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Constraining scalar-tensor modified gravity models

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Origin and Structure of the Universe
The Cluster of Excellence for Fundamental Physics

Overview

- Introduction,
  - f(R) models
- Constraints on f(R) models
  - Solar system constraints
  - Model dependent constraints
  - Cosmological probes
    - Distance based probes
    - Structure formation
- Non-linear regime
  - Perturbation theory
  - PPF formalism
- Weak lensing forecast constraints

DE or modified gravity?

- Late time acceleration typically attributed to dark energy; on large scales acts as a perfect fluid which violates the strong energy condition

$$\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right) u^{\mu} u^{\nu} \geq 0 \qquad w \leq -\frac{1}{3} \qquad w = \frac{p}{\rho}$$

- Could the acceleration be due to new gravitational physics on large scales?

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \text{ + dark energy}$$

Introduce new matter component

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \text{ + modified gravity} = 8\pi G T_{\mu\nu}$$

Modify gravity

- Relax one or more of the assumptions upon which GR is based; introduce additional field(s) which mediate gravity.

f(R) models

- f(R) models $$S=\int\sqrt{-g}\,d^4x\left[\frac{R+f(R)}{16\pi G}+L_{\mathrm{m}}\right]$$
- Equivalent to scalar-tensor gravity; there is an additional scalar field which also mediates gravity (the 'scalaron'.)
- The scalar field is a chameleon; its mass is 'background' dependent.
- Field equations

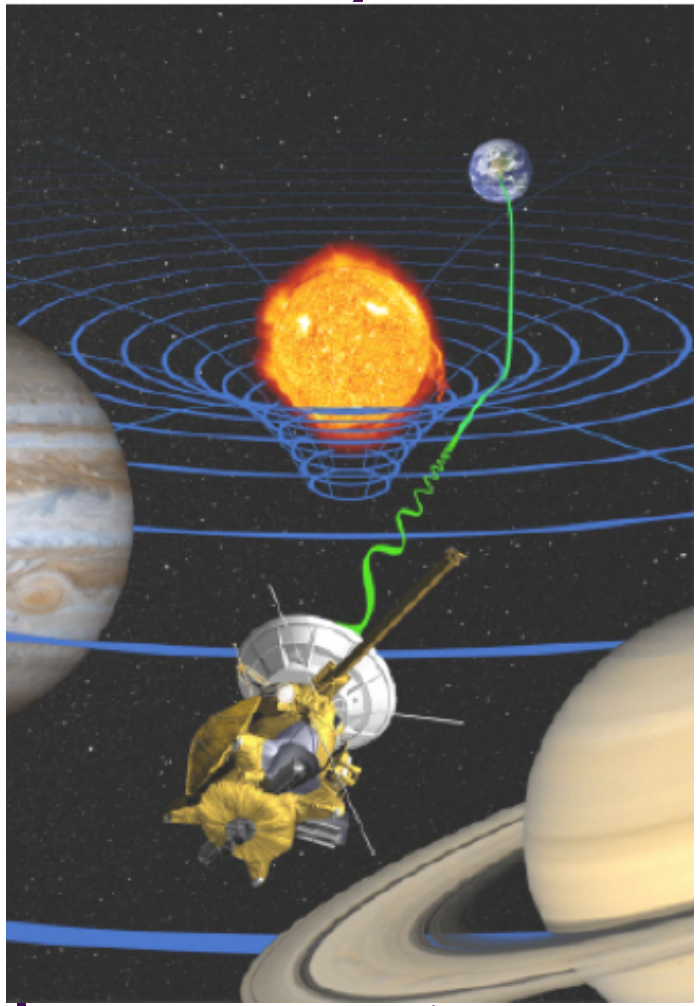
$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+R_{\mu\nu}f_R-\frac{1}{2}g_{\mu\nu}f+[g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu]f_R=8\pi G T_{\mu\nu}$$

General Relativity

$$f_R=\frac{df}{dR}$$

Modified gravity: Observational constraints

- Gravity is well described by General Relativity in the solar system



- Shapiro time delay measured by the Cassini probe

$$|\gamma - 1| < 2.3 \times 10^{-5}$$

B. Bertotti, L. Iess, P. Tortora (2003)

- In f(R) models, this corresponds to the constraint (Hu et al, 2007)

$$f_R < (\gamma - 1)\frac{GM_s}{r_s}$$

$$\frac{GM_s}{r_s} = 2.12 \times 10^{-6}$$

$$|f_R(R_G)| \sim \frac{R_G}{M_{scal}^2} < 4.9 \times 10^{-11}$$

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Modified gravity: Observational constraints

- Solar system constraints are model independent
- Model-specific f(R) constraints at larger scales:

- Strong lensing (T. Smith 2009) $f(R) = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}$ $|f_R(R_{vac})| \leq 10^{-5}$ n=1

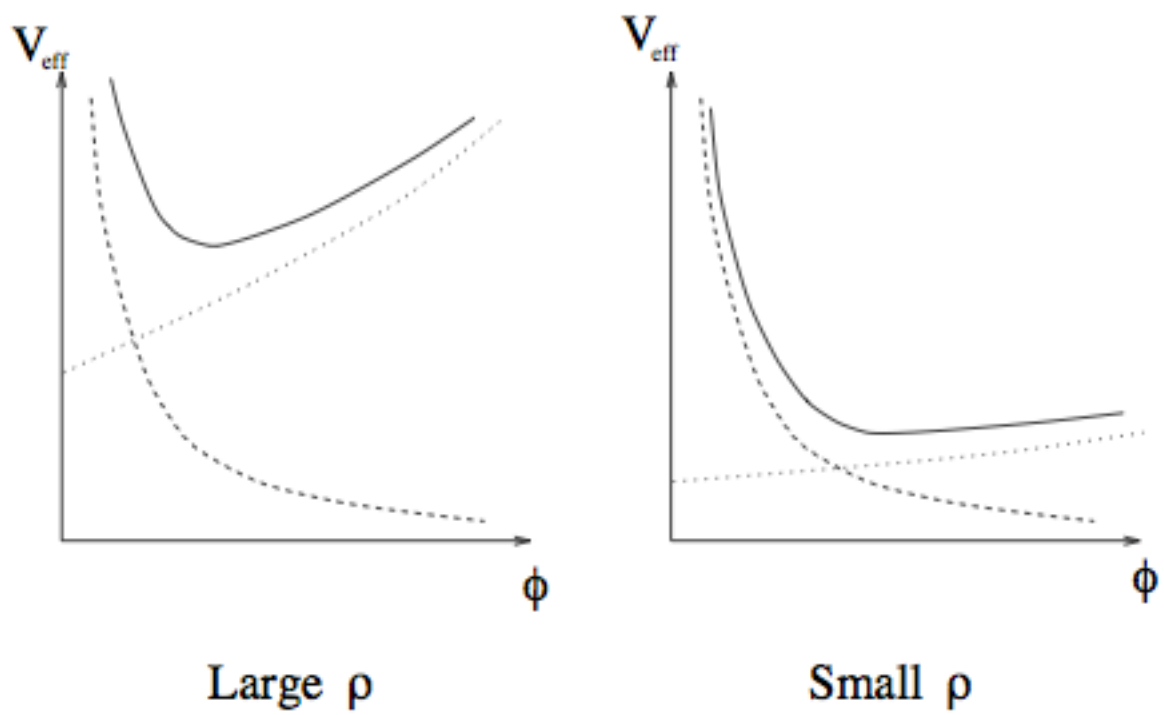
- Combined cosmological data sets (SN1a, BAO,CMB,...) (Lombriser et al. 2010) ΛCDM $B_0 < 1.1 \times 10^{-3}$ $B = \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$

- Structure formation (Linder 2009) $f(R) = -cr\left(1 - e^{-R/r}\right)$ $f_R \lesssim 10^{-3}$

f(R) models

- Solar system tests rule out f(R) models?

- Not necessarily! The scalar field is a chameleon.
- Observational constraints place limits on the scalar field mass, which is background dependent.



Justin Khoury and Amanda Weltman (2003)

- What other constraints can we place on the mass of the scalar field?

Cosmology

- Background cosmology of f(R) models;

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$$3H^2 = 8\pi G(\rho_c + \rho_r) + 3(H^2 + \dot{H})f_R - \frac{f}{2} - 3H\dot{f}_R$$

$$-2\dot{H} - 3H^2 = 8\pi G P_r + \ddot{f}_R + 2H\dot{f}_R + \frac{f}{2} - (\dot{H} + 3H^2)f_R$$

$$\dot{\rho}_c + 3H\rho_c = 0$$

$$\dot{\rho}_r + 4H\rho_r = 0$$

General Relativity

- Fourth order field equations. Simplify with the quasi-static approximation;

$$M^2(a) \simeq \frac{1}{3f_{RR}(R_{GR})} \gg H_{GR}^2$$

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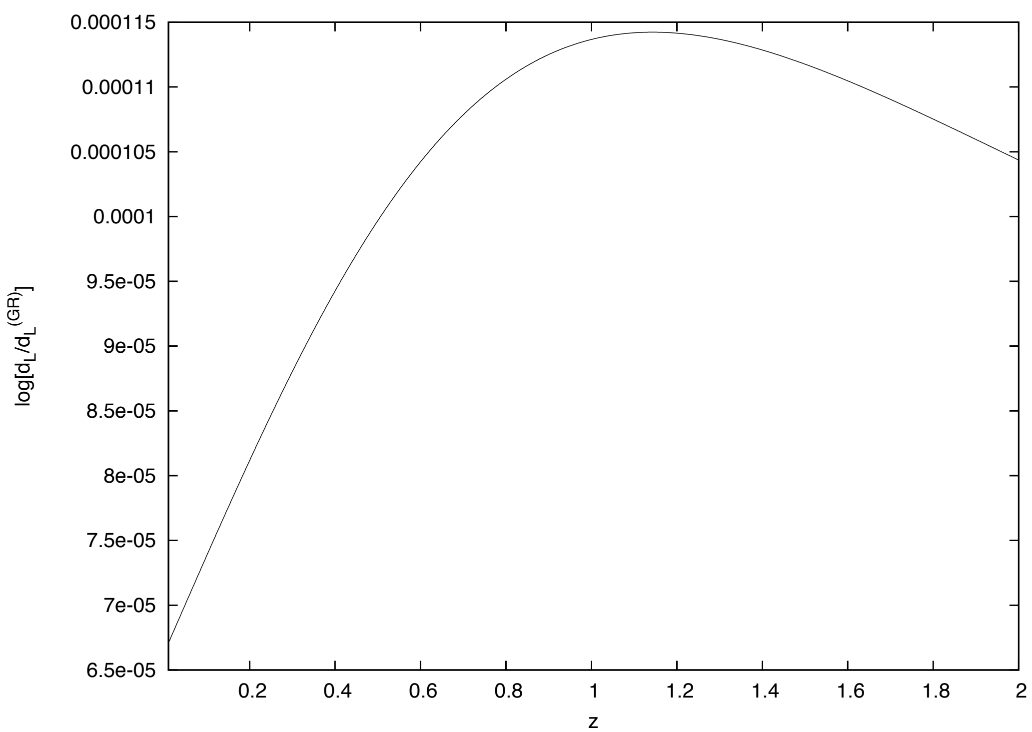
Cosmology

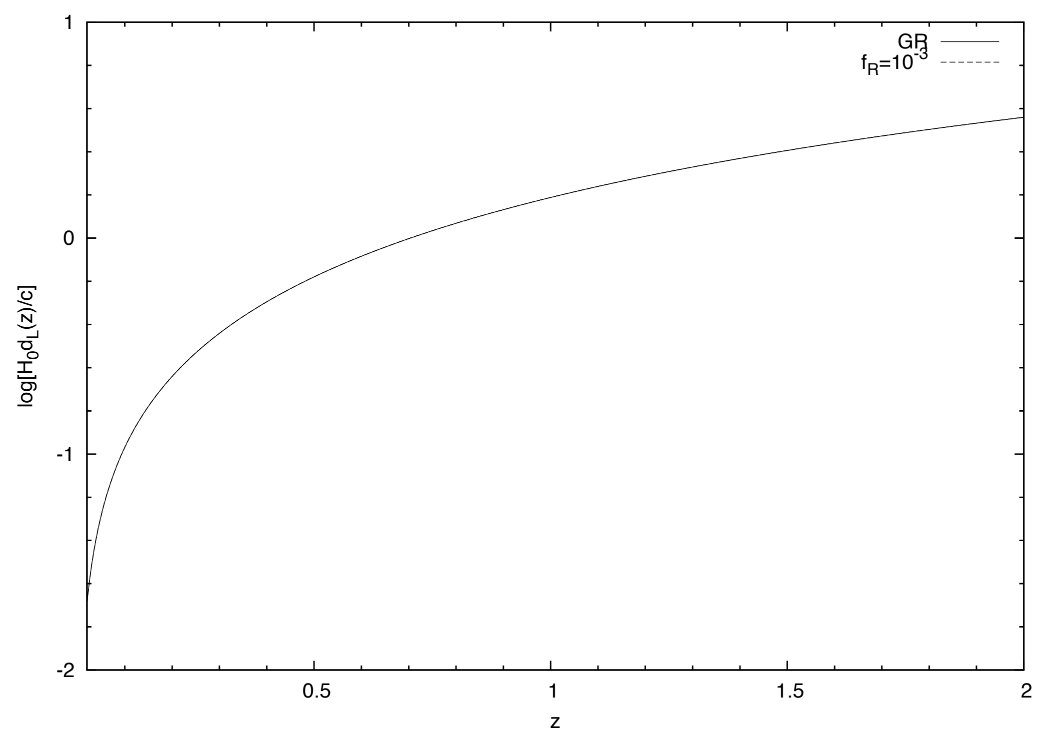
- Background equations in the quasi-static approximation

$$3H^2 = 8\pi G(\rho_c + \rho_r + \rho_\Lambda) + O\left(\frac{H_{GR}^2}{M^2(a)} H_{GR}^2\right)$$

$$\dot{\rho}_c + 3H\rho_c = 0 \qquad \dot{\rho}_r + 4H\rho_r = 0$$

- Deviations from GR expansion history are suppressed by $\left(\frac{H_{GR}^2}{M^2(a)}\right)$





$$f_R(R) = -f_{R0}\frac{R_{vac}}{R}$$

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Modified gravity: Perturbations

- Perturbation equations (scalar perturbations only, Newtonian gauge) Bean et al. 2006

$$ds^2 = a^2\left[-(1+2\psi)d\tau^2 + (1-2\phi)\gamma_{ij}dx^i dx^j\right]$$

$$\delta_c'' + H\delta_c' + k^2\psi - 3\phi'' - 3H\phi' = 0$$

$$\delta_\gamma'' + \frac{1}{3}k^2\delta_\gamma + \frac{4}{3}k^2\psi - 4\phi'' = 0$$

Fluid perturbation equations are unchanged

$$(1+f_R)(\psi-\phi) + f_{RR}\delta R = -\frac{3a^2}{2k^2}8\pi G\Sigma_i(\rho_i + p_i)\sigma_i$$

$$(1+f_R)\left[2k^2\phi + 6H(\phi' + H\psi)\right] + 3f_{RR}H'\delta R - (k^2 f_{RR} + 3Hf_{RR}')\delta R - 3Hf_{RR}\delta R' + f_R'(6H\psi + 3\phi') = -8\pi G a^2\Sigma_i\rho_i\delta_i$$

$$\delta R = \frac{2}{a^2}\left[-6\frac{a''}{a}\psi - 3H\psi' + k^2\psi - 9H\phi' - 3\phi'' - 2k^2\phi\right]$$

Modified gravity: Evolution of perturbations

- To solve these equations, we use the quasi-static limit
- At background level, the mass of the scalar field satisfies

$$M^2 \gg H^2, \dot{H}$$

- The dominant contributions arise from terms involving

$$\mathrm{M}^2 \qquad \frac{k^2}{a^2}$$

- Approximate equations

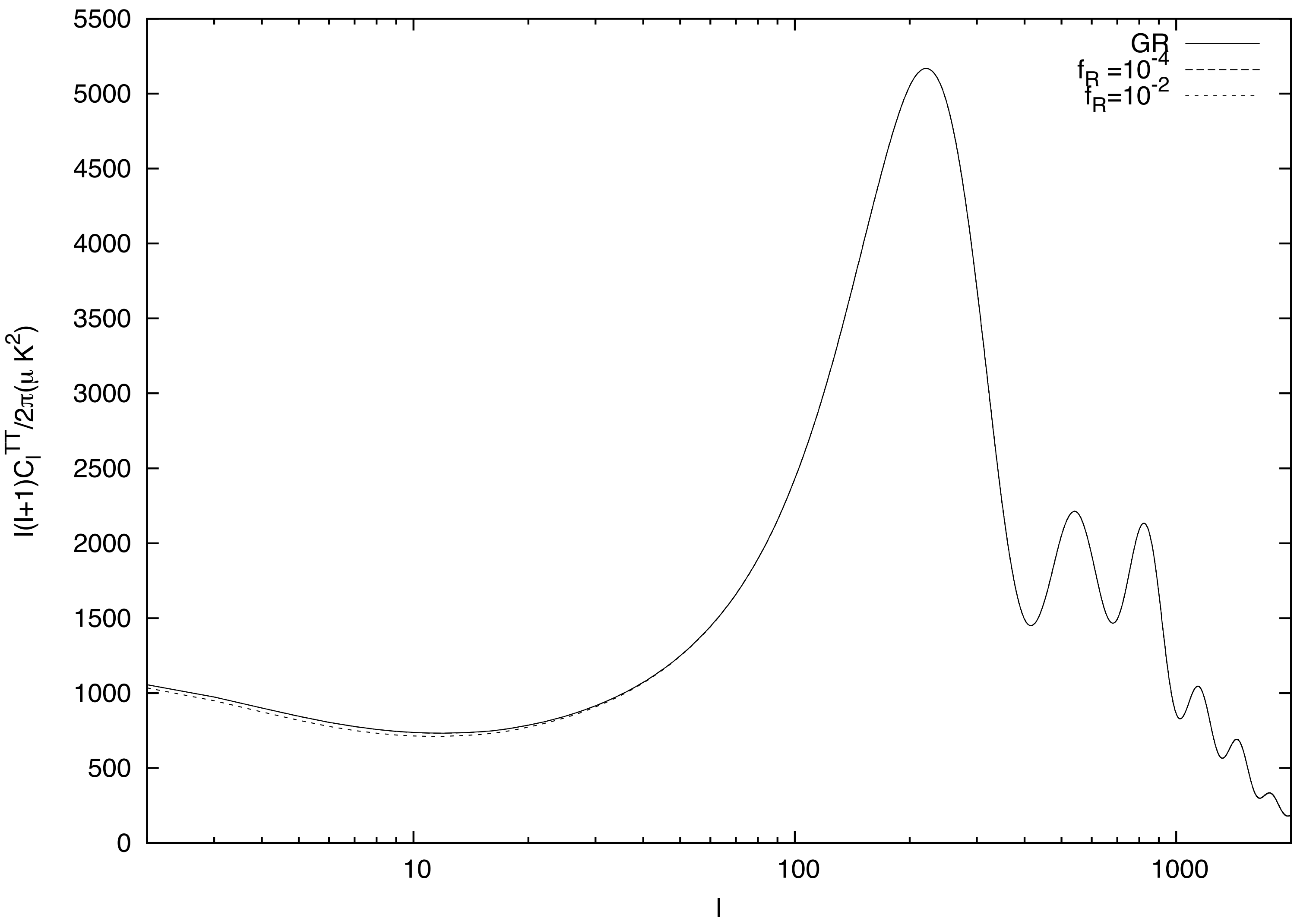
$$k^2\phi = -4\pi G Q(a,k) a^2 \delta_c \qquad Q = \frac{3a^2M^2 + 2k^2}{3a^2M^2 + 3k^2}$$

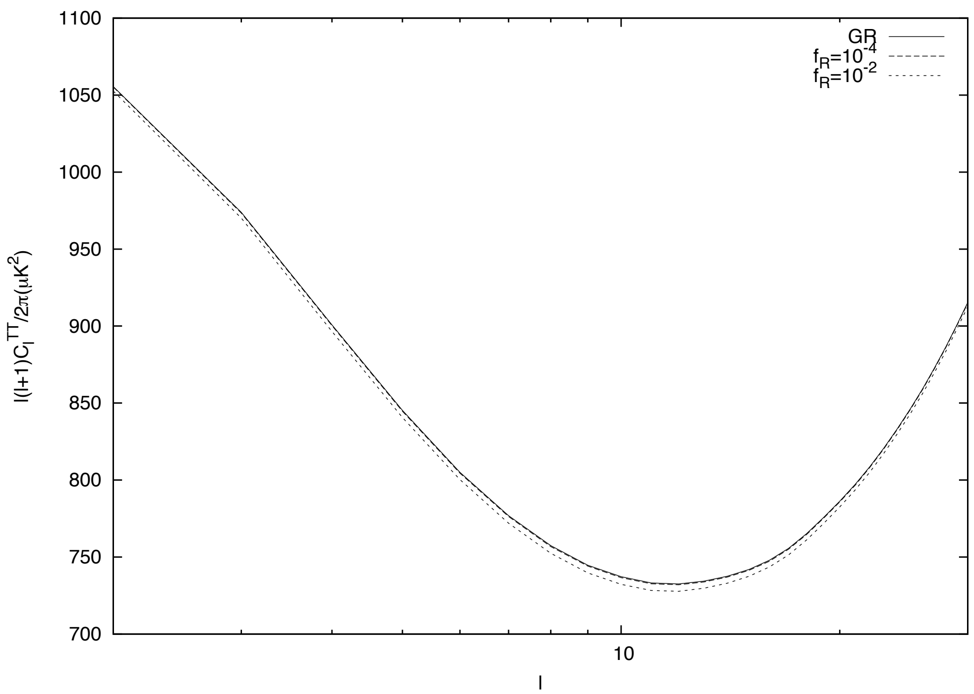
$$\psi = [1 + \eta(a,k)]\phi \qquad \eta = \frac{2k^2}{3a^2M^2 + 2k^2}$$

$$\ddot{\delta}_c + 2H\dot{\delta}_c - 4\pi G \rho_c \theta(a,k)\delta_c = 0 \qquad \theta = \frac{4k^2 + 3a^2M^2}{3k^2 + 3a^2M^2}$$

Cosmological constraints

- CMB angular power spectrum;



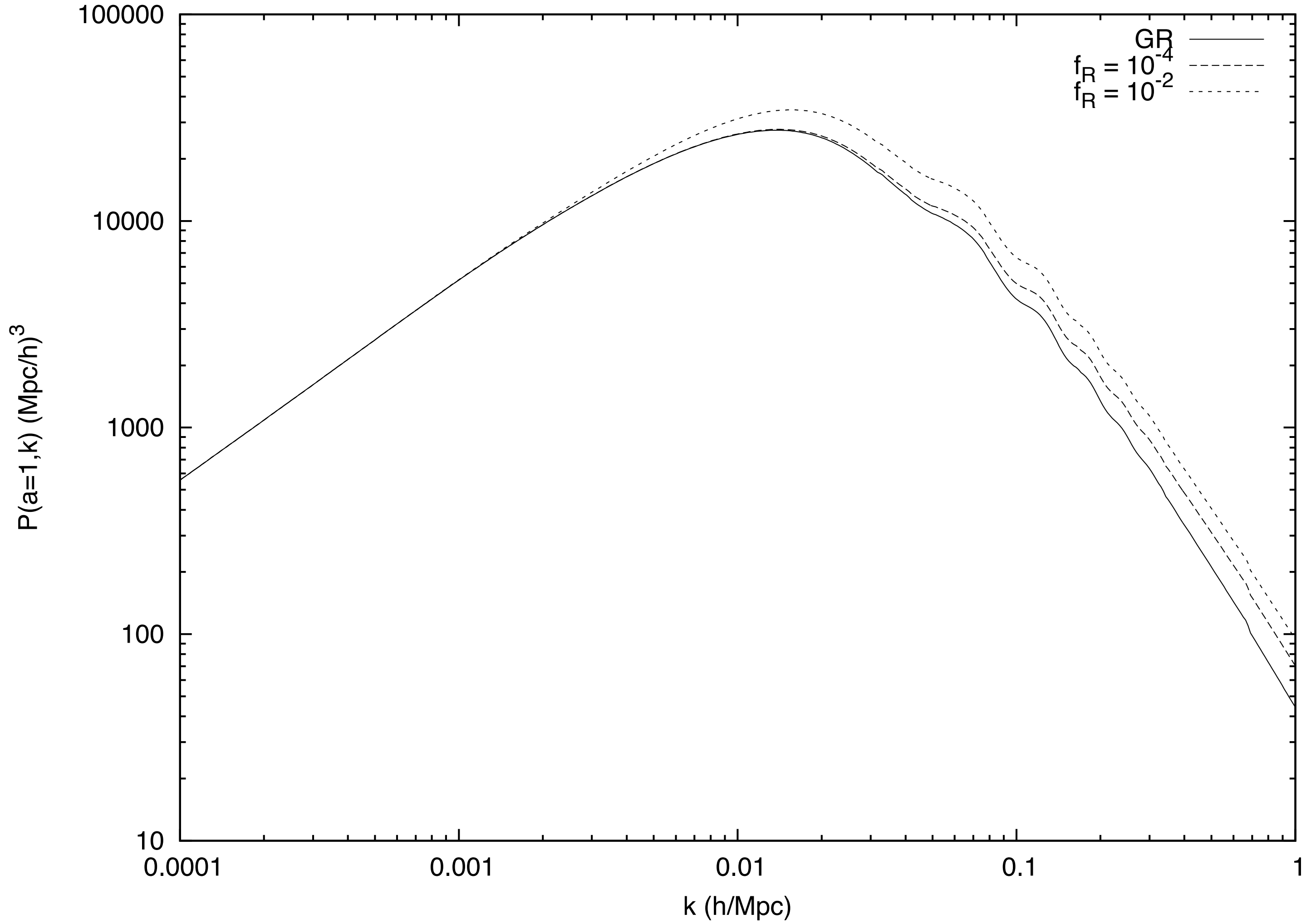


$$f_R(R) = -f_{R0}\frac{R_{vac}}{R}$$

- Very low modified gravity signal in the angular power spectrum, even on large scales.

Modified gravity: Perturbations

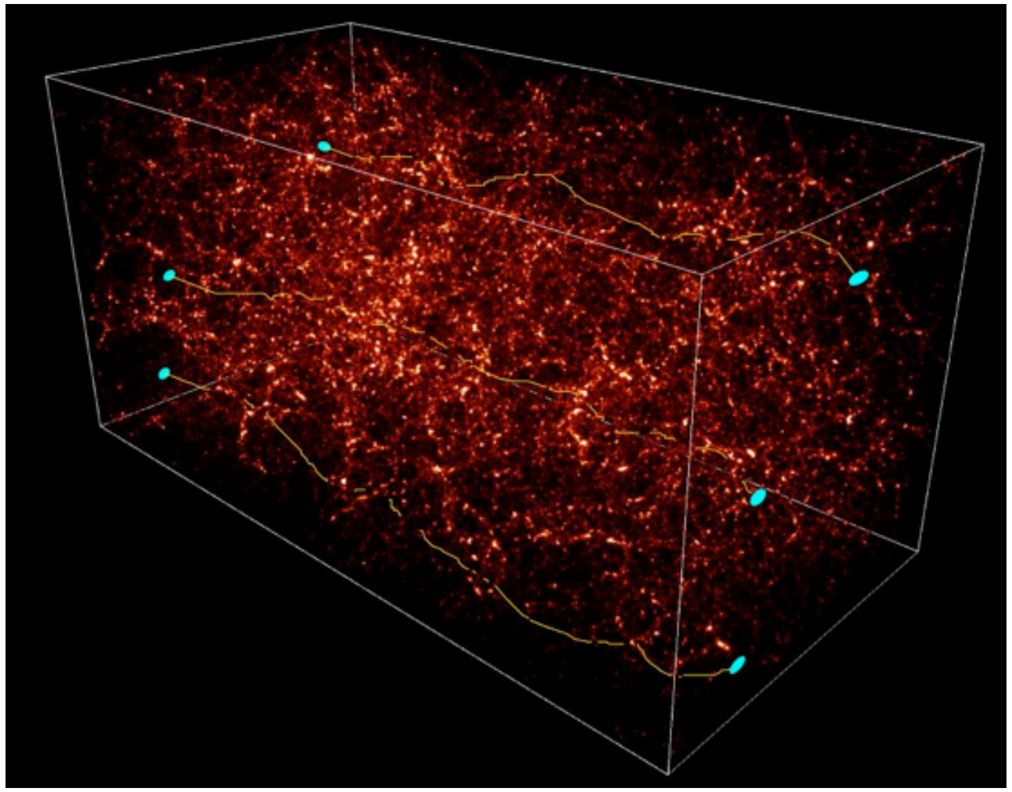
- The matter power spectrum is modified at late times
- $$P(z,k) = P_{\mathrm{GR}}(z=10,k)\left(\frac{\delta_c(z,k)}{\delta_c(z=10,k)}\right)^2$$
- f(R) models generically lead to increased power at 'intermediate' scales



$$f_R(R) = -f_{R0}\frac{R_{vac}}{R}$$

Modified gravity: Perturbations

- We use weak lensing forecast data to place constraints on f(R) models (Appleby Thomas Weller, in prep).
- The weak lensing power spectrum is sensitive to the modified gravity signal in the matter power spectrum



$$P_\kappa(l) = \frac{9\Omega_m^2 H_0^4}{4c^4}\int_0^{\chi_H} \mathrm{d}\chi \left[\frac{g(\chi)}{a(\chi)}\right]^2 P_\delta\left(\frac{l}{\chi},\chi\right)$$

- To maximize the constraining power of weak lensing probes, we must also compute the non-linear power spectrum!

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Modified gravity: Non-linear regime

- Use higher order perturbation theory for the mildly nonlinear regime

$$\Phi_a = \begin{pmatrix} \delta(\tau, \mathbf{k}) \\ -\theta(\tau, \mathbf{k}) \end{pmatrix} \qquad \tau = \log[a]$$

$$\frac{\partial \Phi_a(\tau, \mathbf{k})}{\partial \tau} + \Omega_{ab}\Phi_b(\tau, \mathbf{k}) = \int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^3}\delta^D(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)\gamma_{abc}(\tau, \mathbf{k})\Phi_b(\mathbf{k}_1, \tau)\Phi_c(\mathbf{k}_2, \tau)$$

$$\Omega_{ab} = \begin{pmatrix} 0 & -1 \\ -\frac{4\pi G\rho_m}{H^2}\left(1 + \frac{k^2}{3a^2\Pi(a,\mathbf{k})}\right) & 2 + \frac{\dot{H}}{H^2} \end{pmatrix}$$

$$\Pi(a, \mathbf{k}) = \left(\frac{k^2}{a^2} + \frac{M^2}{3}\right)$$

$$\gamma_{112} = \frac{1}{2}\left(1 + \frac{\mathbf{k}_1.\mathbf{k}_2}{|\mathbf{k}_1|^2}\right)$$

$$\gamma_{121} = \frac{1}{2}\left(1 + \frac{\mathbf{k}_1.\mathbf{k}_2}{|\mathbf{k}_2|^2}\right)$$

$$\gamma_{222} = \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2|\mathbf{k}_1 + \mathbf{k}_2|^2}{|\mathbf{k}_1|^2|\mathbf{k}_2|^2}$$

$$\gamma_{211} = -\frac{1}{12H^2}\left(\frac{8\pi G\rho_m}{3}\right)^2\frac{(k_1 + k_2)^2}{a^2}\frac{M_2(a)}{\Pi(\tau, \mathbf{k}_{12})\Pi(\tau, \mathbf{k}_1)\Pi(\tau, \mathbf{k}_2)}$$

Higher order vertex functions

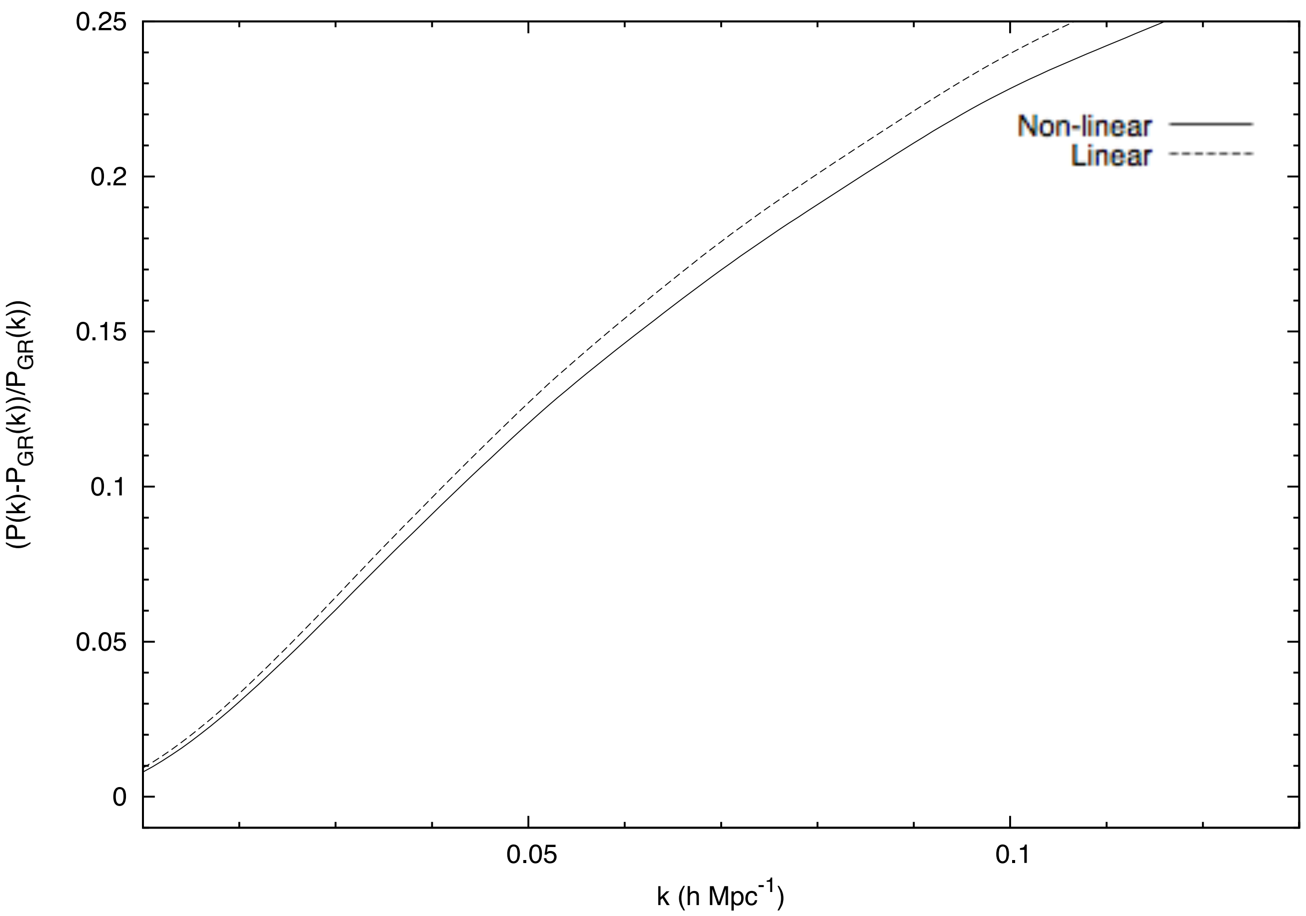
$$\Phi_a = \Phi_a^{(1)} + \Phi_a^{(2)} + \Phi_a^{(3)} + \ldots$$

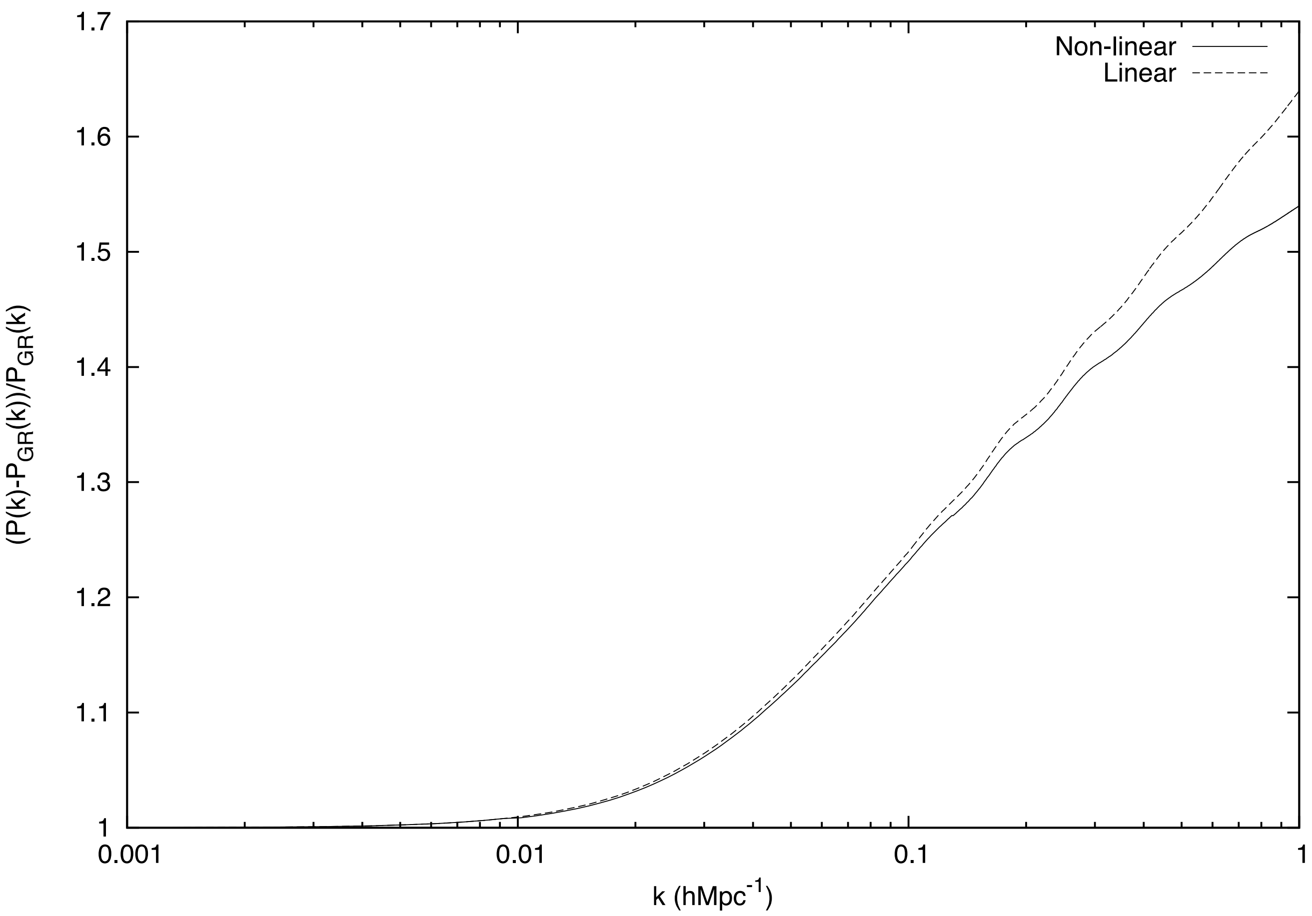
$$P_{ab}(\mathbf{k}, \tau) = P_{ab}^{(11)}(\mathbf{k}, \tau) + P_{ab}^{(22)}(\mathbf{k}, \tau) + P_{ab}^{(13)}(\mathbf{k}, \tau) + \ldots$$

- Extra terms; backreaction of the metric perturbations on the scalar field mass (the chameleon mechanism!)

Modified gravity: Non-linear regime

- We observe the effect of the chameleon mechanism in the mildly non-linear regime





- Can use the mildly non-linear regime to calibrate fully non-linear fitting formulas.

Modified gravity: Non-linear regime

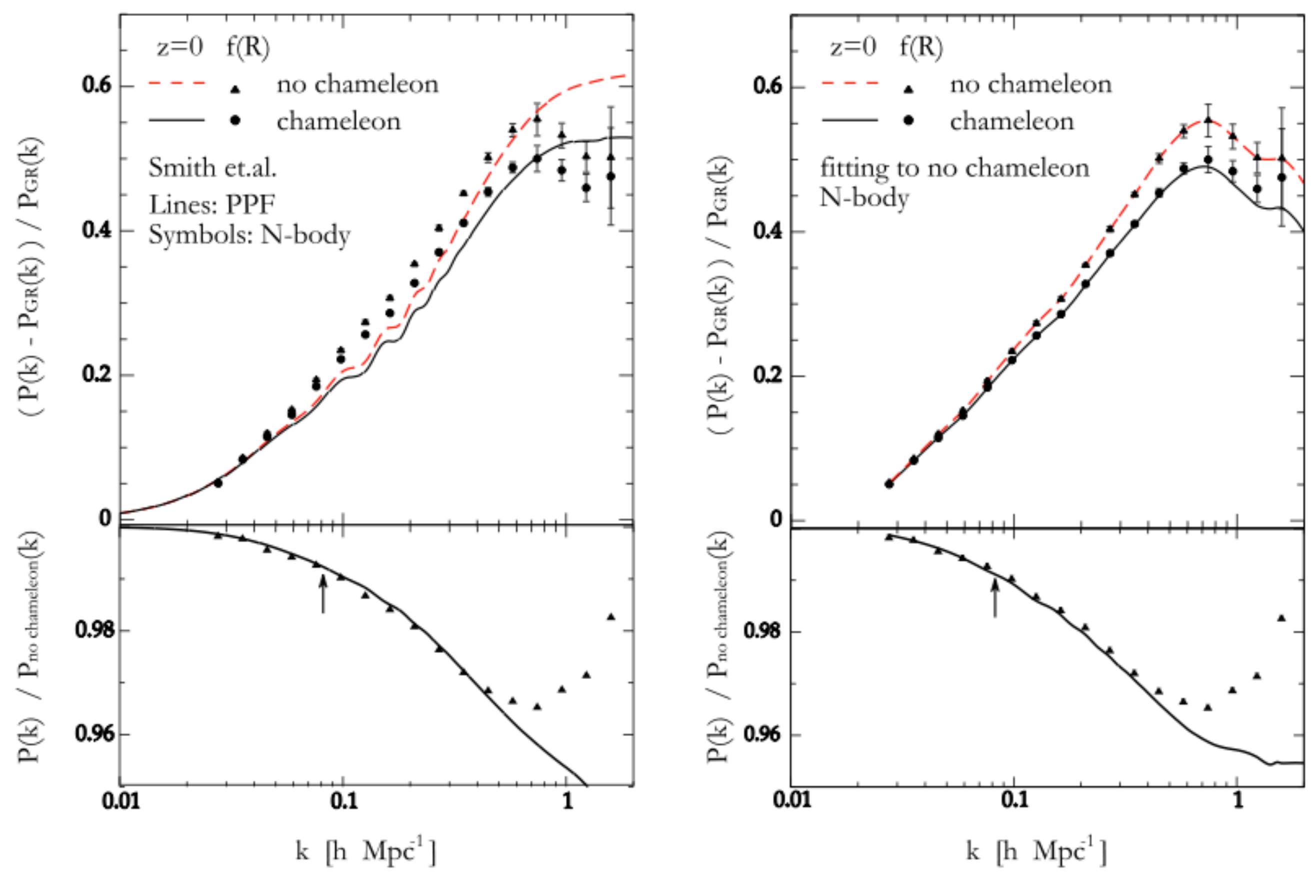
- Perturbation theory cannot be trusted above $k \sim 0.1\, h\ Mpc^{-1}$

- Beyond this, we must use alternative approaches (N-body simulations,...?)

- The PPF formalism,

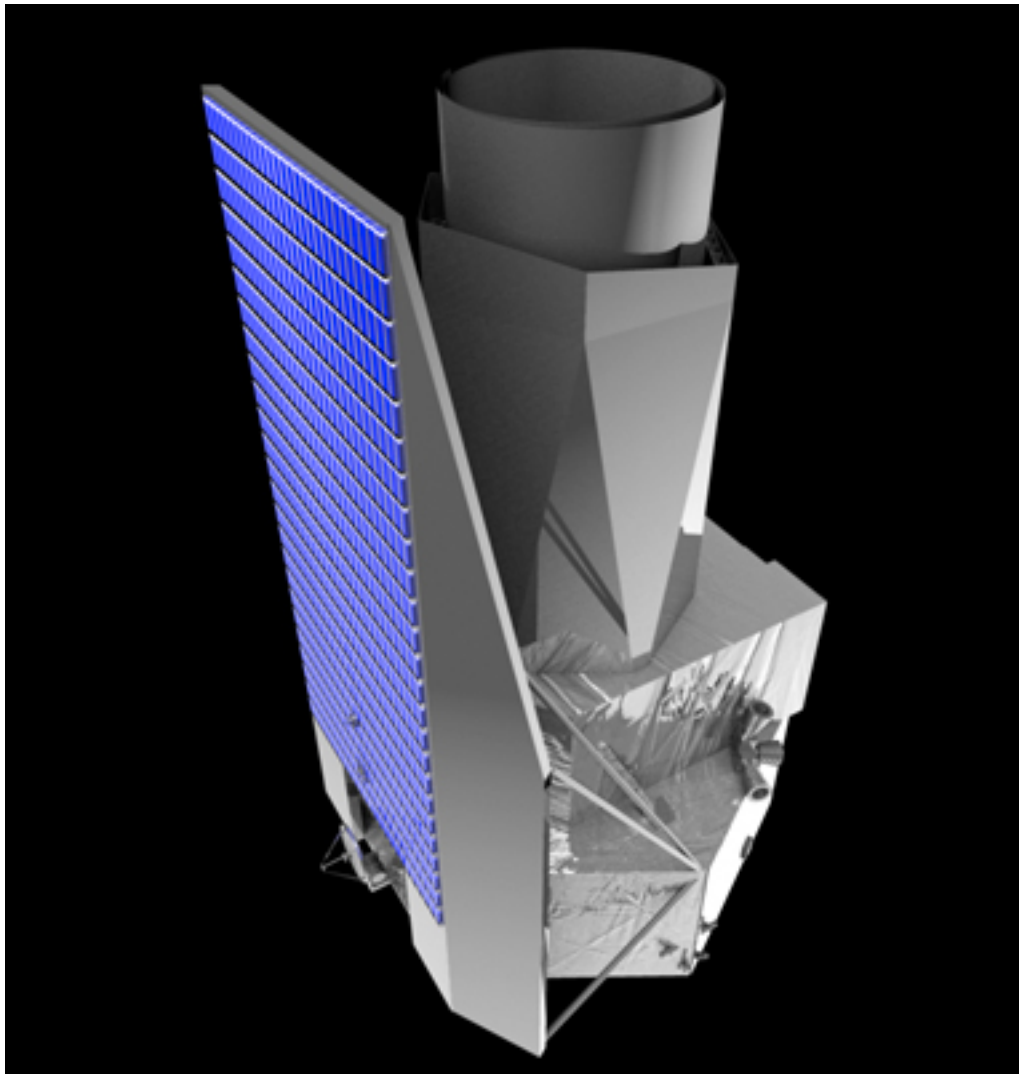
$$P(a,k) = \frac{P_{\mathrm{no-cham}}(a,k) + c_{\mathrm{nl}}(a)\Sigma^2(a,k)P_{\mathrm{GR}}(a,k)}{1 + c_{\mathrm{nl}}(a)\Sigma^2(a,k)}$$

- Requires simulations to calibrate!



Koyama et al. (2009)

Modified gravity: Constraints



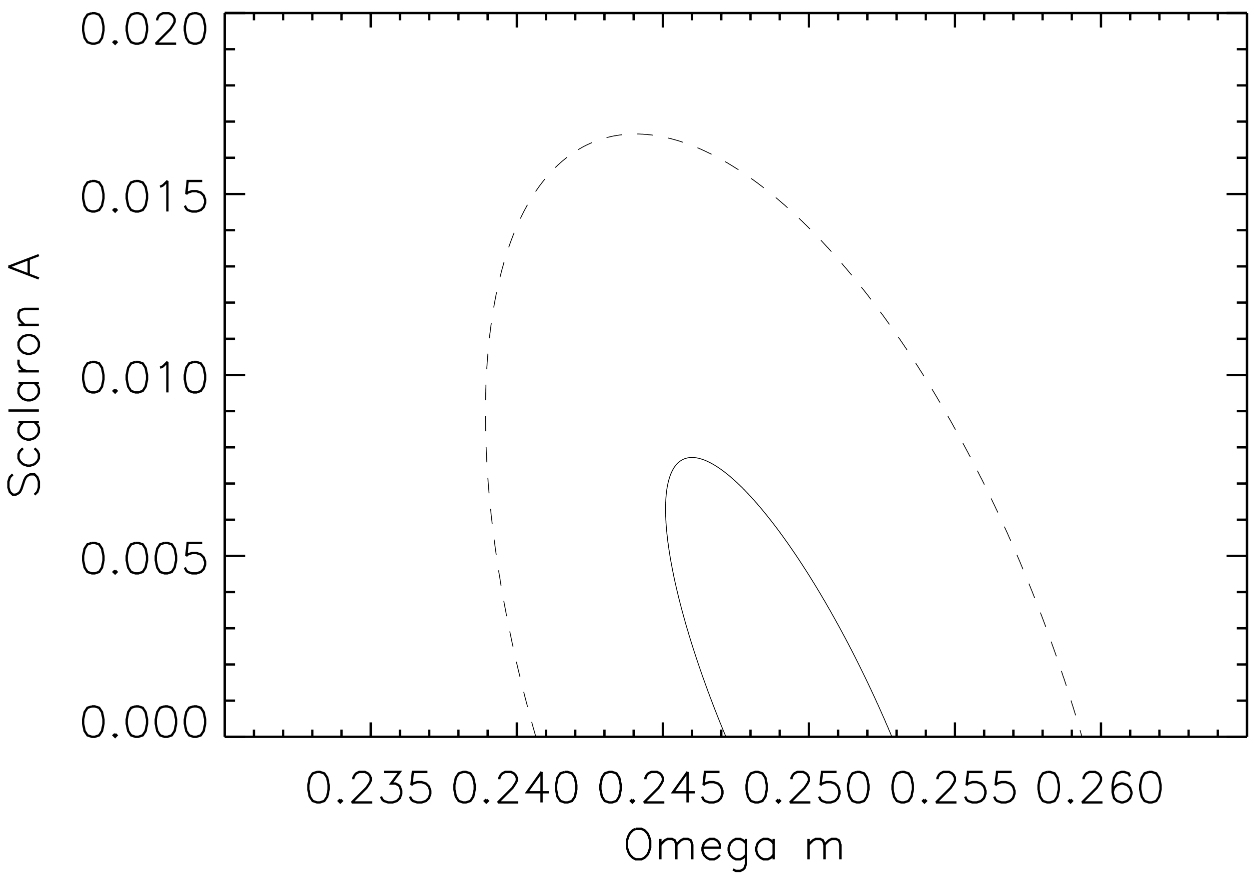
- Euclid weak lensing forecast
- Choose a simple functional form for the mass of the scalaron

$$M(a) = \frac{H_0}{Aa^n + \lambda} \qquad A < 1 \qquad n > 2$$

- Take two cuts
  - Conservative $l_{cut} = 400$ (only consider the linear regime)
  - Include nonlinear physics $l_{cut} = 10000$ (using Smith et al; incorrect!)

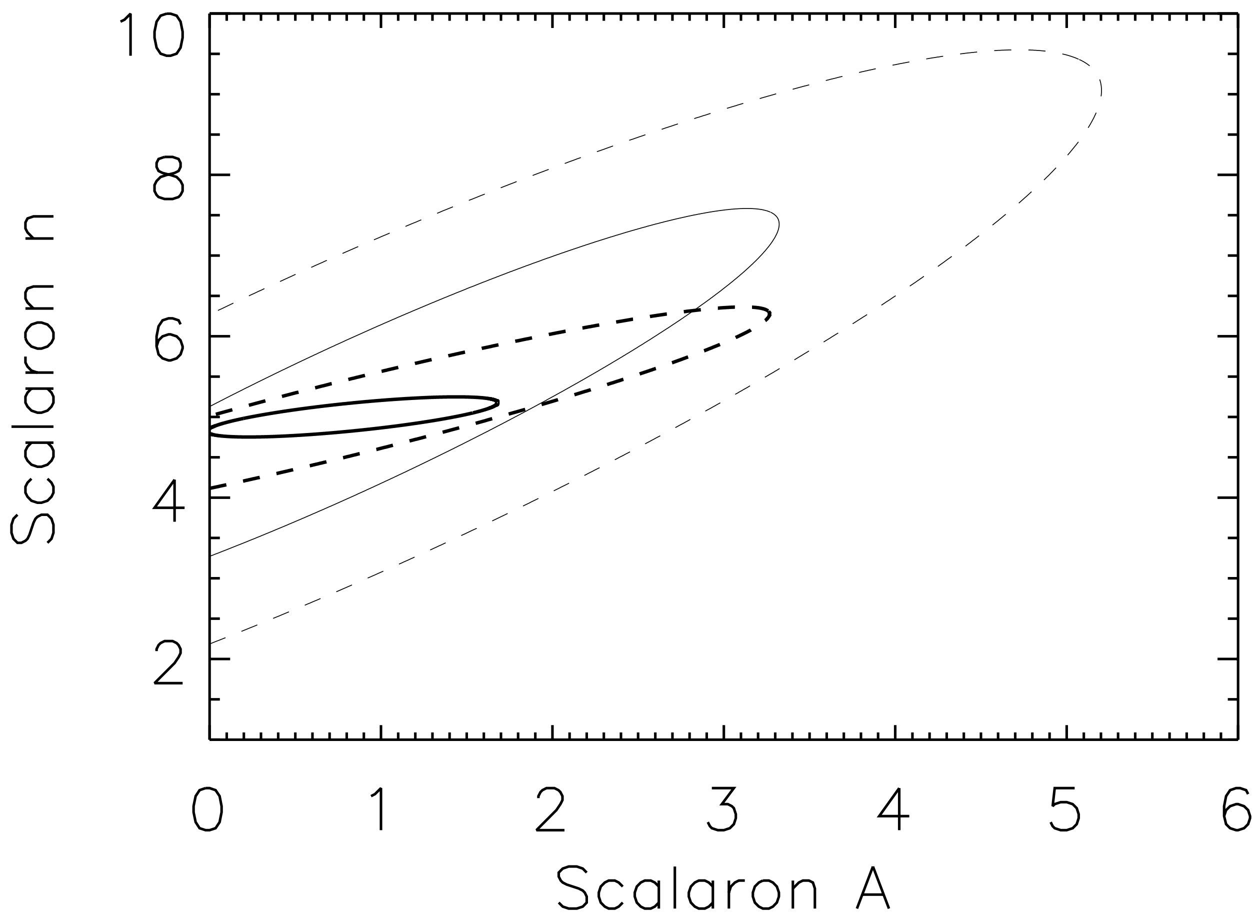
Modified gravity: Forecast constraints

Preliminary!



Fiducial values

$\Omega_{m0} = 0.25$
$\Omega_{b0} = 0.05$
$h = 0.7$
$n_s = 1.0$
$\log[A_s] = \log[2.34e-9]$
$A = 0$
$n = 5(\text{fixed})$



Fiducial values

$\Omega_{m0} = 0.25$
$\Omega_{b0} = 0.05$
$h = 0.7$
$n_s = 1.0$
$\log[A_s] = \log[2.34e-9]$
$A = 0.83$
$n = 5$

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Conclusions

- f(R) models are capable of reproducing the standard expansion history.
- We expect to find modified gravity signals in the growth of structure.
- The linear growth of perturbations is now well understood.
- To fully utilize the constraining power of upcoming surveys, we must have a better understanding of the nonlinear regime.
- Higher order perturbation theory, fitting functions calibrated by simulations, ...?