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Realistic Grand Unification and Nucleon Decay

[SO(10) GUT]

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LHC, Dark Matter and Unification

Outline

- Intro: Shortcomings, Problems & Puzzles of SM / MSSM
  → New Physics
- Motivations for GUT
- Some shortcomings & problems

  --status of min. SUSY SU(5) [p-decay]
- Realistic SO(10) Model with Interesting Predictions :

  GUT breaking, -TD splitting –Proton lifetime

  Calculable thresholds
- Upper bounds on proton lifetimes [in d=5, d=6 modes]
- Realistic Fermion Sector
- Summary

Shortcomings of SM:

SM Interactions : $SU(3)_C \times SU(2)_L \times U(1)_Y$ $\square$ (8 gluons) + (3 + 1 EW Bosons)

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{d} \end{pmatrix} = \mathbf{q}(3, 2, -1/3) \qquad \mathbf{u}^c(\bar{3}, 1, 4/3) \qquad \mathbf{d}^c(\bar{3}, 1, -2/3)$$

$$\begin{pmatrix} \nu \\ \mathbf{e} \end{pmatrix} = \mathbf{l}\,(1, \bar{2}, 1) \qquad \mathbf{e}^c(1, 1, -2)$$

Fractional $U(1)_Y$ charges - Charge quantization ?

Breaking: $SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\langle H \rangle} SU(3)_C \times U(1)_{em}$

Higgs H(1,2, 1) is required (undiscovered yet)

Why/how it is light? –Gauge Hierarchy Problem

Evidences for New Physics:

Atmospheric & Solar Neutrino Data

$$\Delta m^2_{\rm atm} \simeq 2.5 \cdot 10^{-3}\ {\rm eV}^2 \qquad \Delta m^2_{\rm sol} \simeq 7.6 \cdot 10^{-5}\ {\rm eV}^2$$

$$\sin^2\theta_{23} = 0.52 \qquad \sin^2\theta_{12} = 0.312$$

Third mixing angle: $\theta_{13} \lesssim 0.2$

Unknown phase: $\delta_{\rm lept}$

are of great importance for leptonic CP viol.

- **Origin of these scales and mixings?**

Unexplained in SM/MSSM $\leftarrow\ {\rm m}_\nu \lesssim 10^{-4}\ {\rm eV}$ **Without New Physics**

• Charged fermion masses & mixings

Observed Noticeable Hierarchies:

$$\lambda_t \sim 1 \ , \qquad \lambda_u : \lambda_c : \lambda_t \sim \lambda^8 : \lambda^4 : 1$$

$$\lambda_b \sim \lambda_\tau \sim \frac{m_b}{m_t} \tan\beta \ , \qquad \lambda_d : \lambda_s : \lambda_b \sim \lambda^4 : \lambda^2$$

With λ=0.2

$$\lambda_e : \lambda_\mu : \lambda_\tau \sim \lambda^5 : \lambda^2 : 1$$

$$V_{us} \approx \lambda \ , \qquad V_{cb} \approx \lambda^2 \ , \qquad V_{ub} = \lambda^4 - \lambda^3$$

What is origin of these hierarchies?
Is there any relation or sum rule?
Why three families?

Within SM no answer to these questions…

More problems/puzzle of SM:

-- **SM has no candidate for a Dark Matter:**
[what is~85% of total matter?]

-- **Baryon asymmetry** $\Delta B \sim 10^{-10}$

How this asymmetry is generated?
[EW baryogenesis does not give enough assym.]

Additional strong evidences of physics beyond SM

Reasonable extension solving these problems of SM should be found

[perhaps explain also inflation, dark energy...]

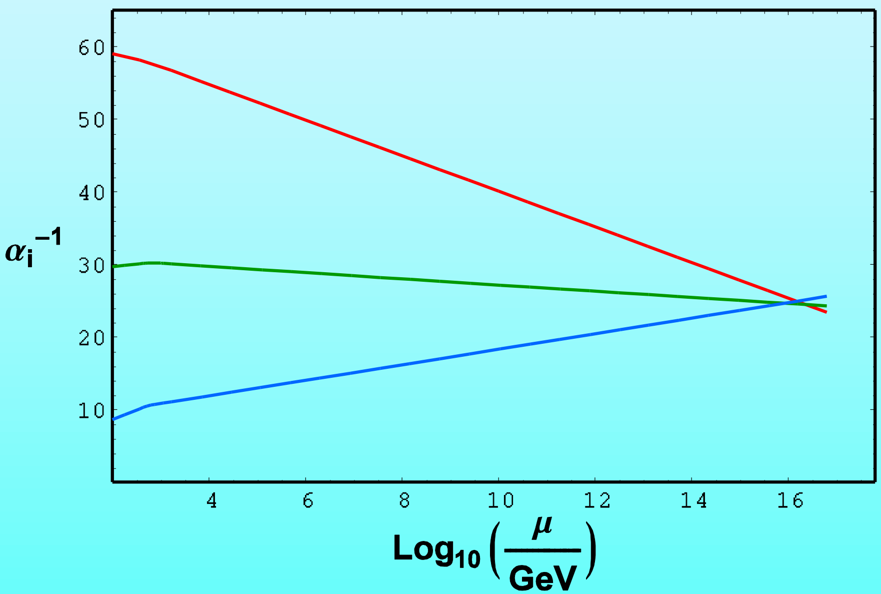
SUSY solves some (not all) problems/puzzles

- **Low scale (~TeV) SUSY**
  - **→ Solution of Hierarchy Problem**

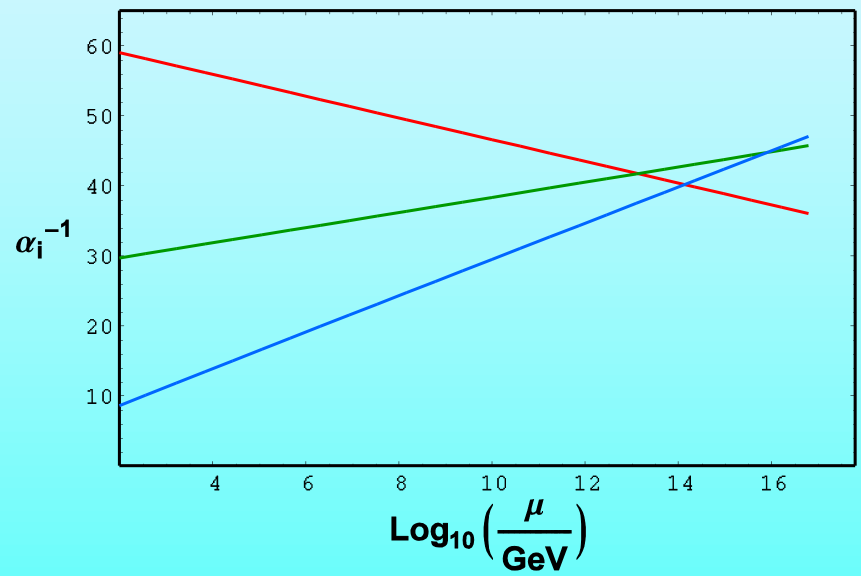
- **In MSSM (R-parity) LSP Neutralino** $(\tilde{\mathrm{H}}_1^0,\ \tilde{\mathrm{H}}_2^0,\ \tilde{\mathrm{B}},\ \tilde{\mathrm{W}}^0) \rightarrow$
  - **→ Dark Matter candidate!**

- **In MSSM→ Good coupling unification**
  - **→ GUT is revamped!**

SUSY Unification vs. non-SUSY



MSSM



SM

SUSY Grand Unification

offers more solutions to some problems/puzzles

- **Matter Unification**

In $SU(5)$: $(q,\ u^c,\ e^c) = \mathbf{10}\ ,\quad (d^c,\ l) = \bar{\mathbf{5}}$

In $SO(10)$: $(q,\ u^c,\ e^c,\ d^c,\ l, \nu_R) = \mathbf{16}$

$\rightarrow$ Charge Quantization - $\frac{Y(q)}{Y(u^c)} = -\frac{1}{4},\ \frac{Y(q)}{Y(e^c)} = \frac{1}{6}\ , \cdots$

And interesting asymptotic relations [in SO(10)]:

$$\lambda_t = \lambda_b = \lambda_\tau\ ,\ m_{\nu D} = m_t \ldots$$

Neutrino masses via see-saw

SO(10) → ν_R

→ Oscillations

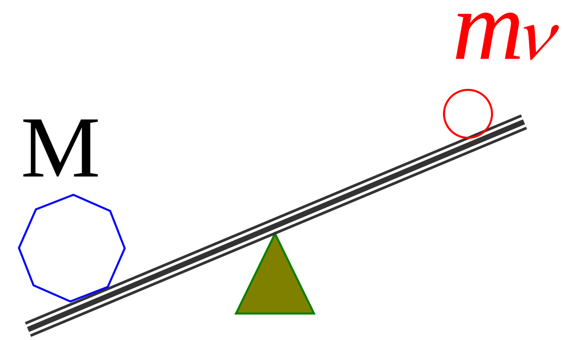
→ Leptogenesis

$\nu^c \equiv N$ □ $(1,1,0)$ ← **SM singlet**

$l\,N\,\langle H \rangle$ $\qquad M\,N\,N$ → **Δ L=2 Lepton number viol.**

$$\begin{pmatrix} 0 & \langle H \rangle \\ \langle H \rangle & M \end{pmatrix}$$

$$m_\nu \sim \frac{\langle H \rangle^2}{M}$$



$$M_N \simeq M$$

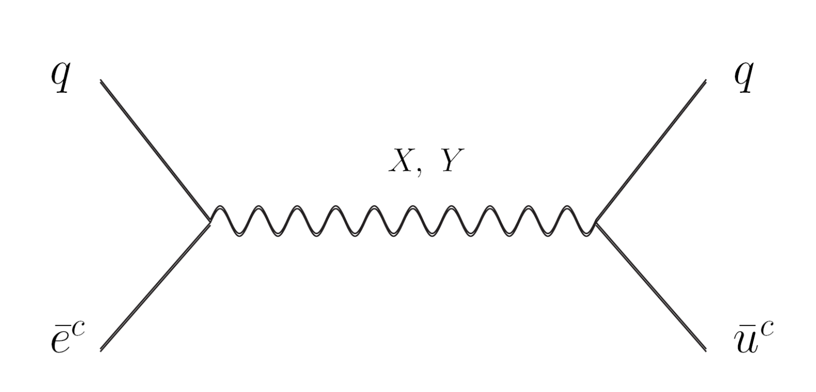
$$M \sim 10^{14}\ \mathrm{GeV} \ \rightarrow\ m_\nu \sim \mathrm{few} \cdot 0.01\ \mathrm{eV}$$

GUT Main Prediction:

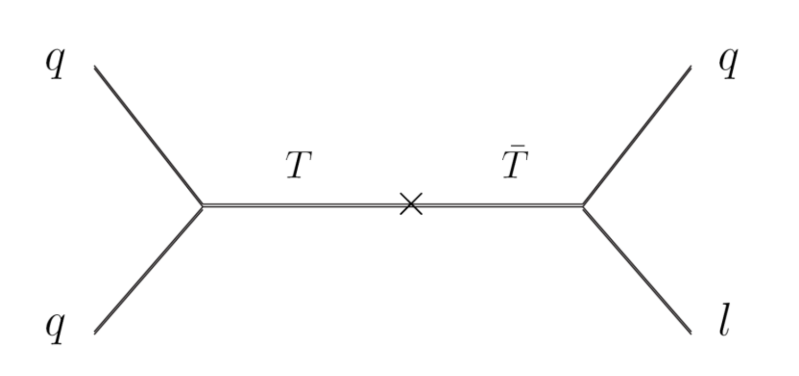
Baryon Number Violation: $\Delta B \neq 0$ → Proton Decay

- **Gauge Mediated d=6 Decay**

$X, Y \subset$ GUT/SM



- **In SUSY: new d=5 Decays**



$T, \bar{T} \subset$ "Unified Higgses" :

$H(5)=(h_u\,, T)\,,\ \bar{H}(\bar{5})=(h_d\,, \bar{T})$

Too Fast in SU(5)

SUSY GUT →

- Charge Quantization,Unification of multiplets $\subset 16\, of\ SO(10)$
- Neutrino Masses (via ν_R of SO(10)); L-violation
- Successful Coupling Unification
- Stab. Hierarchy (Light Higgs) ← low SUSY scale
- Dark Matter Candidate (LSP)
- Baryogenesis via Leptogenesis
- Prediction: B-violation → proton decay

SUSY GUT Problem: Doublet-Triplet (DT) Splitting

SUSY SU(5) example

`Scalar' sector: $\Sigma(24)$, $H(5)=(h_u, T)$, $\bar{H}(\bar{5})=(h_d, \bar{T})$

Colored triplets

$$Wsc = W(\Sigma) + W(\Sigma, H)$$

$$W(\Sigma) = M_\Sigma \Sigma^2 + \lambda_\Sigma \Sigma^3 \leftarrow \text{for GUT breaking (may involve more fields)}$$

$$\langle \Sigma \rangle = V\,\mathrm{Diag}(2,\,2,\,2,\,-3,\,-3) \Rightarrow \quad SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$W(\Sigma, H) = M_H H\bar{H} + \lambda \Sigma H \bar{H} \leftarrow \text{for DT splitting}$$

$$M_D = M_H - 3\lambda V \leq \mathrm{O}(100\,\mathrm{GeV})$$

Fine Tuning

$$M_T = M_H + 2\lambda V \geq M_{GUT} \approx 2 \cdot 10^{16}\,\mathrm{GeV}$$

What is origin of this $\sim 10^{-13}$ hierarchy (tuning) ?
Unexplained in simple SU(5)

SUSY GUT puzzles:

- **GUT Symmetry Breaking? (flat directions/goldstones)**
- **Doublet-Triplet Splitting**
- **How/why mu-Term ~ 100 GeV ? (harder in GUT)**
- **Proton Stability (especially d=5 decay)**
- **Fermion Masses & Mixings (flavor problem)**
- **SUSY FCNC (sflavor problem)**
- **Minimal & Economical System**
  - **Calculability of GUT Threshold Corrections**
  - **Perturbativity all the way up to M_{Planck}**

All these issues are closely related and

Unless *Unified* solution is found,
none of the predictions can be trusted..

Realistic SUSY SO(10)

SO(10) ->
Solution of DT hierarchy via missing VEV

Dimopoulos, Wilczek'81
Babu, Barr'93
Barr, Raby'97

- \`Higgs' System: $H(10) \supset h_u + h_d + T_H + \bar{T}_H \quad \longleftrightarrow \quad 16_3 16_3 H$

$$A(45) + C(16) + \bar{C}(16^*) \quad \text{for} \quad SO(10) \xrightarrow{BR} SU(3)\times SU(2)\times U(1)$$

$$C'(16) + \bar{C}'(16^*) \quad \text{and} \quad Z(1) + S(1)$$

Insuring desirable sym. br. & NO flat dir./pseudo-Golds.

- Additional Symmetries: $U(1)_A \times Z_2$

Symmetry Breaking, All order DT splitting,
mu –term, Nucleon stability (predictions)
Realistic & simple fermion pattern

Some More Studies of Missing VEV Mechanism

Several 45's → Large GUT thresholds

Babu, Barr '93
Chacko, Mohapatra '99

Economical Higgs system With 45+16+16*+10 → Small GUT thresholds

Babu, Pati, Wilczek '99
Barr, Raby '97

Anomalous U(1)→all order Hierarchy, But Several 45's → Large GUT thresholds

Berezhiani, Tavartkiladze '97
Maekawa '01
Maekawa, Yamashita '02

$U(1)_A \times Z_2$ Transformations:

	$A(45)$	$H(10)$	$H'(10)$	$C(16)$	$\bar{C}(\overline{16})$	Z	S	$C'(16)$	$\bar{C}'(\overline{16})$	$16_{1,2}$	16_3
$\mathcal{U}(1)$	0	1	-1	$\frac{k+4}{2k}$	$-\frac{1}{2}$	$\frac{2}{k}$	$\frac{2}{k}$	$\frac{k-4}{2k}$	$-\frac{k+8}{2k}$	-1	$-\frac{1}{2}$
Z_2	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$P_{1,2}$	$+$

`Scalar' Superpotential (fixed):

$$\begin{aligned} W(A) &= M_A \mathrm{tr} A^2 + \frac{\lambda_A}{M_*}\left(\mathrm{tr} A^2\right)^2 + \frac{\lambda'_A}{M_*}\mathrm{tr} A^4 \ , \\ W(A,C,C') &= C\left(\frac{a_1}{M_*}ZA + \frac{b_1}{M_*}C\bar{C} + c_1 S\right)\bar{C}' + C'\left(\frac{a_2}{M_*}ZA + \frac{b_2}{M_*}C\bar{C} + c_2 S\right)\bar{C} \\ W(DT) &= \lambda_1 HAH' + \lambda_{H'}\frac{(S^k, Z^k)}{M_*^{k-1}}(H')^2 + \lambda_2 H\bar{C}\bar{C} + \frac{\lambda_3}{M_*}AH'CC' \ . \end{aligned}$$

Missing VEV Solution: $\langle A\rangle = \mathrm{i}\sigma_2 \otimes \mathrm{Diag}\,(a,\ a,\ a,\ 0,\ 0)$

(incl. FI-term) Fixed VEVs: $\langle A\rangle, \langle C\rangle, \langle \bar{C}\rangle, \langle Z\rangle, \langle S\rangle \neq 0$

Symmetry breaking and VEVs:

Anom. D-term: $$V_D = g^2{}_A(\xi + \Sigma q_\phi |\phi|^2)^2$$

$$c^2 + |z|^2 + |s|^2 = -\frac{k}{2}\xi$$

At least one VEV is fixed. This trigger all remaining VEVs:

F=0 directions: $$-\frac{3a_1}{M_*}za + \frac{b_1}{M_*}c^2 + c_1 s = 0\ , \qquad -\frac{3a_2}{M_*}za + \frac{b_2}{M_*}c^2 + c_2 s = 0$$

$$c,\ \ s,\ \ z \neq 0$$

No flat directions, no pseudo-goldstones
Only Light MSSM states including doublets are massless

DT Splitting to All Orders

$$M_{D,T} = \begin{array}{c} \\ \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_C \\ \bar{5}_{C'} \end{array} \begin{array}{c} \begin{array}{cccc} 5_H & 5_{H'} & 5_{\bar{C}} & 5_{\bar{C}'} \end{array} \\ \begin{pmatrix} 0 & \eta_{D,T}\lambda_1 a & \lambda_2 c & 0 \\ -\eta_{D,T}\lambda_1 a & M_{H'} & 0 & 0 \\ 0 & 0 & 0 & \kappa_{D,T} Y_1 \\ 0 & Y_{D,T} & \kappa_{D,T} Y_2 & M_{C'} \end{pmatrix} \end{array}$$

$$\text{with} \qquad \eta_D = 0\ , \quad \eta_T = 1\ , \quad \kappa_D = 3\ , \quad \kappa_T = 2\ .$$

$\Rightarrow$ massless doublet pair M(hu, hd)=0,

and *all* M(triplets)-heavy

In SUSY limit $\quad \mu - \text{term} = 0$

After SUSY breaking $\quad \mu \sim m_{\text{susy}}$

μ-Term Generation after SUSY breaking

$$\langle A(45)\rangle \sim \begin{pmatrix} M_{\rm GUT} & & & & \\ & M_{\rm GUT} & & & \\ & & M_{\rm GUT} & & \\ & & & m_{\rm susy} & \\ & & & & m_{\rm susy} \end{pmatrix}$$

$$HAH' \quad \Longrightarrow \quad \mu \sim m_{\rm susy}$$

This can always happen with
linear terms ($\sim m_{\rm susy}$) ***in potential***

μ -Term Generation by Triggered VEV $\sim m_{\rm susy}$

GUT VEV Fields: $\Phi_i^0 = v_i + \Phi_i \qquad \langle \Phi_i \rangle = 0$

$$W(\Phi_i) = \tfrac{1}{2}\Phi_i M_\Phi^{ij} \Phi_j + \cdots$$

$$\Phi^\dagger M_\Phi^\dagger M_\Phi \Phi$$

Soft SUSY br. terms

$$m_{susy} M_G^2 \phi_{C'}(\nu^c) \qquad m_{susy} M_G^2 \phi_{\bar{C}'}(\bar{\nu}^c)$$

Two categories of fields: $\Phi_i = (\hat{\Phi}_{\hat{i}}, \tilde{\Phi}_{\tilde{i}})$

$$V = \hat{\Phi}^\dagger \hat{M}_\Phi^\dagger \hat{M}_\Phi \hat{\Phi} + \tilde{\Phi}^\dagger \tilde{M}_\Phi^\dagger \tilde{M}_\Phi \tilde{\Phi} + \hat{\Phi}^\dagger \mathcal{M}_\Phi \tilde{\Phi} + \tilde{\Phi}^\dagger \mathcal{M}_\Phi^\dagger \hat{\Phi} + m_{susy}(\hat{\Phi}^\dagger \hat{\rho} + \hat{\rho}^\dagger \hat{\Phi})$$

$$\langle \hat{\Phi}_{\hat{i}} \rangle = -m_{susy} \left[\hat{M}_\Phi^\dagger \hat{M}_\Phi - \mathcal{M}_\Phi (\tilde{M}_\Phi^\dagger \tilde{M}_\Phi)^{-1} \mathcal{M}_\Phi^\dagger \right]^{-1}_{\hat{i}\hat{j}} \hat{\rho}_{\hat{j}}$$

$$\langle \tilde{\Phi}_{\tilde{i}} \rangle = -\left[(\tilde{M}_\Phi^\dagger \tilde{M}_\Phi)^{-1} \mathcal{M}_\Phi^\dagger \right]_{\tilde{i}\hat{j}} \langle \hat{\Phi}_{\hat{j}} \rangle$$

←Triggered VEV $\sim m_{\rm susy}$

Applying for present SO(10):

Gauge choice

$$Z=(z+Z_0)e^{\Omega}\ ,\qquad S=(s+S_0)e^{\Omega}\ ,\qquad \phi_C(\nu^c)=(c+\frac{\Delta}{\sqrt{2}})e^{\Delta'+\Omega}$$

$$\phi_{\bar C}(\bar\nu^c)=(c+\frac{\Delta}{\sqrt{2}})e^{-\Delta'}\ ,\quad \phi_{C'}(\nu^c)=\phi' e^{\Delta'-\Omega}\ ,\quad \phi_{\bar C'}(\bar\nu^c)=\bar\phi' e^{-\Delta'-2\Omega}$$

$$A(45)=\langle A(45)\rangle\ \oplus\ \frac{1}{\sqrt{24}}\eta_a\ \oplus\ \frac{1}{\sqrt{8}}\eta_b\ ,$$

$$M_\Phi=\begin{array}{c} \\ \phi' \\ \bar\phi' \\ \eta_b \\ \eta_a \\ \Delta \\ S_0 \end{array}\begin{array}{c}\begin{array}{cccccc} \phi' & \bar\phi' & \eta_b & \eta_a & \Delta & S_0 \end{array} \\ \begin{pmatrix} 0 & 0 & \frac{a_2 zc}{M_* 2\sqrt{8}} & -\frac{3a_2 zc}{M_* 2\sqrt{6}} & \frac{\sqrt{2}b_2c^2}{M_*} & c_2 c \\ 0 & 0 & \frac{a_1 zc}{M_* 2\sqrt{8}} & -\frac{3a_1 zc}{M_* 2\sqrt{6}} & \frac{\sqrt{2}b_1c^2}{M_*} & c_1 c \\ \frac{a_2 zc}{M_* 2\sqrt{8}} & \frac{a_1 zc}{M_* 2\sqrt{8}} & M_{\eta_b} & 0 & 0 & 0 \\ -\frac{3a_2 zc}{M_* 2\sqrt{6}} & -\frac{3a_1 zc}{M_* 2\sqrt{6}} & 0 & M_{\eta_b} & 0 & 0 \\ \frac{\sqrt{2}b_2c^2}{M_*} & \frac{\sqrt{2}b_1c^2}{M_*} & 0 & 0 & 0 & 0 \\ c_2 c & c_1 c & 0 & 0 & 0 & 0 \end{pmatrix}\end{array}$$

←Singlet mass matrix.
All massive!

State identification: $\hat\Phi^T=(\phi'\ ,\ \bar\phi')\qquad \tilde\Phi^T=(\eta_b,\ \eta_a,\ \Delta_0,\ S_0)$

VEV in B-L direction: $\langle\eta_b\rangle\simeq 3m_{susy}$

$$\lambda_1 HAH'\rightarrow\lambda_1\frac{\langle\eta_b\rangle}{\sqrt{8}}h_u h_d\sim m_{susy}h_u h_d\qquad\qquad \mu\sim m_{\rm susy}\sim {\rm TeV}$$

GUT Spectrum (all heavy)

Well defined spectrum:

5-plets:

$$M_{D,T} = \begin{array}{c} \\ \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_C \\ \bar{5}_{C'} \end{array} \begin{array}{c} \begin{array}{cccc} 5_H & 5_{H'} & 5_{\bar{C}} & 5_{\bar{C}'} \end{array} \\ \begin{pmatrix} 0 & \eta_{D,T}\lambda_1 a & \lambda_2 c & 0 \\ -\eta_{D,T}\lambda_1 a & M_{H'} & 0 & 0 \\ 0 & 0 & 0 & \kappa_{D,T} Y_1 \\ 0 & Y_{D,T} & \kappa_{D,T} Y_2 & M_{C'} \end{pmatrix} \end{array}$$

$$\text{with} \qquad \eta_D = 0 \ , \quad \eta_T = 1 \ , \quad \kappa_D = 3 \ , \quad \kappa_T = 2 \ .$$

$$\frac{M_{D_1} M_{D_2} M_{D_3}}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} = \frac{9}{4 M_{\text{eff}} \cos\gamma} \ , \qquad \text{with} \qquad \frac{1}{M_{\text{eff}}} = \left(M_T^{-1}\right)_{11} = \frac{M_{H'}}{\lambda_1^2 a^2}$$

10-plets:

$$M(\Psi^{10}) = \begin{array}{c} \\ \Psi^{10}_A \\ \Psi^{10}_C \\ \Psi^{10}_{C'} \end{array} \begin{array}{c} \begin{array}{ccc} \overline{\Psi}^{\overline{10}}_A & \overline{\Psi}^{\overline{10}}_{\bar{C}} & \overline{\Psi}^{\overline{10}}_{\bar{C}'} \end{array} \\ \begin{pmatrix} M_\Psi & 0 & X_1 \\ 0 & 0 & \kappa_\Psi Y_1 \\ X_2 & \kappa_\Psi Y_2 & M'\eta \end{pmatrix} \end{array}$$

$$\text{with} \qquad \Psi = (u^c, q, e^c) \ , \quad \kappa_\Psi = (2, 1, 0) \ , \quad M_\Psi$$

10's fragments masses:

Matter: $$\mathcal{U}_1^c\mathcal{U}_2^c = Y_1Y_2(4+\tilde{p}^2)\ ,\quad \mathcal{Q}_1\mathcal{Q}_2 = Y_1Y_2(1+\tilde{p}^2)\ ,\quad \mathcal{E}_1^c\mathcal{E}_2^c = Y_1Y_2\hat{p}^2$$

$$\tilde{p} = p \qquad \tilde{p}^2 = \frac{|X_1|^2}{|Y_1|^2} = \frac{|X_2|^2}{|Y_2|^2}\ ,\quad \hat{p}^2 = \tilde{p}^2\left|1-\frac{M_\Sigma M_{C'}}{2X_1X_2}\right|$$

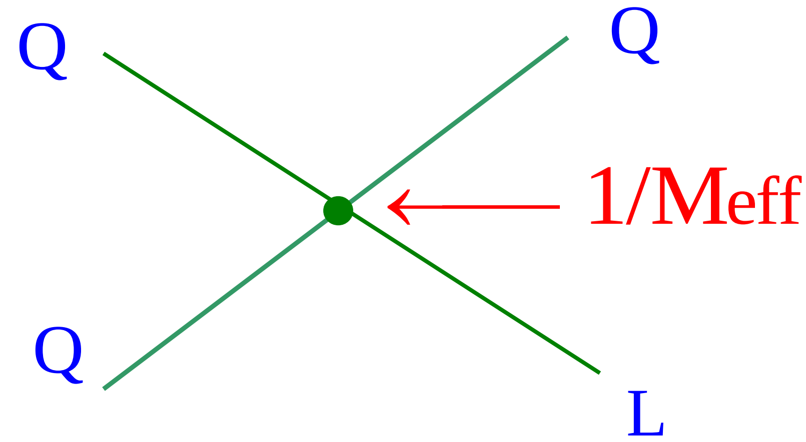
Gauge: $$M^2(X,Y) = g^2a^2 \equiv M_X^2\ ,\quad M^2(X',Y') = M_X^2(1+p^2)$$

$$M^2(V_{u^c,\bar{u}^c}) = M_X^2(4+p^2)\ ,\quad M^2(V_{e^c,\bar{e}^c}) = M_X^2p^2 \qquad p^2 = \frac{4c^2}{a^2}$$

3-matter + 1 vector superfield ←→ N=4 SYM multiplet

Gauge $\beta -$ function $= 0$ **←Great reduction of thresholds**

M$_{\text{eff}}$**:** In d=5 decay



In RG Equations →

$$\frac{M_{D_1} M_{D_2} M_{D_3}}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} = \frac{9}{4 M_{\text{eff}} \cos\gamma}$$

$\cos\gamma \approx \tan\beta/60$

$$\alpha_{\mathrm{U}}^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\Lambda}{M_Z} + \Delta_{i,\,w}^{(2)} + \Delta_i^{\mathrm{GUT}}$$

Calculable Thresholds!

EW/SUSY Scale Thresholds

$(m_{\tilde{q}}, M_{\tilde{W}})$(GeV)	$(m_0, m_{1/2})$(GeV)	$(\Delta^{(2)}_{1,\,w}\;, \Delta^{(2)}_{2,\,w}\;, \Delta^{(2)}_{3,\,w})$
(600, 130)	(450.7, 158)	(0.2113, 0.05328, 0.7747)
(1200, 130)	(1039, 239.5)	(0.4761, 0.3124, 1.037)
(1200, 200)	(1134, 156.4)	(0.4717, 0.4100, 1.178)
(1800, 300)	(1564, 355.5)	(0.6148, 0.6483, 1.4624)
(1800, 125)	(1761, 149.2)	(0.616, 0.4471, 1.1757)
(1500, 250)	(1301, 297.6)	(0.5505, 0.5409, 1.3343)
(1500, 125)	(1452, 150)	(0.5536, 0.3826, 1.107)

RG and Gauge Coupling Unification

$$\ln\frac{M_{\rm eff}\cos\gamma}{M_Z}=\frac{5\pi}{6}\left(3(\alpha_2^{-1}+\Delta^{(2)}_{2,\,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,\,w}-\frac{1}{4\pi})-(\alpha_1^{-1}+\Delta^{(2)}_{1,\,w})\right)$$

$$-\ln\frac{4\kappa^{5/2}}{9}+\ln\frac{(4+p^2)^{3/2}(1+\tilde{p}^2)^2}{(4+\tilde{p}^2)^{3/2}(1+p^2)^2}+\ln\frac{p}{\hat{p}}\ ,$$

$$\ln\frac{(M_X^2M_\Sigma)^{1/3}}{M_Z}=\frac{\pi}{18}\left(5(\alpha_1^{-1}+\Delta^{(2)}_{1,\,w})-3(\alpha_2^{-1}+\Delta^{(2)}_{2,\,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,\,w}-\frac{1}{4\pi})\right)$$

$$+\frac{1}{6}\ln\kappa-\frac{1}{6}\ln\frac{(4+p^2)(1+\tilde{p}^2)^2}{(4+\tilde{p}^2)(1+p^2)^2}-\frac{1}{3}\ln\frac{p}{\hat{p}}\ .$$

Model Gives: $p=\tilde{p}$; with $\hat{p}/p \sim 10^{-4}$, $\cos\gamma \approx 1/20$

$\alpha_3(M_Z)=0.1176$, $M_{\rm eff}=\text{few}\times 10^{19}$ GeV

$(M_X{}^2 M_\Sigma)^{1/3} \approx 10^{16}$ GeV

[Compare with SU(5) - Hisano, Murayama,Yanagida'93**]**

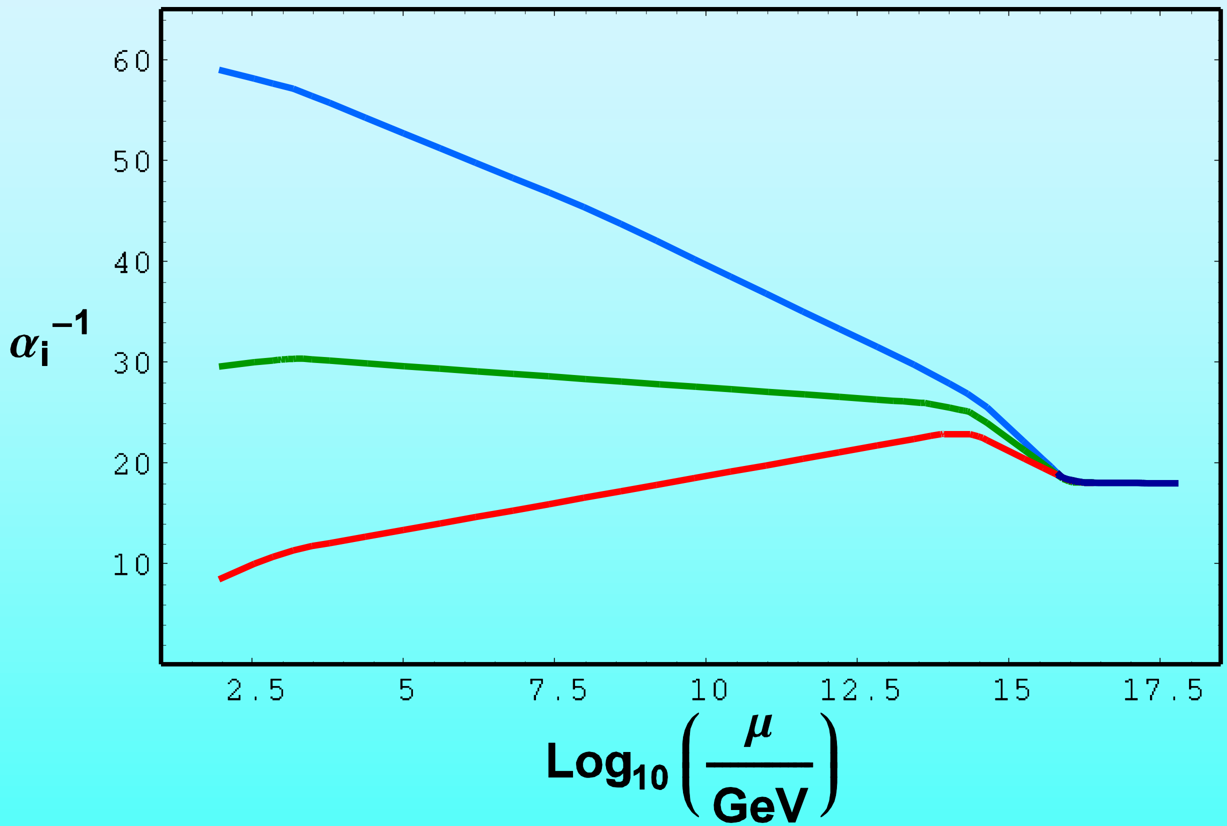
Large $M_{\rm eff}$ suppress Nucleon decay:

$$\Gamma^{-1}_{d=5}(p \to \bar{\nu} K^+) = 3\times 10^{33}{\rm yrs} \left(\frac{0.012{\rm GeV}^3}{|\beta_H|}\right)^2 \left(\frac{6.8}{\bar{A}^{\alpha}_S}\right)^2 \left(\frac{1.25}{R_L}\right)^2 \left(\frac{M_{\rm eff}}{4.51 \times 10^{19}{\rm GeV}}\right)^2 \times$$

$$\times \left(\frac{m_{\tilde{q}}}{1.5{\rm TeV}}\right)^4 \left(\frac{190{\rm GeV}}{m_{\tilde{W}}}\right)^2 \left(\frac{3.1}{K^{\nu}_{d=5}}\right) \; .$$

And compatible with coupling unification..

Gauge Coupling Unification



Compare with SUSY SU(5)

$\mathrm{M_{eff}} \cos\gamma \to \mathrm{M_T}$ □ d=5 decay $(p \to \bar{\nu} K^{+})$

- **Minimal renorm. SUSY SU(5) →**

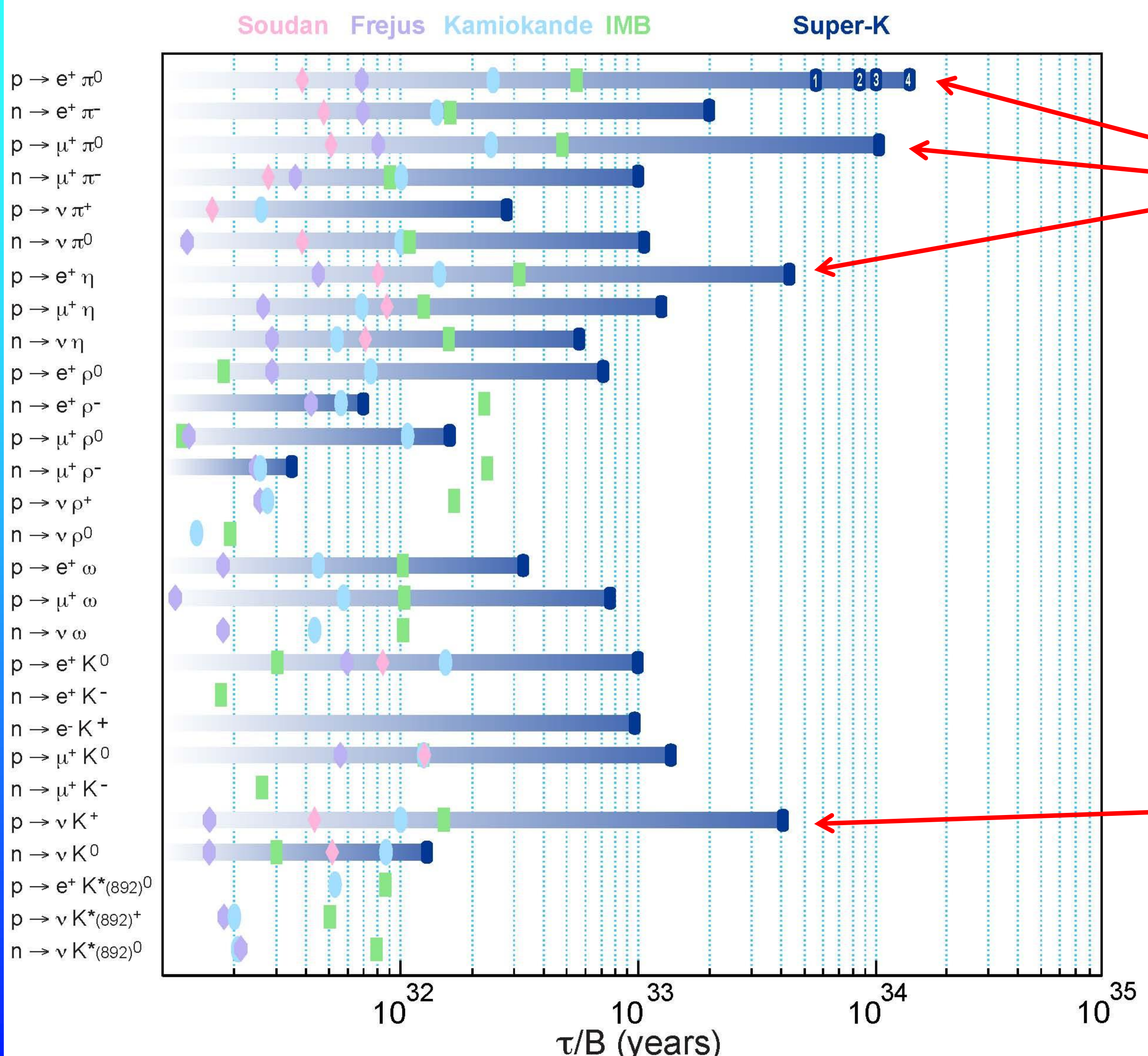
$$\Gamma^{-1}_{d=5}(p \to \bar{\nu} K^{+}) \simeq 1.2 \cdot 10^{31}\ \mathrm{yrs} \times \left(\frac{0.012\mathrm{GeV}^3}{\beta_H}\right)^2 \left(\frac{7}{\bar{A}^{\alpha}_{S}}\right)^2 \left(\frac{1.25}{R_L}\right)^2 \times$$

$$\times \left(\frac{M_T}{2 \cdot 10^{16}\mathrm{GeV}}\right)^2 \left(\frac{m_{\tilde{q}}}{1.5\mathrm{TeV}}\right)^4 \left(\frac{190\mathrm{GeV}}{M_{\tilde{W}}}\right)^2 ,$$

For $\tau_{exp}(p \to \bar{\nu} K^{+}) \gtrsim 4 \cdot 10^{33}$ yrs. **One needs** $M_T \gtrsim 3.6 \cdot 10^{17}$ GeV

How large can be M_T ?

Exp. limits on Nucleon lifetime [*from talk of M. Miura,BLV2011*]



In d=5 suppressed by λu

GUT's dominant d=5 mode

- **Minimal renorm. SUSY SU(5) →**

RG: $$\ln\frac{M_T}{M_Z}=\frac{5\pi}{6}\left(3(\alpha_2^{-1}+\Delta^{(2)}_{2,\,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,\,w}-\frac{1}{4\pi})-(\alpha_1^{-1}+\Delta^{(2)}_{1,\,w})\right)$$

$$\alpha_3(M_Z)=0.12\quad\rightarrow\quad M_T\simeq 7\cdot 10^{14}\ \mathrm{GeV}\Longrightarrow\ \tau(p\rightarrow\overline{\nu}K^+)\simeq 1.5\cdot 10^{28}\ \mathrm{yrs}$$

$$M_T\gtrsim 2\cdot 10^{16}\ \mathrm{GeV}\quad\Longrightarrow\quad\alpha_3(M_Z)\gtrsim 0.132$$

[← also problem of unification]

In gross conflict with experiments:

$$\alpha_3^{exp}(M_Z)=0.1176\pm 0.002$$

$$\tau_{exp}(p\rightarrow\overline{\nu}K^+)\gtrsim 4\cdot 10^{33}\ \mathrm{yrs.}$$

- **One solution:**

SUSY SU(5) with high dim. Ops.→

Large threshold corrections->

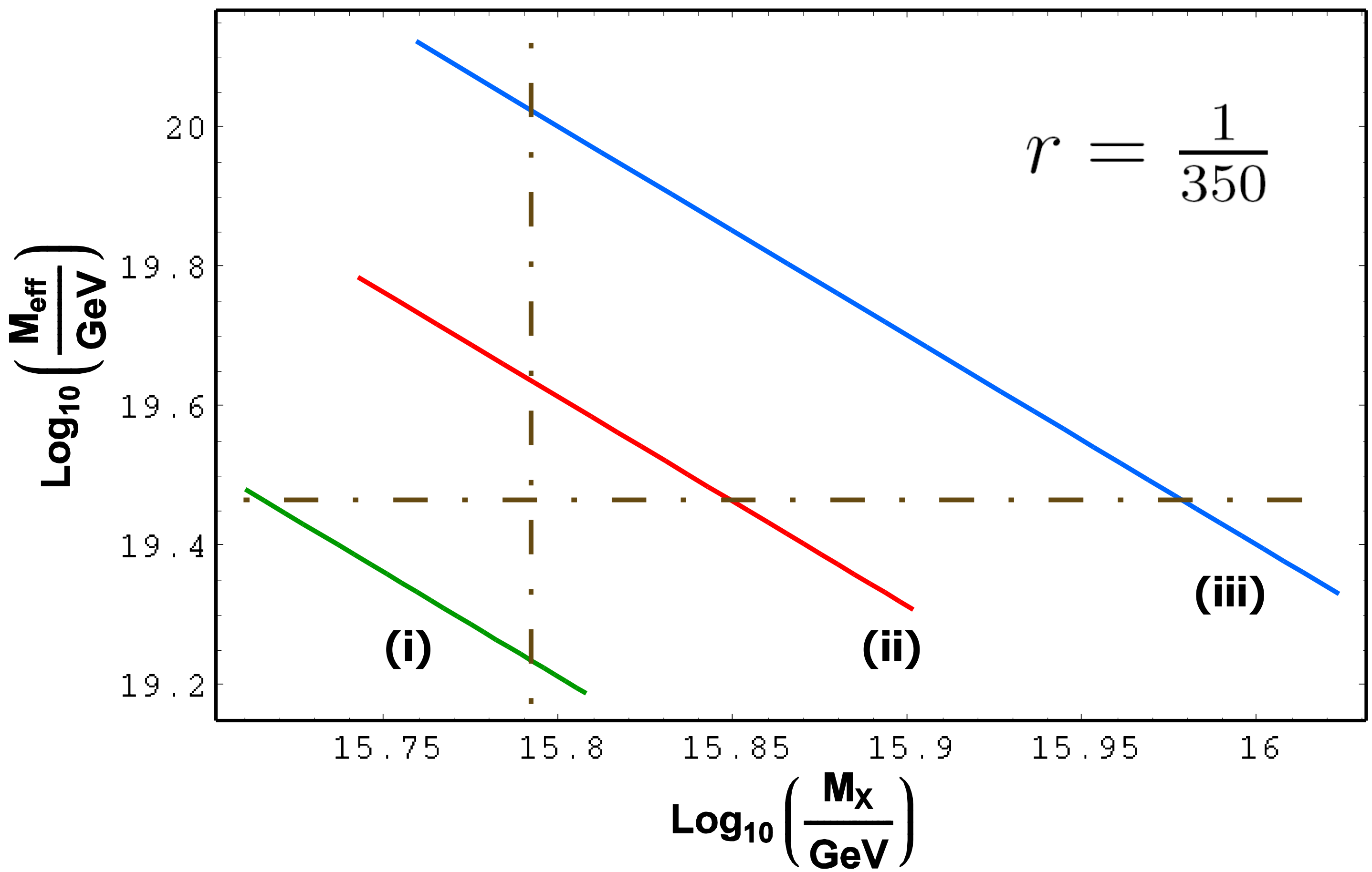
(Bajc, Perez, Senjanovic '02)

$$\mathrm{M_T}\geq 10^{17}\ \mathrm{GeV}$$

Considered SO(10) has no these problems

Correlation Between M_{eff} & M_X

$$M_{\rm eff} \simeq 10^{19}{\rm GeV}\cdot\left(\frac{10^{16}{\rm GeV}}{M_X}\right)^3\left(\frac{1/100}{r}\right)\left(\frac{0.63\times 3}{\eta_\gamma \tan\beta}\right)\left\{\frac{\exp[2\pi(\Delta^{(2)}_{2,\,w}-\Delta^{(2)}_{3,\,w}-\delta\alpha_3^{-1})]}{2.24\cdot 10^{-2}}\right\}$$



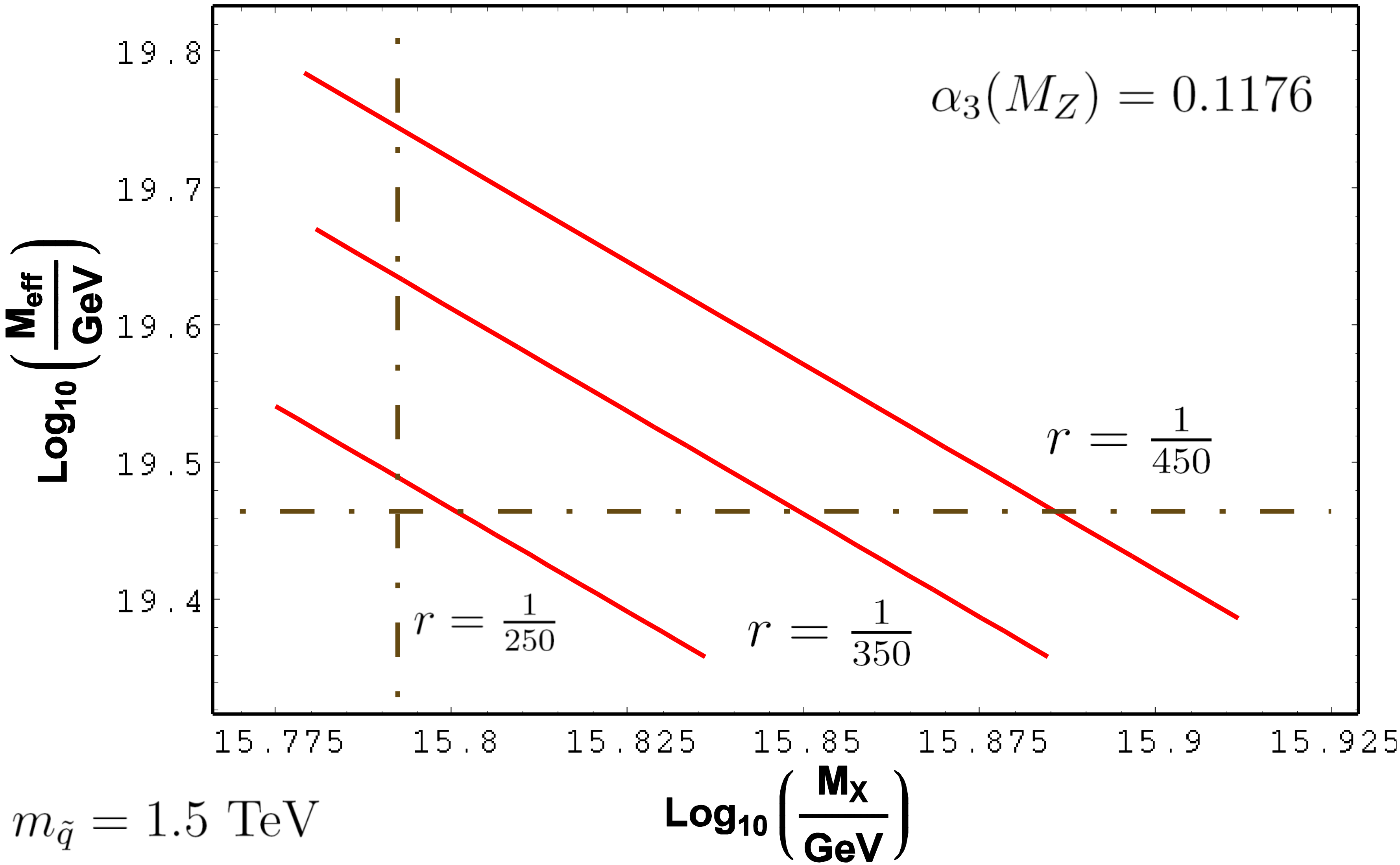
$r \equiv M_\Sigma / M_X$

$m_{\tilde{q}} = 1.5$ TeV

$M_{\tilde{W}} = 190$ GeV

(i) $\alpha_3 = 0.1156$ **(ii)** $\alpha_3 = 0.1176$ **(iii)** $\alpha_3 = 0.1196$

Correlation Between M_{eff} & M_X



$m_{\tilde{q}} = 1.5 \text{ TeV}$

$M_{\tilde{W}} = 190 \text{ GeV}$

→Correlation Between d=5 & d=6 Proton Decays and Upper Bounds on Lifetimes

Symmetries give/suggest ranges: $\frac{1}{10} \lesssim r\,(= M_{\Sigma}/M_X) \lesssim \frac{1}{500}$

$$1.4 \cdot 10^{19}\,\mathrm{GeV} \le \mathrm{M}_{\mathrm{eff}} \le 10^{20}\,\mathrm{GeV}$$

\`Naturalness' Requirement: $m_{\tilde{q}} \lesssim 1.5\ \mathrm{TeV}$

Empirical Bound: $M_{\tilde{W}} \gtrsim 190\ \mathrm{GeV}$

& Correlation Leads to:

$$\Gamma^{-1}_{d=6}(p \to e^{+}\pi^{0}) \lesssim 1.4 \times 10^{35}\ \mathrm{yrs}$$

$$\Gamma^{-1}(p \to \overline{\nu} K^{+}) \leq 6.5 \times 10^{34}\ \mathrm{yrs}$$

Potentially observable with improvement of exp. sensitivity by factor 10

Comment: For calculating d=5 Proton decay, well defined Yukawa sector is important

Yukawa Sector with Q4 Flavor Symmetry

Q4 → Interesting (predictive) Textures, Solves SUSY FCNC problem

Pouliot, Seiberg '93
(*) Babu, Kubo '05

$$\vec{q} \equiv (\hat{q}_1, \hat{q}_2) \qquad m^2_{\rm susy}\left(|\tilde{q}_1|^2 + |\tilde{q}_2|^2\right)$$

$$Q_4\!: \overrightarrow{16} \equiv (16_1, 16_2) \sim \mathbf{2}, \qquad 16_3 \sim \mathbf{1} \qquad \overrightarrow{X}, \overrightarrow{Y} \sim \mathbf{2} - \mathrm{Flavons}$$

(*):

$$\mathbf{1}' \times \mathbf{1}' = \mathbf{1}'' \times \mathbf{1}'' = \mathbf{1}''' \times \mathbf{1}''' = \mathbf{1} \qquad \mathbf{1}' \times \mathbf{1}'' = \mathbf{1}'''$$

$$\mathbf{1}'' \times \mathbf{1}''' = \mathbf{1}' \qquad \mathbf{1}''' \times \mathbf{1}' = \mathbf{1}''$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} = (x_1 y_2 - x_2 y_1)_{\mathbf{1}} \oplus (x_1 y_1 + x_2 y_2)_{\mathbf{1}'} \oplus (x_1 y_2 + x_2 y_1)_{\mathbf{1}''} \oplus (x_1 y_1 - x_2 y_2)_{\mathbf{1}'''}$$

Q4 Breaking: No flat directions and massless modes

$$\langle \overrightarrow{X} \rangle = (x_1, 0) \qquad \langle \overrightarrow{Y} \rangle = (y_1, 0) \qquad\qquad x_1, y_1 \neq 0$$

Additional Singlets: $S_1 \sim 1', \quad S_2 \sim 1, \quad \hat{X}, \hat{Y} \sim 1''$

For VEV fixing and mass generation

$$W_{\text{Flavon}} = S_1 \left(\frac{Z^6}{M_*^6} \overrightarrow{X}^2 + \frac{1}{M_*^4} \overrightarrow{X}^6 \right) + S_2 \left(\frac{1}{M_*^2} \overrightarrow{X}^2 \overrightarrow{Y}^2 + \frac{1}{M_*^8} Z^{10} \right) + \hat{X} \overrightarrow{X}^2 + \hat{Y} \overrightarrow{Y}^2$$

$$F_Z = F_{S_{1,2}} = F_{\overrightarrow{X}} = F_{\overrightarrow{Y}} = 0$$

$$\langle S_1 \rangle = \langle S_2 \rangle = 0 \qquad x_1 \sim z \left(\frac{z}{M_*} \right)^{1/2} \sim \frac{M_*}{40} \qquad y_1 \sim z^5/(x_1 M_*^3) \sim \frac{M_*}{200}$$

$$\hat{X} \overrightarrow{X}^2 + \hat{Y} \overrightarrow{Y}^2 \rightarrow \hat{X} X_1 X_2 + \hat{Y} Y_1 Y_2$$

All states are heavy

Effective Yukawa Interactions are fixed by symmetries → Predictive

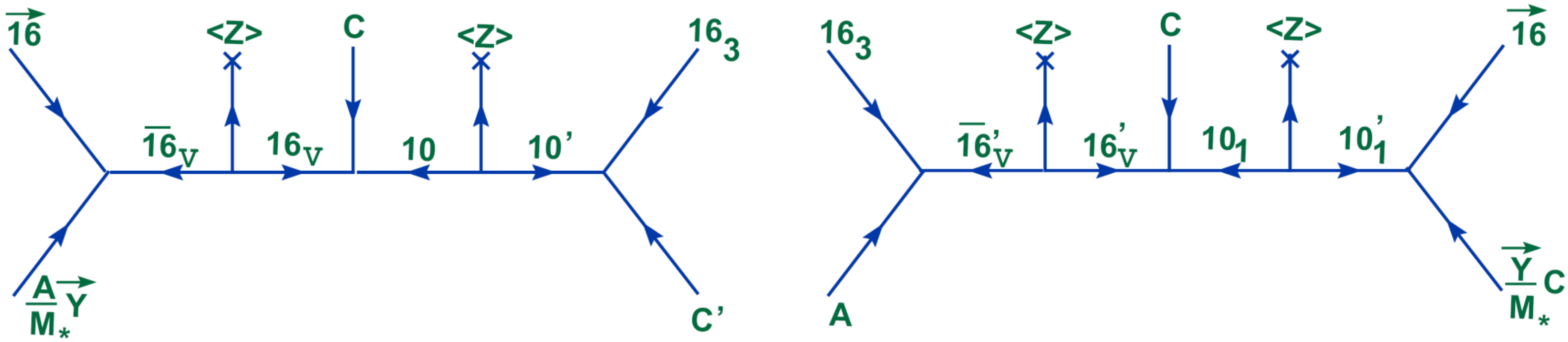
$$W^{(D)}_{\rm Yukawa} = 16_3 16_3 H + \frac{\overrightarrow{X}}{M_*}\overrightarrow{16}16_3 H + \frac{SZ^2A}{M_*^4}\overrightarrow{16}\overrightarrow{16}H + \frac{Z^3C}{M_*^4}\overrightarrow{16}\overrightarrow{16}C' +$$

$$\frac{AC\overrightarrow{Y}}{M_*\langle Z\rangle^2}\left(\overrightarrow{16}\cdot 16_3 + 16_3\cdot\overrightarrow{16}\right)C' + \frac{\overrightarrow{X}^2}{M_*^3}(\overrightarrow{16}^2)CC' \ .$$

$$\langle\overrightarrow{X}\rangle \simeq \left(\tfrac{M_*}{40}, 0\right) \qquad \langle\overrightarrow{Y}\rangle \simeq \left(\tfrac{M_*}{200}, 0\right)$$

Can be obtained by integrating of heavy states

Generation of Effective Yukawa Couplings



This 16, 10-plet exchange gives Clebsch 3 for lepton vs. quark
→ Good relation

Planck Scale induced d=5 p-decay is strongly suppressed

By model's symmetries: **QQQL –type ops.**

$$Z\overrightarrow{X}\,\overrightarrow{Y}^2\,\overrightarrow{16}^3\,16_3,\;\; S\overrightarrow{Y}^2\,\overrightarrow{16}^2\,16_3^2,\;\; Z^2 S\,\overrightarrow{Y}\,\overrightarrow{16}\,16\,16_3^3$$

Emerged terms $q_1 q_2 q_2 l_i / M_*$ **have suppression** $\lesssim \mathrm{few} \times 10^{-9}$

Adequate suppression

Mass Matrices

$$Y_u = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \end{array} \begin{array}{c} \begin{array}{ccc} u_1^c & u_2^c & u_3^c \end{array} \\ \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \sigma \\ 0 & \sigma & 1 \end{pmatrix} \end{array} \lambda_t$$

$$Y_d = \begin{array}{c} \\ d_1 \\ d_2 \\ d_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1^c & d_2^c & d_3^c \end{array} \\ \begin{pmatrix} 0 & \epsilon' + \eta' & 0 \\ -\epsilon' - \eta' & \xi_{22}^d & \sigma + \epsilon \\ 0 & \sigma + \overline{\epsilon} & 1 \end{pmatrix} \end{array} \lambda_b$$

$$Y_e = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{ccc} e_1^c & e_2^c & e_3^c \end{array} \\ \begin{pmatrix} 0 & -3\epsilon' - \eta' & 0 \\ 3\epsilon' + \eta & 3\xi_{22}^d & \sigma + 3\overline{\epsilon} \\ 0 & \sigma + 3\epsilon & 1 \end{pmatrix} \end{array} \lambda_b$$

$$Y_\nu = \begin{array}{c} \\ \nu_1 \\ \nu_2 \\ \nu_3 \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1^c & \nu_2^c & \nu_3^c \end{array} \\ \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & \sigma \\ 0 & \sigma & 1 \end{pmatrix} \end{array} \lambda_t$$

$$\lambda_b = \lambda_t \cos \gamma$$

$$\sigma = 0.0508, \epsilon = -0.0188 + 0.0333i \qquad \overline{\epsilon} = 0.106 + 0.0754i$$

Input:

$$\epsilon' = 1.56 \cdot 10^{-4}, \eta' = -0.00474 + 0.00177i, \xi^d_{22} = 0.014e^{4.1i}$$

At low energies: **(perform RG running)**

Input: $\tan\beta = 3,\ m_t(m_t) = 160\ \text{GeV} \qquad m_\tau(M_Z) = 1.746\ \text{GeV}$

→

- $m_e = 0.51\ \text{MeV} \qquad m_\mu = 105.66\ \text{MeV} \qquad m_\tau = 1.776\ \text{GeV}$

- $m_u(2\ \text{GeV}) = 3.55\ \text{MeV},\ m_c(m_c) = 1.15\ \text{GeV}$

- $m_b(m_b) = 4.67\ \text{GeV}$
  $m_d(2\ \text{GeV}) = 6.45\ \text{MeV},\ m_s(2\ \text{GeV}) = 137.6\text{MeV}$

- **At M(Z) scale:**

$$|V_{us}| = 0.225 \ , \quad |V_{cb}| = 0.0414 \ , \quad |V_{ub}| = 0.0034 \ , \quad |V_{td}| = 0.00878$$

CP violation:

- $\overline{\eta} = 0.334 \ , \quad \overline{\rho} = 0.12$

$$\overline{\rho} + i\overline{\eta} = -V_{ud}V_{ub}^{*}/(V_{cd}V_{cb}^{*})$$

$$(\ \sin 2\beta = 0.663\ )$$

All in good agreement with experiments

RG from GUT scale down to low energies:

$$\left.\frac{m_{u,c}}{m_t}\right|_{m_t} = \eta_t^3 \frac{m^0_{u,c}}{m^0_t} \qquad m_u(2\ \mathrm{GeV}) = 1.8 m_u(m_t),\ m_c(m_c) = 2.11 m_c(m_t)$$

$$\eta_t = 1.1097 \qquad \eta_t = \exp\left[\frac{1}{16\pi^2}\int_{M_Z}^{M_G} \lambda_t^2(\mu)\, d\ln\mu\right]$$

$$\frac{m_b}{m_\tau} = \eta_t^{-1} R_{b\tau}\ \frac{m^0_b}{m^0_\tau} \qquad R_{b\tau} = \exp\left[\frac{1}{3\pi}\int_{M_Z}^{M_G} (4\alpha_3(\mu) - \alpha_1(\mu))\, d\ln\mu\right]$$

$$\left.\frac{m_{d,s}}{m_b}\right|_{M_Z} = \left.\frac{m_{d,s}}{m_b}\right|_{m_t} = 1.1159 \frac{m^0_{d,s}}{m^0_b}$$

$$V_{\alpha\beta} = \eta_t\ V^0_{\alpha\beta} \quad \text{for} \quad \alpha\beta = (ub,\ cb,\ td,\ ts) \qquad V_{\alpha\beta} = V^0_{\alpha\beta} \quad \alpha\beta = (ud,\ us,\ cd,\ cs,\ tb)$$

$$|V_{us}| = |V^0_{us}| \qquad |V_{cb}| = \eta_t |V^0_{cb}| \qquad |V_{ub}| = \eta_t |V^0_{ub}| \qquad \overline{\eta} = \overline{\eta}^0$$

'Majorana' Couplings:

$$W_{\text{Yukawa}}^{(M)} = \frac{Z^{k-4}\overrightarrow{Y}^2}{M_*^{k-3}M_{N''}^2}\overrightarrow{16}^2\bar{C}^2 + \frac{Z^{k-2}\overrightarrow{Y}}{M_*^{k-2}M_{N'}^2}\overrightarrow{16}16_3\bar{C}^2 + \frac{Z^{k-1}S}{M_*^{k-1}M_N^2}16_3^2\bar{C}^2$$

$$M_R = \begin{array}{c c} & \begin{array}{ccc} \nu_1^c & \nu_2^c & \nu_3^c \end{array} \\ \begin{array}{c} \nu_1^c \\ \nu_2^c \\ \nu_3^c \end{array} & \left(\begin{array}{ccc} b & 0 & 0 \\ 0 & b & a \\ 0 & a & 1 \end{array} \right) \end{array} M_0$$

← **3 parameters**

- **Input:** $\{(a,\, b) \quad M_0\}$ **or** $\{\sqrt{\Delta m^2_{\text{atm}}},\, \theta_{12} \text{ and } \theta_{23}\}$

- **Output/prediction:** $\sqrt{\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}} \simeq m_2/m_3 \text{ and } \theta_{13}$

Neutrino Masses & Mixings

$$m_\nu = m_D \frac{1}{M_{\nu^C}} m^T$$

- **With input:**

$$M_0 = 1.89 \cdot 10^{13}\ \text{GeV}$$

$$a = 0.0252 e^{-0.018i},\ b = 1.61 \cdot 10^{-6} e^{-1.592i}$$

$$\sqrt{\Delta m^2_{\text{atm}}} \approx 0.05\ \text{eV}$$

$$\theta_{12} \simeq 30^o$$

$$\theta_{23} \simeq 43^o$$

- **Output/prediction:**

$$\sqrt{\Delta m^2_{\text{sol}} / \Delta m^2_{\text{atm}}} \simeq m_2/m_3 \simeq 0.13$$

$$\theta_{13} \simeq 3.6^o$$

Summary

- Introduction / Motivations

  Presented minimal/economical SUSY SO(10)

- Minimal Higgs System, Simple GUT Breaking
  and Natural DT Splitting to all orders

- Natural mu-term Generation

- Calculable GUT Threshold Corrections

Summary (continued)

- Stabile Proton with Predictive (& Testable) Correlation Between d=5 & d=6 Decay Modes
  Upper bounds on decay modes

- Simple, realistic fermion pattern by Q4 flavor symmetry, and No SUSY FCNC problem

- Realistic Neutrino sector

Thank You