$$

$$m_{\chi_1^\pm} - m_{\chi_1^0} = \frac{1}{2} M_Z^2 \left(\frac{s_W^2}{M_1} + \frac{c_W^2}{M_2} \right) + \frac{1}{2} M_Z^2 \left(\frac{s_W^2}{M_1} - \frac{c_W^2}{M_2} \right) \epsilon \sin 2\beta + \mathcal{O}(\frac{M_Z^3}{M_2^2})$$

+ for $|M_1| < \mu$, Bino or Higgsino LSP candidate

$b \to s\gamma$



(a) 54 (b) 210

(c) 770 (d) O-II

Figure 5: Contour plots of the branching ratio $\mathrm{Br}(\overline{B} \to X_s\gamma)$ in the m_0—$M_{1/2}$ plane. This is for the hypersurface in parameter space with $\tan\beta = 10$, $A_0 = 0$ and $\mu > 0$. Solid (black) contours are shown for $\mathrm{Br}(\overline{B} \to X_s\gamma) \times 10^4 = 3.52 + n\sigma$, where $n \in \mathbb{Z}$ and the error $\sigma = 0.34$ combines all experimental and theoretical errors in quadrature. The (red) dashed and (red) dot-dashed contours indicate where the Higgs mass, m_{h^0}, is 111 GeV and 114 GeV, respectively, whilst the dotted contours correspond to the $|\Delta| = 5$ and $|\Delta| = 10$ contours from Fig 2.

$g-2$

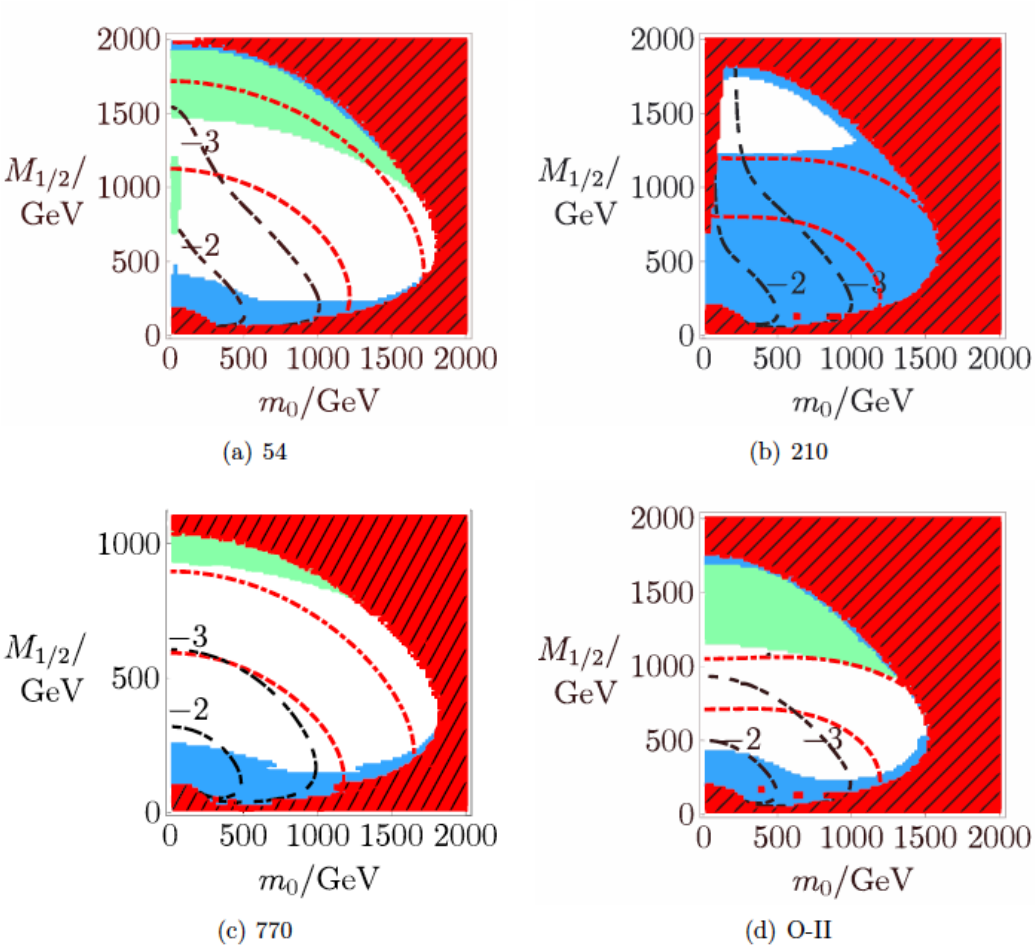
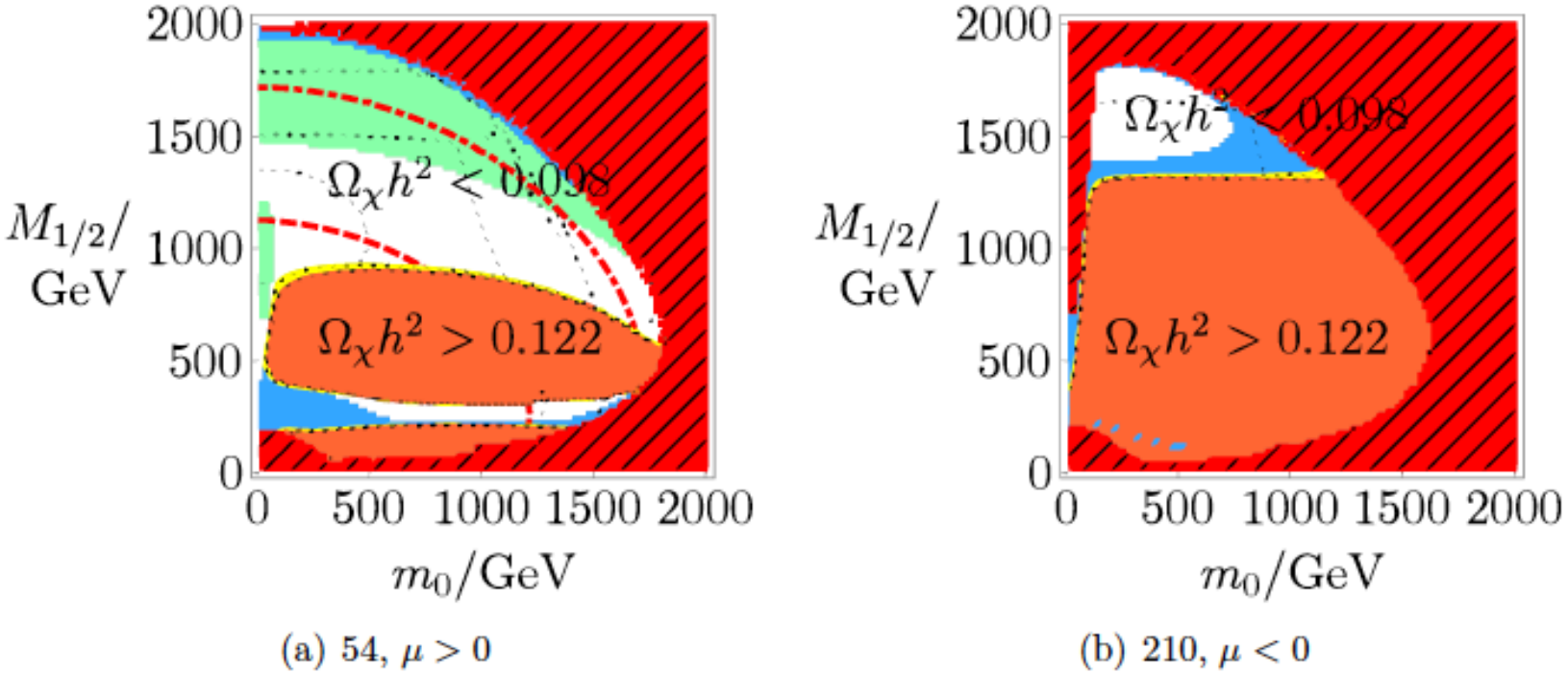


Figure 7: Contour plots of δa_μ, in the m_0—$M_{1/2}$ plane. This is for the hypersurface in parameter space with $\tan\beta = 10$, $A_0 = 0$ and $\mu > 0$. Dashed (black, with alternating long and short dashing) contours are shown for $\delta a_\mu \times 10^{10} = 27.5 + n\sigma'$, where $n \in \mathbb{Z}$ and the error $\sigma' = 8.1$ combines all experimental and theoretical errors in quadrature. The (red) dashed and (red) dot-dashed contours indicate where the Higgs mass, m_{h^0}, is 111 GeV and 114 GeV, respectively, whilst the dotted contours correspond to the $|\Delta| = 5$ and $|\Delta| = 10$ contours from Fig 2.

$\Longrightarrow$ split squarks and Higgs from sleptons ???

Dark Matter

$$\Delta < 5, 5 \leq \Delta^{\Omega} \leq 15$$



(a) 54, $\mu > 0$ (b) 210, $\mu < 0$

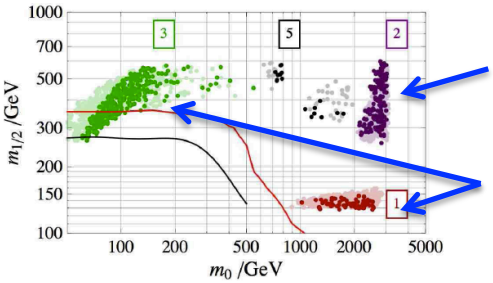
Figure 9: Contour plots of $\Omega_\chi h^2$ in the 54 and 210 models. These are for the hypersurface in parameter space with $\tan\beta = 10$ and $A_0 = 0$. The narrow, yellow region, which lies between the regions of over- and under- abundance, has a dark matter abundance that satisfies, within 3σ, the constraint given in Eq (47). The (red) dashed and (red) dot-dashed contours indicate where the Higgs mass, m_{h^0}, is 111 GeV and 114 GeV, respectively.

Summary

- Hierarchy problem $\Longrightarrow$ SUSY breaking structure and/or further states

- CMSSM $m_i = M_0 \quad Max\left[\Delta_{EW}, \Delta_{\Omega}\right] = 15(29), \quad m_h = 114(116) \pm 2GeV$

  Complementary DM & LHC searches



(a) $m_h > 111$ GeV

DM $\quad \Delta \le 100 \quad$ Sensitivity $\times (10 - 100)$

LHC $\quad \left(\text{Full region } LHC\ 14TeV\ 10\,fb^{-1}\right)$

(Gauge mediation $\Delta > 100$)

- NMSSM $\quad$ Reduced $\Delta \Longrightarrow$ GNMSSM $\Longrightarrow Z_{4R}, Z_{8R}$

  SUSY states can be (slightly)heavier

  $m_h \rightarrow 130GeV$

- Gaugino focus point $\quad M_i = \eta_i M_{1/2}$

  Characteristic η_i

  Light $\chi^{0,\pm}$

  $\delta(b \rightarrow s\gamma)$ significant

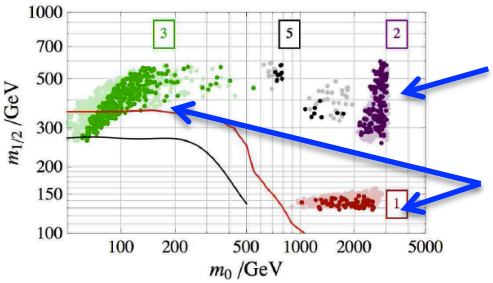
  $\delta(g-2)$ small

Summary

- Hierarchy problem $\Longrightarrow$ SUSY breaking structure and/or further states

- CMSSM $m_i = M_0 \quad Max\left[\Delta_{EW}, \Delta_{\Omega}\right] = 15(29), \quad m_h = 114(116) \pm 2GeV$

  Complementary DM & LHC searches



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  † invisible Higgs a distinct possibility

- Gaugino focus point

…Light Higgs search may† provide the first crucial test!

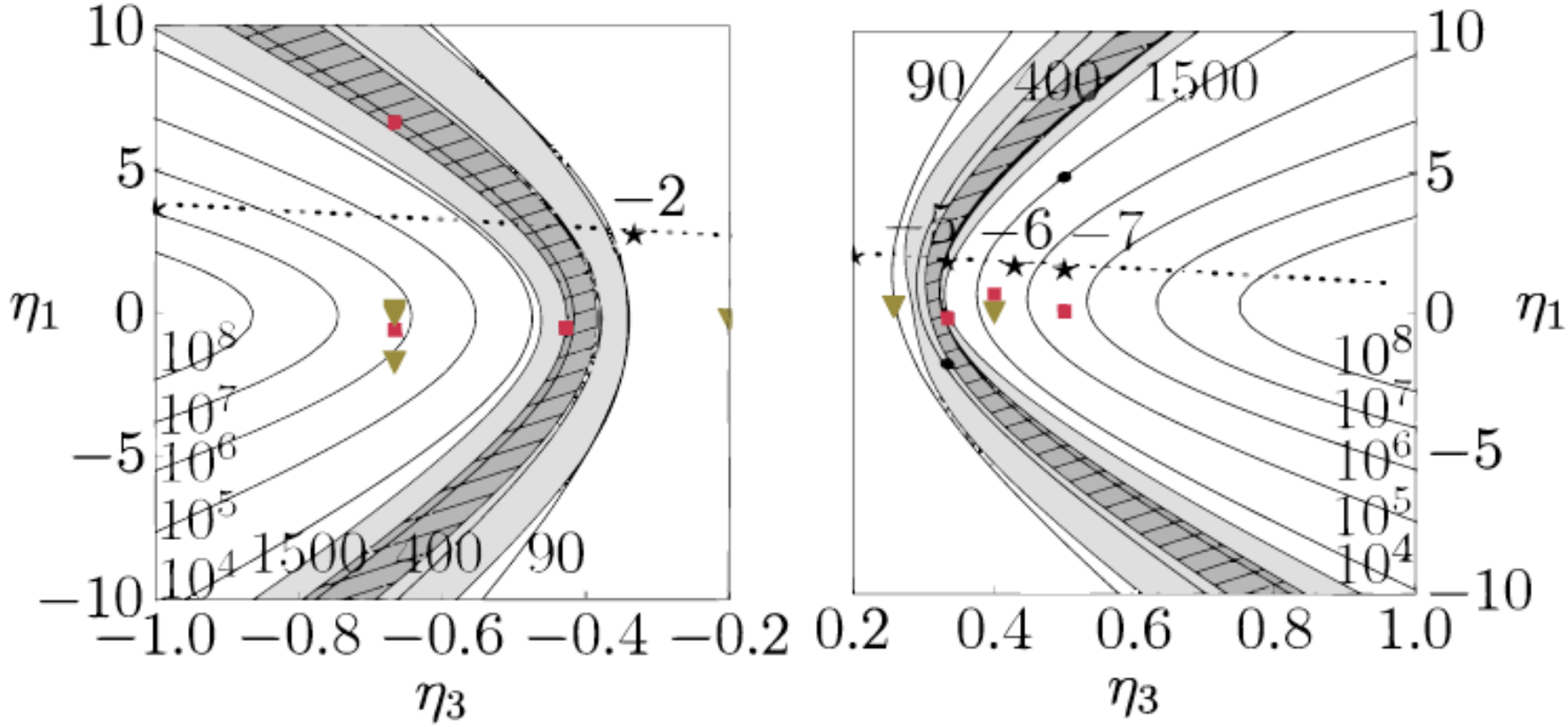
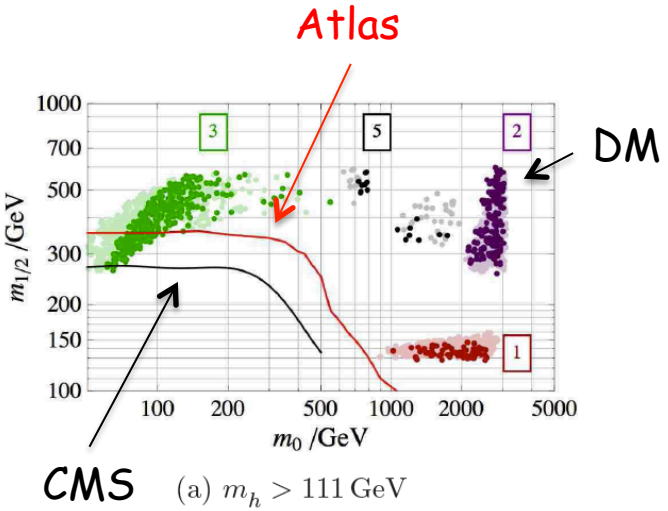


Figure 1: A contour plot of the gaugino focus point scale Q^M (GeV), as a function of the gaugino mass ratios η_1 and η_3. The light grey (dark grey, hatched) region indicates the ratios that will permit $\Delta_M \lesssim 10$ for $M_S = 600$ GeV ($M_S = 1$ TeV). The filled circles, squares and triangles indicate the mass ratios predicted by the $SU(5)$, $SO(10)$ and E_6 GUT models, respectively, that are discussed in Section 3. The stars give the prediction of a string model with moduli-dominated SUSY breaking (the O-II model [29]), for various integer values of the parameter δ_{GS}. The ratios that are permitted within models with mixed Moduli-Anomaly breaking lie upon the dashed line.

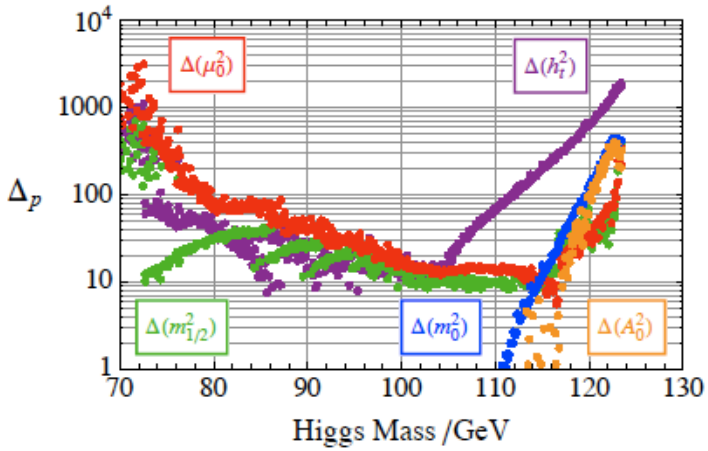
LHC – Regions of low fine tuning $\Delta < 100$:

	SUG0	SUG1	SUG2	SUG3	SUG5
m_0	1455	1508	2270	113	725
$m_{1/2}$	160	135	329	383	535
A_0	238	1492	30	-220	1138
$\tan\beta$	22.5	22.5	35	15	50
μ	191	433	187	529	581
$m_{\tilde{g}}$	482	414	900	898	1252
$m_{\tilde{u}_L}$	1469	1509	2331	826	1315
$m_{\tilde{t}_1}$	876	831	1423	602	1000
$m_{\tilde{\chi}_1^+}$	106	104	168	293	416
$m_{\tilde{\chi}_2^0}$	108	104	181	293	416
$m_{\tilde{\chi}_1^0}$	60	53	123	155	222
Δ	9	50	45	68	84
$\Omega_{\tilde{\chi}_1^0}h^2$	0.41	0.13	0.10	0.13	0.10
$\mathrm{BR}(b \to s\gamma) \times 10^4$	3.4	3.7	3.4	3.2	3.2
$\mathrm{BR}(B_s \to \mu^+\mu^-) \times 10^9$	3.0	2.9	2.9	3.4	1.7
$\delta a_\mu \times 10^{10}$	4.5	3.2	3.2	22.5	16.6
$\sigma^{\mathrm{SI}}_{\chi p}$ (pb) $\times 10^{10}$	108	5	432	24	101
$\sigma^{(LO)}$(7 TeV) (pb)	8	12	0.9	0.4	0.02
$\sigma^{(LO)}$(14 TeV) (pb)	40	75	3	5	0.4



Effect of focus point limited by h_t :

$$\Delta'(h_t) \simeq \left| \frac{\Delta h_t}{M_Z} \frac{\partial M_Z}{\partial h_t} \right| \text{ gives lower limit on } a_0$$



$\Delta(p)^{-1}$ is probability p lies in range $p, p+\delta p$

For measured parameter, probability p lies in range,

compatible with measured value is: $P \sim \frac{\delta p}{\sigma_p} = \Delta(p)^{-1} \frac{p}{\sigma_p}$

Ciafolini, Strumia
Romanino, Strumia

- (General) Gauge mediation in the MSSM

$$M_{\tilde{\lambda}_i}(M_{mess}) = k_i \frac{\alpha_i(M_{mess})}{4\pi} \Lambda_G$$

$$m^2_{\tilde{f}}(M_{mess}) = 2 \sum_{i=1}^{3} C_i k_i \frac{\alpha_i^2(M_{mess})}{(4\pi)^2} \Lambda_S^2$$

No focus point

$$k_i = (\frac{5}{3}, 1, 1)$$

$$k_i \alpha_i \left(M_{GUT}\right) = 1, \quad i = 1,2,3$$

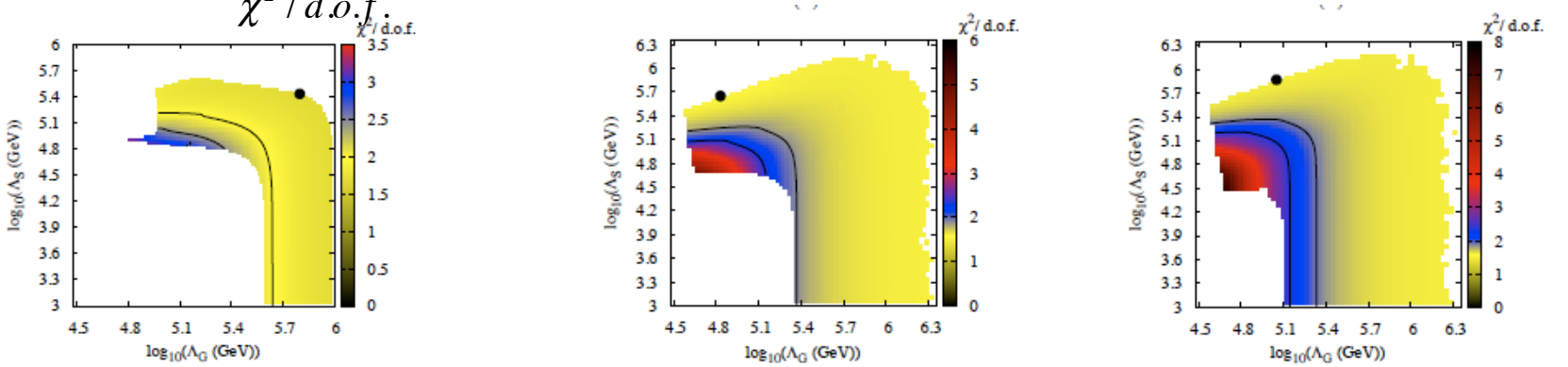
(Ordinary gauge mediation $\Lambda_G = \Lambda_S$)

Meade, Seiberg, Shih

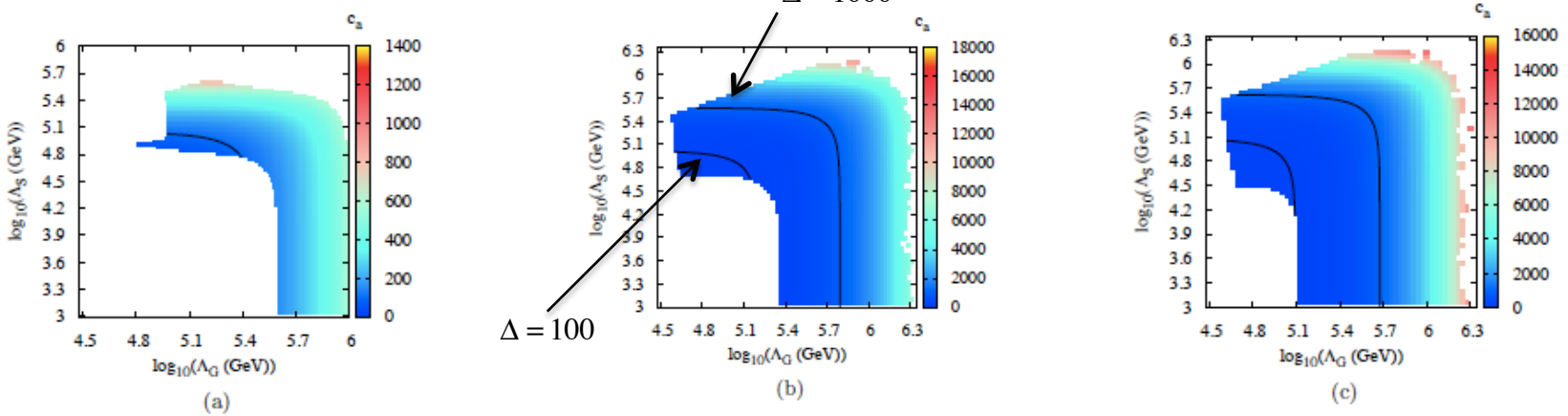
Fine tuning in General Gauge Mediation

$B \to X_s \gamma, B \to \tau\mu, B \to \mu^+\mu^-, B \to D\tau\mu,$
$D_s \to \mu\nu, D_s \to \tau\nu, K \to \mu\nu / \pi \to \mu\nu, \Delta_{0-}$

$\chi^2 / d.o.f.$



Δ



$M_{Messenger}$ $10^6 GeV$ $10^{10} GeV$ $10^{14} GeV$

$\Delta > 100$ no focus point

Abel, Dolan, Jaeckel, Khoze
(Giusti, Romanino, Strumia)