

Non-linear supersymmetry and MSSM

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- ① Non-linear SUSY and goldstino
- ② Description of low energy couplings
I.A.-Tuckmantel '04; Komargodski-Seiberg '09
- ③ Non-linear MSSM
I.A.-Dudas-Ghilencea-Tziveloglou '10
- ④ Extended supersymmetry and brane dynamics
Bagger-Galperin '97; I.A.-Derendinger-Maillard '08
Ambrosetti-I.A.-Derendinger-Tziveloglou '09 + '10

Non-linear supersymmetry $\Rightarrow$ goldstino mode χ

Volkov-Akulov '73

Why study goldstino interactions:

- Effective field theory of SUSY breaking at low energies $m_\chi \ll m_{\text{susy}}$
e.g. gauge mediation dominant vs gravity mediation

χ : longitudinal gravitino with $m_\chi \simeq \frac{m_{\text{susy}}^2}{M_{\text{Planck}}} \lesssim m_{\text{soft}} \ll m_{\text{susy}}$

$M_{\text{Planck}} \rightarrow \infty$: SUGRA decoupled

massless χ coupled to matter $\sim 1/m_{\text{susy}}$

- Brane dynamics: half SUSY of the bulk broken but NL realized
 $\Rightarrow$ strongly constrain coupling of brane to bulk fields

Non-linear SUSY transformations: [5]

$$\delta\chi_\alpha = \frac{\xi_\alpha}{\kappa} + \kappa \Lambda_\xi^\mu \partial_\mu \chi_\alpha \quad \Lambda_\xi^\mu = -i (\chi \sigma^\mu \bar{\xi} - \xi \sigma^\mu \bar{\chi})$$

κ : goldstino decay constant (SUSY breaking scale) $\kappa = 1/(\sqrt{2}m_{susy}^2)$

Goldstino interactions: 3 formulations

- Standard realization

Volkov-Akulov '73, Clark-Love '96, Clark-Lee-Love-Wu '98

- Superfield formalism

Ivanov-Kapustnikov '78, Samuel-Wess '83

Brignole-Feruglio-Zwirner '97, Luty-Ponton '98, I.A.-Tuckmantel '04

- Constrained superfields

Rocek-Tseytlin '78, Lindstrom-Rocek '79, Komargodski-Seiberg '09

Standard realization

Define the 'vierbein': $E_\mu^a = \delta_\mu^a + \kappa^2 t_\mu^a$ $t_\mu^a = i\chi \overset{\leftrightarrow}{\partial}_\mu \sigma^a \bar{\chi}$

$\delta(\det E) = \kappa \partial_\mu \left(\Lambda_\xi^\mu \det E \right) \Rightarrow$ invariant action:

$$S_{VA} = -\frac{1}{2\kappa^2} \int d^4x \det E = -\frac{1}{2\kappa^2} - \frac{i}{2} \chi \sigma^\mu \overset{\leftrightarrow}{\partial}_\mu \bar{\chi} + \dots$$

Generalization to matter and gauge fields:

$$S_{eff} = \int d^4x \det E \mathcal{L}_{SM}(\phi) \quad \text{invariant if } \delta\phi = \kappa \Lambda_\xi^\mu \partial_\mu \phi \quad \text{and so } \mathcal{L}_{SM}$$

However problem with derivatives $\Rightarrow$ define SUSY covariant ones:

$$\mathcal{D}_a \phi \equiv (E^{-1})_a^\nu D_\nu \phi \quad \mathcal{F}_{ab} \equiv (E^{-1})_a^\lambda (E^{-1})_b^\rho F_{\lambda\rho}$$

$$\mathcal{L}_{eff} = \det E \mathcal{L}_{SM}(\phi, \mathcal{D}_\mu \phi) = \mathcal{L}_{SM}(\phi, D_\mu \phi) + \kappa^2 t^{\mu\nu} T_{\mu\nu} + \dots$$

universal coupling to stress-tensor but NOT the most general inv action

Superfield formalism

Recipe: $\phi(x) \rightarrow \Phi(x, \theta, \bar{\theta}) \equiv \phi(\tilde{x}) \quad \tilde{x}^\mu = x^\mu + \Lambda_\theta^\mu(\tilde{x})$ [3] [10]

$$= \phi(x) + \kappa \Lambda_\theta^\mu \partial_\mu \phi + \dots \Rightarrow$$

Goldstino (spinor) superfield: $\mathcal{G}_\alpha = \frac{\theta_\alpha}{\kappa} + \chi_\alpha(\tilde{x})$

space-time derivatives: use the 'vierbein' $E(\tilde{x})$

$$\text{e.g. } \mathcal{F}_{ab}(x, \theta, \bar{\theta}) \equiv \left[(E^{-1})^\lambda_a (E^{-1})^\rho_b F_{\lambda\rho} \right] (\tilde{x})$$

List of lowest dim operators

2 operators of dim 6 linear in χ [8]

$$S_1 = C_1 \int d^4x \, \kappa F_{\mu\nu} \psi \sigma^\mu \partial^\nu \bar{\chi} + h.c. \quad S_2 = C_2 \int d^4x \, \kappa (\psi \partial_\alpha \chi) D^\alpha \phi + h.c.$$

Quadratic in χ : 1 operator of dim 7

$$S_7 = C_7 \int d^4x \, \kappa^{3/2} \phi_1 \phi_2 \partial_\mu \chi J_{(\frac{1}{2},0)}^{\mu\nu} \partial_\nu \chi + h.c. \quad J_{(\frac{1}{2},0)}^{\mu\nu} = \frac{i}{4} \sigma^{[\mu} \bar{\sigma}^{\nu]}$$

+ 5 operators of dim 8

$$S_3 = C_3 \int d^4x \, \kappa^2 (\psi_1 \partial^\mu \chi) (\bar{\psi}_2 \partial_\mu \bar{\chi}) + h.c. \quad S_4 = C_4 \int d^4x \, \kappa^2 (\psi_1 \psi_2) (\partial_\mu \chi \partial^\mu \chi) + h.c.$$

$$S_5 = C_5 \int d^4x \, \kappa^2 \phi_1 \overleftrightarrow{D}_\mu \phi_2 i \partial_\alpha \chi \sigma^\mu \partial^\alpha \bar{\chi} + h.c.$$

$$S_6 = C_6 \int d^4x \, \kappa^2 \partial^\alpha \chi \sigma^\mu \partial^\nu \bar{\chi} \partial_\alpha F_{\mu\nu} + h.c.$$

$$S_8 = C_8 \int d^4x \, \kappa^2 \phi_1 \phi_2 \phi_3 (\partial_\mu \chi J_{(\frac{1}{2},0)}^{\mu\nu} \partial_\nu \chi) + h.c.$$

D-brane examples

Type II (closed) strings on 4d Minkowski $M_4 \times X_6$ internal 6d manifold

X_6 flat $\Rightarrow N = 8$ SUSY ; X_6 Calabi-Yau $\Rightarrow N = 2$ SUSY

Single stack of N D p -branes $\Rightarrow$ half SUSY is spontaneously broken $p \geq 3$

$(p - 3)$ dims wrapped around cycles in $X_6 \Rightarrow 4d$ effective field theory

- Gauge group: $G = U(N)$ (generically)
- SUSY: half remains unbroken Q_e ; other half NL realized Q_o
broken SUSY commutes with $G \Rightarrow$ goldstino = $U(1)$ gaugino of Q_e

Intersecting branes: all SUSY is generally broken except for special angles

e.g. $\theta_1 + \theta_2 + \theta_3 = 0$ for $X_6 = T^2 \times T^2 \times T^2$

goldstino becomes a combination of the 2 gauginos

1) Goldstino decay constant: sum of brane tensions

$$\frac{1}{2\kappa^2} = T_1 + T_2 \quad T_i = \frac{M_s^4}{4\pi^2 g_i^2} N_i$$

2) Goldstino couplings: only 3 non-vanishing up to order κ^2 [6]

$$C_1 = \sqrt{2} \quad ; \quad C_2 = 2 \quad ; \quad C_3 = 2$$

- universal coefficients independent of brane-angles
- C_3 : fixes the field theory ambiguity of 4-fermion operator

Brignole-Feruglio-Zwirner '97, I.A.-Benakli-Laugier '01

Constrained superfields

spontaneous global SUSY: no supercharge but still conserved supercurrent

⇒ superpartners exist in operator space (not as 1-particle states)

⇒ constrained superfields: 'eliminate' superpartners

Goldstino: chiral superfield X_{NL} satisfying $X_{NL}^2 = 0 \Rightarrow$ [21]

$$\begin{aligned} X_{NL}(y) &= \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F & y^\mu &= x^\mu + i\theta\sigma^\mu\bar{\theta} \\ &= F\Theta^2 & \Theta &= \theta + \frac{\chi}{\sqrt{2}F} \end{aligned}$$

$$\mathcal{L}_{NL} = \int d^4\theta X_{NL}\bar{X}_{NL} - \frac{1}{\sqrt{2}\kappa} \left\{ \int d^2\theta X_{NL} + h.c. \right\} = \mathcal{L}_{VA}$$

$$F = \frac{1}{\sqrt{2}\kappa} + \dots$$

Constrained matter superfields

- Fermions: Q_{NL} satisfying $Q_{NL}X_{NL} = 0$ (eliminate sfermions) $\Rightarrow$

$$Q_{NL} = \sqrt{2} \left(\psi - \frac{F_Q \chi}{F} \right) \Theta + F_Q \Theta^2$$

- Complex scalars: H_{NL} with $X_{NL}\bar{H}_{NL} = \text{chiral}$ (eliminate 'higgsinos') [5]

$$\Rightarrow H_{NL} = H(\hat{y}) \quad \hat{y} = y^\mu + i\sqrt{2}\theta\sigma^\mu\bar{\chi}(\hat{y})/\bar{F}(\hat{y})$$

- Gauge fields: V_{NL} convenient gauge choice: $X_{NL}V_{NL} = 0$

eliminate gauginos: $X_{NL}W_{NL} = 0$ field strength $W = -\frac{1}{4}\bar{D}^2DV$

$$\Rightarrow V_{NL} = -\Theta \left(\sigma^m V_m + \frac{D}{|F|^2} \chi \bar{\chi} \right) \bar{\Theta} + \frac{1}{2} \Theta^2 \bar{\Theta}^2 D + \text{derivatives}$$

Goldstino couplings to matter supermultiplets

Before: goldstino coupling to non-SUSY matter $E \ll m_{\text{soft}}, m_{\text{susy}}$
sparticle masses $\nearrow$ $1/\sqrt{\kappa} \nwarrow$
 $\rightarrow$ constrained matter superfields

However if $m_{\text{soft}} \lesssim E \ll m_{\text{susy}} \Rightarrow$ linear SUSY in matter sector

$\rightarrow$ goldstino coupling to ordinary matter superfields

constrained X_{NL} coupled to MSSM

Komargodski-Seiberg '09

Assumption in the following: gaugino, higgsino, slepton masses $\ll m_{\text{susy}}$

results independent on squark masses $\Rightarrow$ can be much higher

Equivalence theorem $\Rightarrow$ leading goldstino couplings:

$$\sim \int \partial_\mu \chi J^\mu = - \int \chi \partial_\mu J^\mu \leftarrow \text{supercurrent}$$

Equations of motion: $\partial_\mu J^\mu \sim$ soft terms

$\rightarrow$ generalization to non-linear terms: superfield formalism

Usually parametrization of soft SUSY terms using auxiliary spurion:

Non-linear MSSM: replace $S \rightarrow \sqrt{2\kappa} m_{\text{soft}} X_{NL} = \frac{m_{\text{soft}}}{m_{\text{susy}}} X_{NL}$ $S = m_{\text{soft}} \theta^2$

F -auxiliary in X_{NL} : dynamical field with no derivatives to be solved

$$-\bar{F} = m_{\text{susy}}^2 + \frac{B_\mu}{m_{\text{susy}}^2} h_1 h_2 + \frac{A_u}{m_{\text{susy}}^2} \tilde{u}_R \tilde{q} h_2 + \dots$$

$\Rightarrow$ compact form for all goldstino couplings at linear and non-linear level

Non-linear MSSM

with

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{X_{NL}} + \mathcal{L}_H + \mathcal{L}_m + \mathcal{L}_{AB} + \mathcal{L}_g$$

$$\mathcal{L}_H = \sum_{i=1,2} \frac{m_i^2}{m_{susy}^4} \int d^4\theta X_{NL}^\dagger X_{NL} H_i^\dagger e^{V_i} H_i$$

$$\mathcal{L}_m = \sum_{\Phi} \frac{m_{\Phi}^2}{m_{susy}^4} \int d^4\theta X_{NL}^\dagger X_{NL} \Phi^\dagger e^V \Phi \quad ; \quad \Phi = Q, U^c, D^c, L, E^c$$

$$\begin{aligned} \mathcal{L}_{AB} = & \frac{1}{m_{susy}^2} \int d^2\theta X_{nl} (A_u H_2 Q U^c + A_d Q D^c H_1 + A_e L E^c H_1) \\ & + \frac{B\mu}{m_{susy}^2} \int d^2\theta X_{NL} H_1 H_2 + h.c. \end{aligned}$$

$$\mathcal{L}_g = \sum_{i=1}^3 \frac{1}{8g_i^2} \frac{m_{\lambda_i}}{m_{susy}^2} \int d^2\theta X_{NL} \text{Tr} [W^\alpha W_\alpha]_i + h.c.$$

Higgs potential is modified:

$$V = V_{MSSM} + 2\kappa^2 |m_1^2| h_1|^2 + m_2^2 |h_2|^2 + B\mu h_1 h_2|^2 + \mathcal{O}(\kappa^4) \Rightarrow$$

$m_{1,2}, B\mu$: soft mass parameters, μ : higgsino mass

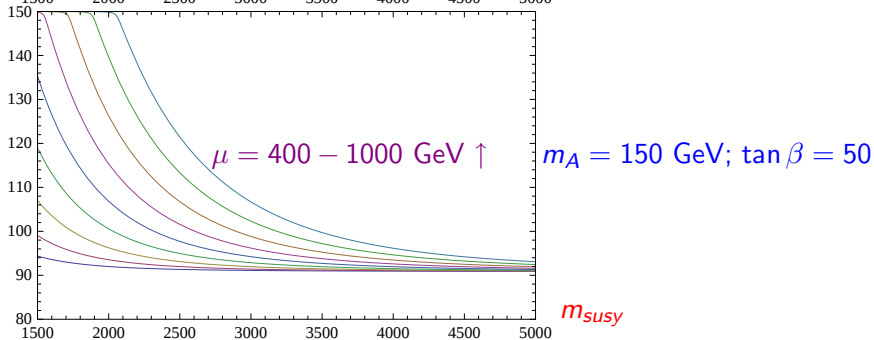
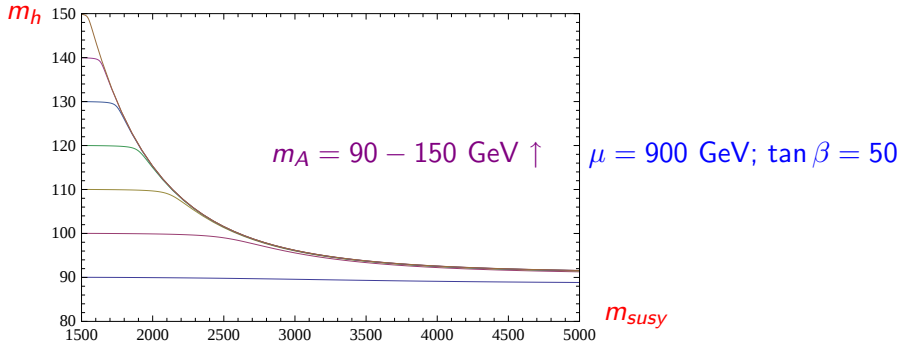
Classical value of light higgs mass can be increased above the LEP bound
for $m_{susy} \sim$ a few TeV

large $\tan \beta$ limit: $m_h^2 = m_Z^2 + \frac{v^2}{2m_{susy}^2} (2\mu^2 + m_Z^2)^2 + \dots$

$\rightarrow$ e.g. $\mu = 900$ GeV, $m_{susy} = 2$ TeV $\Rightarrow m_h = 114.4$ GeV

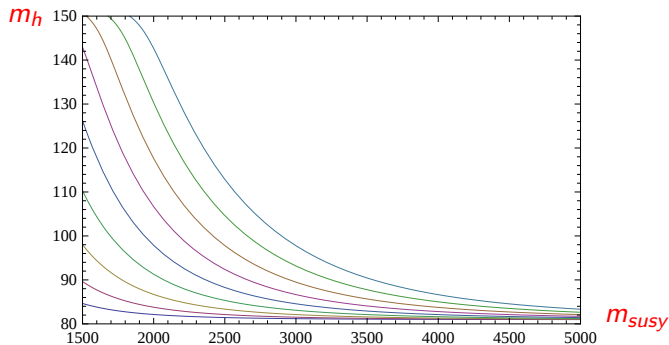
Quartic higgs coupling increases for large soft masses $\Rightarrow$ [18]

MSSM 'little' fine tuning of the EW scale is alleviated

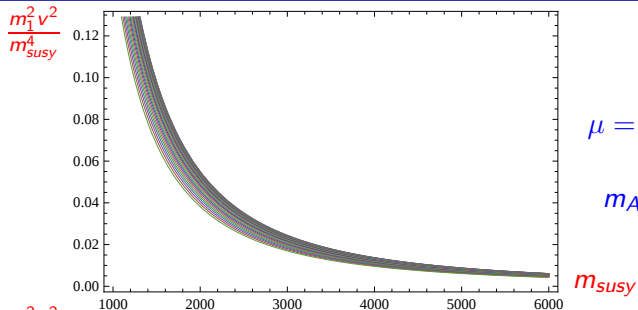


Mild dependence on $\tan \beta$

$\mu = 400 - 1000 \text{ GeV} \uparrow$ $m_A = 150 \text{ GeV}; \tan \beta = 5$

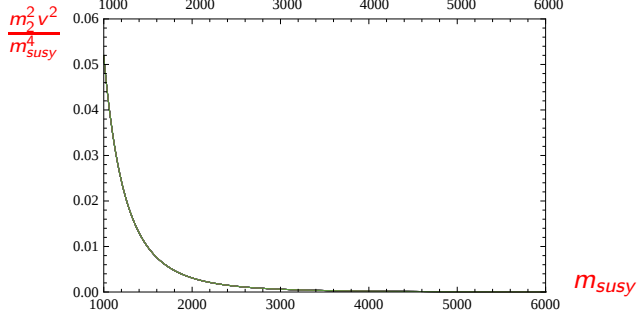


Validity of perturbative expansion: $m_i^2 v^2 / m_{\text{susy}}^4 \ll 1$



$$\mu = 900 \text{ GeV}; \tan \beta = 50$$

$$m_A = 90 - 650 \text{ GeV} \uparrow \text{ [14]}$$



Invisible decays of Higgs and Z boson

Other relevant couplings at order $\mathcal{O}(\kappa)$: $\frac{1}{m_{susy}^2} \times$

$$\left(m_1^2 \chi \psi_{h_1^0} h_1^{0*} + m_2^2 \chi \psi_{h_2^0} h_2^{0*} \right) + B\mu \left(\chi \psi_{h_2^0} h_1^0 + \chi \psi_{h_1^0} h_2^0 \right) +$$

$$\sum_{i=1,2} \frac{m_{\lambda_i}}{\sqrt{2}} \tilde{D}_i^a \chi \lambda_i^a + \sum_{i=1}^2 \frac{m_{\lambda_i}}{\sqrt{2}} \chi \sigma^{\mu\nu} \lambda_i^a F_{\mu\nu, i}^a + h.c.$$

$\Rightarrow$ invisible higgs decay $h \rightarrow \chi + \text{NLSP}$ if NLSP is light enough

otherwise inverse decay studied in the past

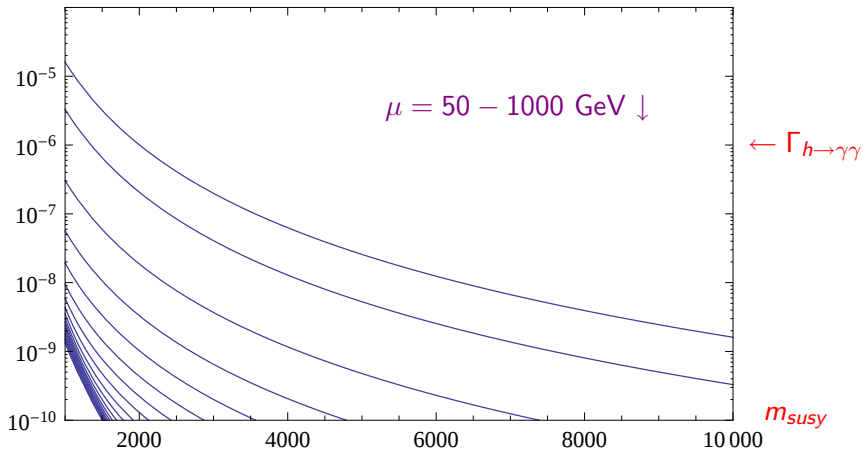
taking also into account the goldstino components of higgsinos/gauginos

from SUSY terms: $\sim h_i^0 \lambda \tilde{h}_i^0$

Similarly $Z \rightarrow \chi + \text{NLSP} \Rightarrow m_{susy} \gtrsim 400\text{--}700 \text{ GeV}$ from Z-width

$$\Delta\Gamma_Z \lesssim 2.3 \text{ MeV}$$

$\Gamma_{h \rightarrow \chi + \text{NLSP}}$ $m_A = 150 \text{ GeV}; \tan \beta = 50; (m_{\lambda_1}, m_{\lambda_2}) = (70, 150) \text{ GeV}$



Goldstino in multiplet of $N = 1$ SUSY: **vector or chiral?**

brane dynamics $\Rightarrow$ Maxwell goldstino multiplet

gauge chiral multiplet $|_{N=2} \mathcal{W} = (\text{vector } W + \text{chiral } X)_{N=1}$ [23]

$$\mathcal{W}(y, \theta, \tilde{\theta}) = X(y, \theta) + i\sqrt{2}\tilde{\theta}W(y, \theta) - \tilde{\theta}^2 \left[\frac{1}{4}\overline{DD}X(y, \theta) + \frac{1}{2\kappa} \right]$$

allow partial SUSY breaking $N = 2 \rightarrow N = 1$ 

$$\delta^* X = i\sqrt{2}\eta^\alpha W_\alpha \quad \delta^* W_\alpha = \frac{i}{\sqrt{2\kappa}}\eta_\alpha + \dots \leftarrow \text{linear SUSY}$$

$$\mathcal{L}_{Maxwell}^{N=2} = -\frac{1}{8} \int d^2\theta d^2\tilde{\theta} \mathcal{W}^2 + h.c. = \int d^2\theta \left[\frac{1}{2} W^2 - \frac{1}{4} X \overline{DD}X - \frac{1}{2\kappa} X \right] + h.c.$$

Partial SUSY breaking: non trivial prepotential $f(\mathcal{W})$

I.A.-Partouche-Taylor '96

DBI action

Non-linear $N = 2$ constraint: $\mathcal{W}_{NL}^2 = 0$

$$\Rightarrow X^2 = 0 \quad , \quad XW_\alpha = 0 \quad , \quad WW - \frac{1}{2}X\overline{DDX} = \frac{1}{\kappa}X \quad [9]$$

$$X = \kappa W^2 - \kappa^3 \bar{D}^2 \frac{W^2 \overline{W}^2}{1 + A_+ + \sqrt{1 + 2A_+ + A_-^2}} \quad A_\pm = \frac{\kappa^2}{2} \left(D^2 W^2 \pm \bar{D}^2 \overline{W}^2 \right) = \pm A_\pm^*$$

$$\begin{aligned} \Rightarrow \mathcal{L}_{NL}^{N=2} &= \frac{1}{4\kappa} \int d^2\theta X + h.c. \\ &= \frac{1}{8\kappa^2} \left(1 - \sqrt{-\det(\eta_{\mu\nu} + 2\sqrt{2}\kappa F_{\mu\nu})} \right) + \dots = \mathcal{L}_{\text{DBI}} \leftarrow \text{D-brane} \end{aligned}$$

The FI-term is also invariant under NL SUSY

$$\mathcal{L}_{FI} = \xi \int d^4\theta V; \quad W = -\frac{1}{4}\bar{D}^2 DV; \quad \delta^* V = \frac{i}{2\kappa} (\eta D + \bar{\eta} \bar{D}) \theta^2 \bar{\theta}^2 + \dots$$

$$\Rightarrow \mathcal{L}_{Max}^{NL} = \mathcal{L}_{NL}^{N=2} + \mathcal{L}_{FI} \quad [24]$$

Coupling to bulk hypermultiplets e.g. the universal dilaton

at least one isometry $\rightarrow$ single-tensor multiplet (RR 2-forms)

offers an off-shell formulation of matter (hyper) multiplet

$N = 2$ tensor $\mathcal{Y} = (\text{tensor } L + \text{chiral } \Phi)_{N=1} \quad D^2 L = \bar{D}^2 L = 0$

general action: $\mathcal{L}_{ST} = \int d^4\theta \mathcal{H}(L, \Phi, \bar{\Phi})$ with $(\partial_L^2 + 2\partial_\Phi \partial_{\bar{\Phi}}) \mathcal{H} = 0$

L supersymmetrizes the field-strength of a 2-index tensor $H = dB_2$

analog of $W = -\frac{1}{4}\bar{D}^2 DV$ for vector multiplet $F = dA$

To supersymmetrize the 2-index gauge potential B :

$L = D^\alpha \ell_\alpha + h.c.$ ℓ_α : chiral spinor superfield analog of V

$N = 2 \Rightarrow$ add auxiliary chiral superfield $Y \sim \theta^2 \epsilon \cdot C_4 \leftarrow$ 4-form

Single-tensor multiplet

$$\mathcal{Y}(y, \theta, \tilde{\theta}) = Y(y, \theta) + i\sqrt{2}\tilde{\theta}\ell(y, \theta) - \frac{i}{2}\tilde{\theta}^2\Phi(y, \theta) \quad [20]$$

Bononic field content:

$N = 1$ superfield	Field	Gauge invariance	Number of fields
ℓ_α	$B_{\mu\nu}$	$\delta B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}$	$6_B - 3_B = 3_B$
	ℓ		1_B
Φ	Φ		2_B
	F_Φ	$\delta C_{\mu\nu\rho\sigma} = 4\partial_{[\mu}\Lambda_{\nu\rho\sigma]}$	2_B (auxiliary)
Y	$C_{\mu\nu\rho\sigma}$		$1_B - 1_B = 0_B$

coupling to $N = 2$ vector $\mathcal{W}$ via Chern-Simons interaction:

$$\mathcal{L}_{CS} \sim g \int d^2\theta d^2\tilde{\theta} \mathcal{Y} \mathcal{W} = g \int d^2\theta \left(\ell^\alpha W_\alpha + \frac{1}{2} \Phi X - \frac{i}{2\kappa} Y \right) + h.c.$$

$\Rightarrow$ global SUSY limit of D-brane coupling to bulk hypermultiplets

$N = 2$ NL QED and novel super-higgs mechanism

General action: $\mathcal{L}_{tot}^{NL} = \mathcal{L}_{CS} + \mathcal{L}_{Max}^{NL}(W) + \mathcal{L}_{ST}(L, \Phi) \Rightarrow$ [21]

superhiggs mechanism without gravity:

Maxwell goldstino $\mathcal{W}_{NL}(W)$ is 'absorbed' by $N = 2$ tensor $\mathcal{Y}(L, \Phi)$

→ $N = 1$ massive vector (W, L) + massless chiral Φ :

- tensor of L + vector of $W \rightarrow$ massive vector
- scalar of $L \in$ same massive vector multiplet
- goldstino + fermion of $L \rightarrow$ Dirac spinor

System identical to Higgs phase of $N = 2$ NL QED (up to $\mathcal{L}_{ST}$)

$\Phi \sim Q_1 Q_2$ $L \sim |Q_1|^2 - |Q_2|^2$ (Q_1, Q_2) : charged hypermultiplet

adding mass-term $m \int d^2\theta \Phi \Rightarrow$ also Coulomb phase for $\xi = 0$

Vacuum structure of $N = 2$ NL QED

3 parameters: κ, ξ, m

- $m = 0$: Higgs phase and super-higgs without gravity

$$\langle Q_1 \rangle = v \text{ (real) arbitrary} \quad \langle Q_2 \rangle = \sqrt{\xi + v^2}$$

- $m \neq 0, \xi = 0$: Coulomb phase with $N = 2$ NL SUSY unbroken
- $m \neq 0, \xi \neq 0$: $N = 1$ (linear) SUSY is also broken

Ambrosetti-I.A.-Derendinger-Tziveloglou '09

Conclusions

Non-linear supersymmetry: powerful tool for studying:

- low energy SUSY breaking $E \ll m_{\text{susy}} \sim 1/\sqrt{\kappa}$
Volkov-Akulov action and goldstino χ couplings to matter
standard coupling to stress-tensor *not* the most general
- $m_{\text{soft}} \lesssim E \ll m_{\text{susy}}$: goldstino $\equiv$ spurion coupled to supermultiplets
→ Non-linear MSSM : narrow but interesting region
 - new quartic higgs coupling $\Rightarrow$ • can increase the higgs mass
 - reduce the MSSM fine tuning of the EW scale
- brane effective actions $\Rightarrow$ brane dynamics
 $N = 2$ NL SUSY $\Rightarrow$ DBI action and couplings to bulk fields
vacuum structure of NL QED and superhiggs without gravity