2nd Bethe Center Workshop - Bad Honnef, 07.10.10

INFLATION IN NO-SCALE SUGRA

Laura Covi

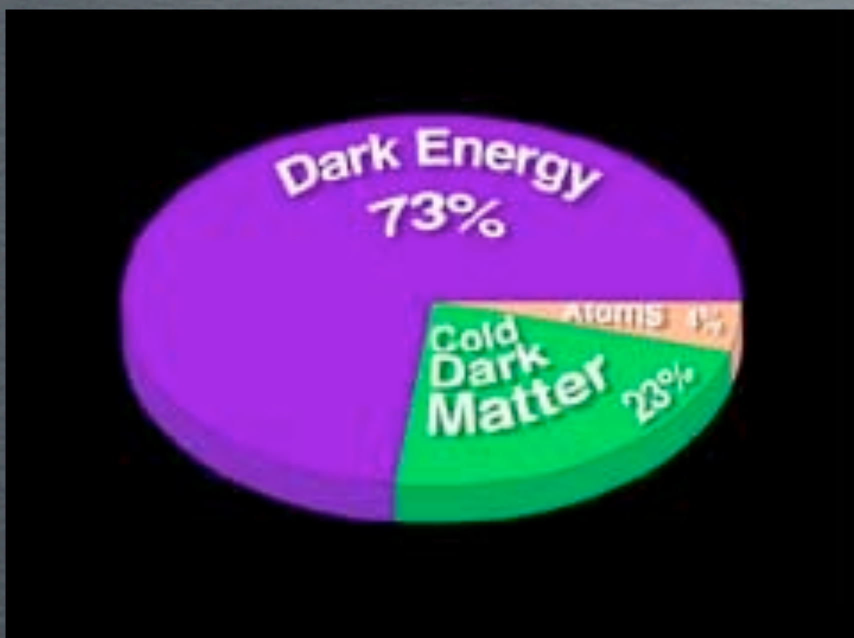
based on C6 project with J. Louis, M. Gomez-Reino, C. Gross, G. Palma, C. Scrucca, C. Burrage & J. Baacke, N. Kevlishvili

Outline

- Introduction & Motivation:
  The present Universe
- Part I: de Sitter solutions in no-scale SUGRA
- Part II: Inflation and the gravitino mass
- Part III: Hybrid inflation and quantum corrections
- Outlook

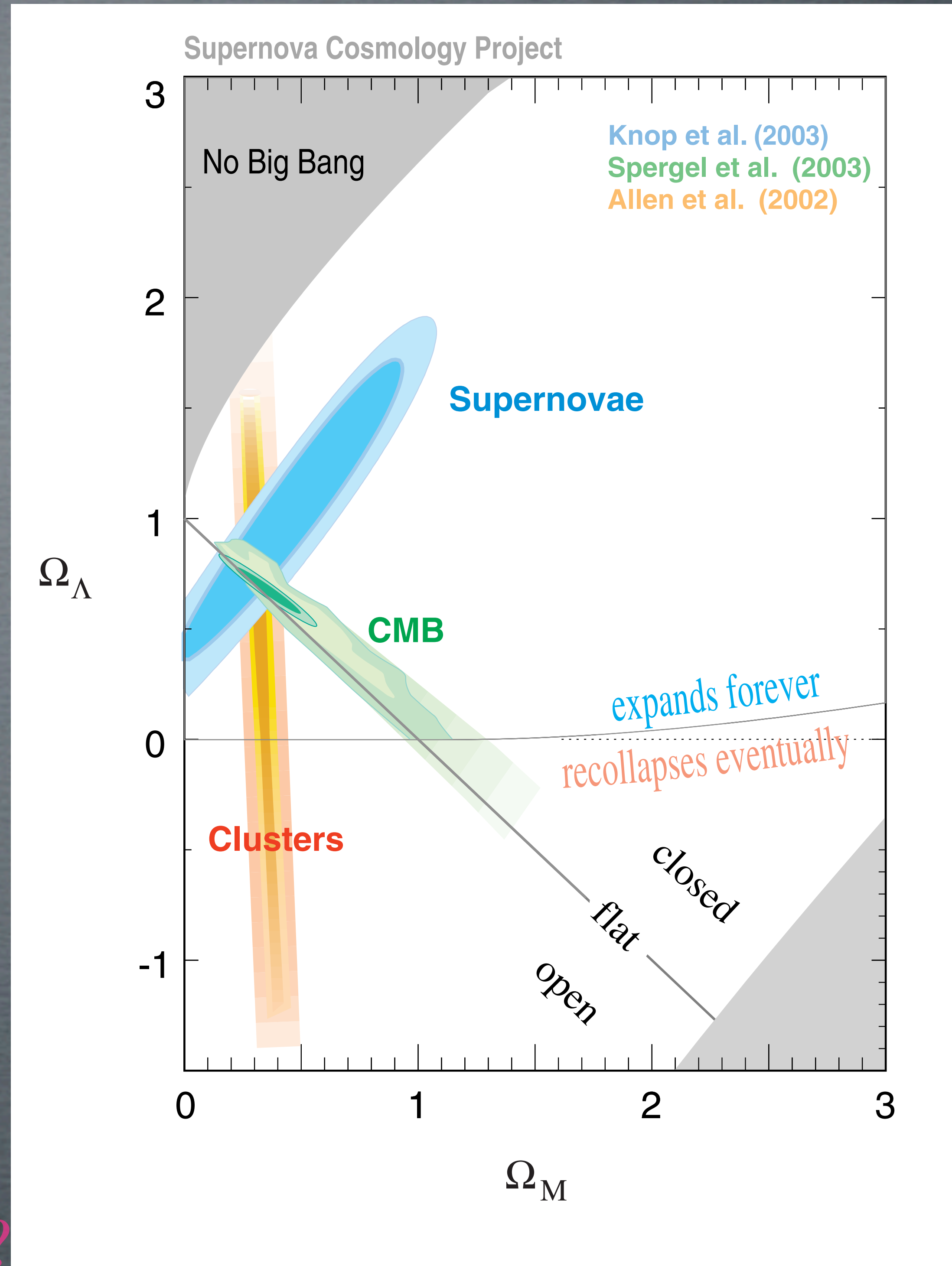
Introduction & Motivation

PRESENT ENERGY CONTENT

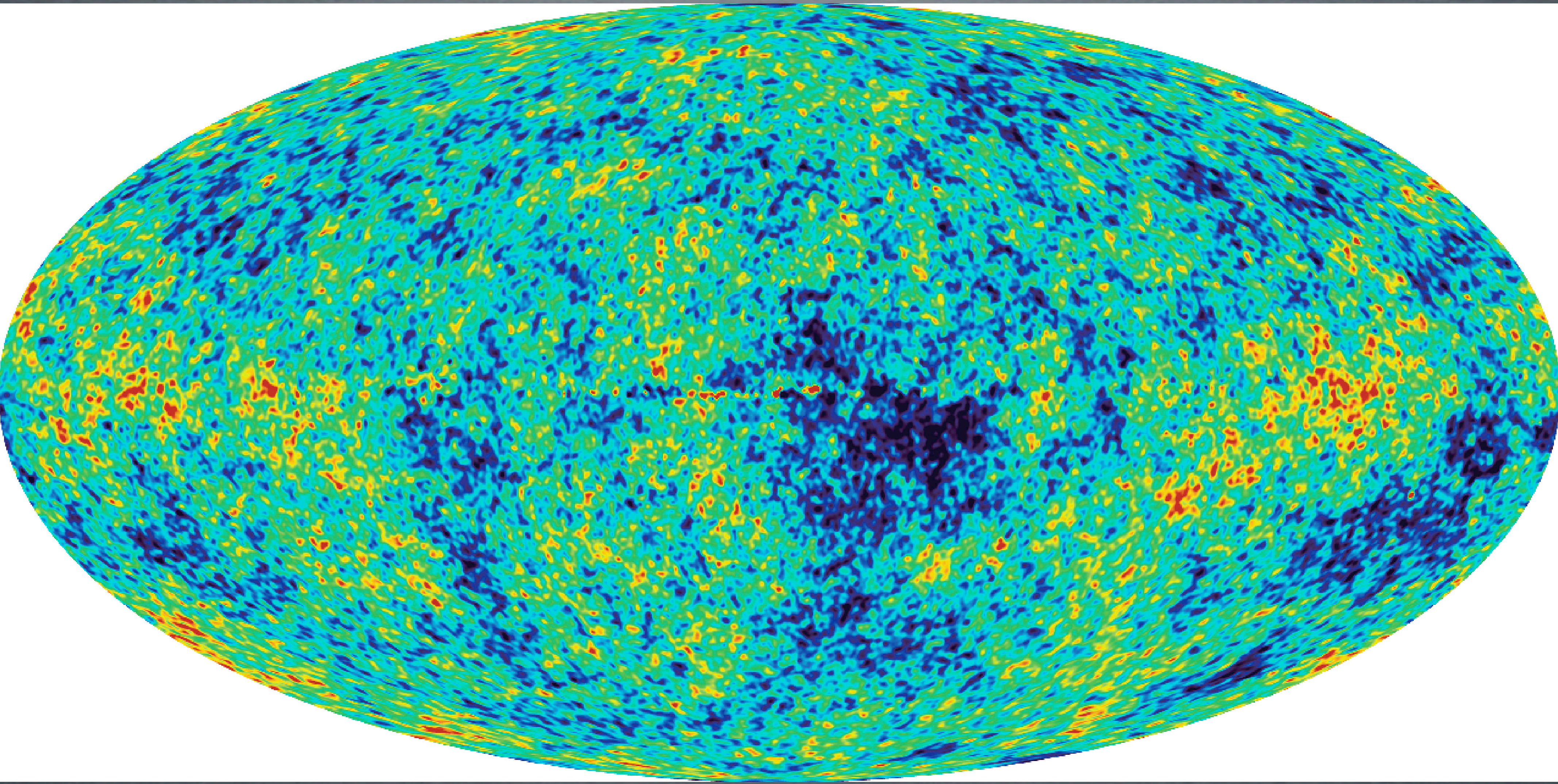


with traces of photons, neutrinos & ... ?

What is DARK ENERGY ???



The Universe is NOT perfectly homogeneous !



[WMAP 06]

Tiny ripples on the black body spectrum at level of 0.01%...

Why is the universe flat, homogeneous & isotropic ?

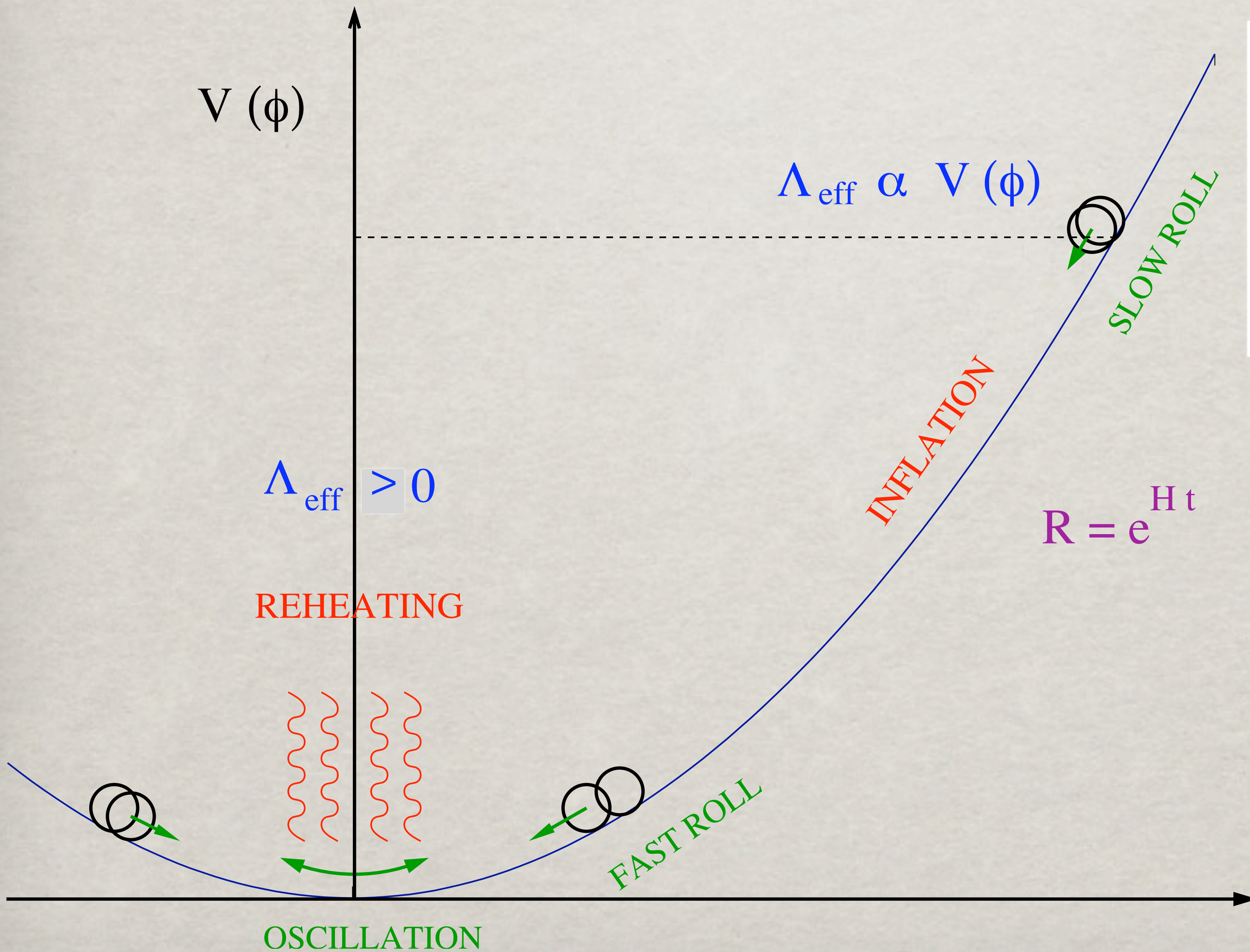
What caused the tiny ripples, which are origin of structure?

↓

INFLATION

early phase of exponential expansion

Inflation: driven by a scalar field ϕ



$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$|\eta| = \left|\frac{V''}{V}\right| \ll 1$$

$R = e^{H t}$ Quasi - de Sitter

de Sitter ?

Power spectrum of the fluctuations

Testing inflation: Single field inflation $\Longleftrightarrow$ Flat Potential $V(\phi)$

The scalar power spectrum is given by
$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{12\pi^2 M_P^6} \frac{V^3}{V'^2}\bigg|_{k=aH} \qquad \propto k^{n-1}$$

and its spectral index is:
$$n(k) - 1 = \frac{d\log(\mathcal{P}_{\mathcal{R}})}{d\log(k)}\bigg|_{k=aH} = 2\eta - 6\epsilon + \ldots$$

For gravity waves the situation is simpler since in fact they are perfectly massless... : the gravity waves are generated by fluctuations in the metric, i.e. $h_{ij} = \delta g_{ij}$.

The tensor power spectrum is given by
$$\mathcal{P}_{grav}(k) = \frac{1}{6\pi^2} \frac{V}{M_P^4}\bigg|_{k=aH}$$

and its spectral index is
$$n_{grav}(k) = \frac{d\log(\mathcal{P}_{grav})}{d\log(k)}\bigg|_{k=aH} = -2\epsilon + \ldots$$

WANTED: de Sitter !

- A positive cosmological constant, i.e. a (possibly metastable) de Sitter state provides at the moment the best fit to the data...
- A quasi de Sitter solution describe very well an inflationary phase since the slow roll parameters have to be small...
- Try to find a model which starts and finishes in a de Sitter vacuum !

WHY SUPERGRAVITY ?

- Theoretically attractive: supersymmetry gives gauge unification, solves hierarchy problem,etc...
- Provides a coherent framework to study different signal in high energy physics, astrophysics and cosmology.
- It is surely necessary to extend supersymmetry to supergravity to discuss cosmology !
- Allows extension to string theory...:
  the low energy 4D limit of some string theories is a N=1 supergravity of the no-scale type.

(QUASI)DE SITTER IN SUGRA

- A de Sitter or quasi-de Sitter phase is needed to account for the present cosmological constant and for inflation

- But in SUGRA the absolute minima are either anti-de Sitter or Minkowski... and do not break SUSY !

$$V = e^K (K^{i\bar{j}}(W_i + K_i W)(\bar{W}_{\bar{j}} + K_{\bar{j}}\bar{W}) - 3|W|^2)$$

- Also inflation is difficult → η problem
  the SUGRA potential is usually steep with $V'' \sim V$
  as long as one does not resort to some tuning...
  ... SLOW ROLL inflation not easy to realise !

[Copeland et al 94; Guth, Randall & Thomas 94, ...]

(QUASI)DE SITTER IN SUGRA

- A de Sitter or quasi-de Sitter phase is needed to account for the present cosmological constant and for inflation

- But in SUGRA the absolute minima are either anti-de Sitter or Minkowski... and do not break SUSY !

$$V = e^K (K^{i\bar{j}}(W_i + K_i W)(\bar{W}_{\bar{j}} + K_{\bar{j}} \bar{W}) - 3|W|^2$$

- Also inflation is difficult → η problem
  the SUGRA potential is usually steep with $V'' \sim V$ as long as one does not resort to some tuning...
  ... SLOW ROLL inflation not easy to realise !

[Copeland et al 94; Guth, Randall & Thomas 94, ...]

(QUASI)DE SITTER IN SUGRA

- A de Sitter or quasi-de Sitter phase is needed to account for the present cosmological constant and for inflation

- But in SUGRA the absolute minima are either anti-de Sitter or Minkowski... and do not break SUSY !

$$V = e^K (K^{i\bar{j}} (W_i + K_i W)(\bar{W}_{\bar{j}} + K_{\bar{j}} \bar{W}) - 3|W|^2)$$

- Also inflation is difficult → η problem
  the SUGRA potential is usually steep with $V'' \sim V$
  as long as one does not resort to some tuning...
  ... SLOW ROLL inflation not easy to realise !

[Copeland et al 94; Guth, Randall & Thomas 94, ...]

(QUASI)DE SITTER IN SUGRA

- A de Sitter or quasi-de Sitter phase is needed to account for the present cosmological constant and for inflation

- But in SUGRA the absolute minima are either anti-de Sitter or Minkowski... and do not break SUSY !

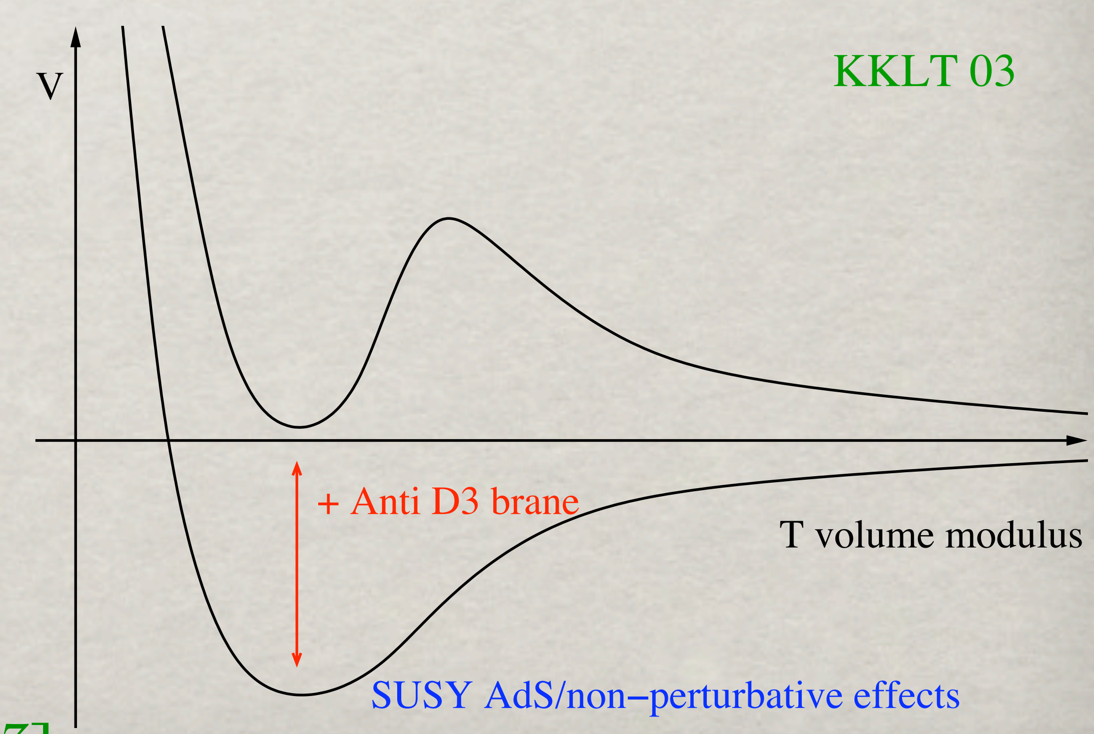
$$V = e^{K}\left(K^{i\bar{j}}(W_i + K_i W)(\bar{W}_{\bar{j}} + K_{\bar{j}}\bar{W}) - 3|W|^2\right)$$

- Also inflation is difficult → η problem
the SUGRA potential is usually steep with $V'' \sim V$
as long as one does not resort to some tuning...
... SLOW ROLL inflation not easy to realise !

[Copeland et al 94; Guth, Randall & Thomas 94, ...]

de Sitter vacua and moduli stabilisation

- One of the historical problems of string theory is to stabilise all the moduli fields.
- Progress in the last years: possible to stabilise most moduli using flux compactifications !
- But in some models one has to rely to explicit SUSY breaking terms to stabilise all the moduli and up-lift the vacuum (e.g. KKLT...)



[Kachru, Kallosh, Linde & Trivedi 03]

Part I: de Sitter in no-scale SUGRA

SUGRA and scalar fields

Thanks to the Kaehler symmetry the scalar potential can be written very simply as a function of a single function

$$G(\Phi,\bar{\Phi}) = K(\Phi,\bar{\Phi}) + \ln\left[W(\Phi)\right] + \ln\left[\bar{W}(\bar{\Phi})\right]$$

i.e. the potential is $V(\Phi,\bar{\Phi}) = e^{G(\Phi,\bar{\Phi})}(G_i G^i - 3)$

where $G_i = \partial_{\Phi_i} G(\Phi,\bar{\Phi})$ is the derivative w.r.t. fields

and indices are lowered and raised by the metric and its inverse

$$g_{i\bar{j}} = \partial_{\Phi_i}\partial_{\Phi_{\bar{j}}} G(\Phi,\bar{\Phi}) \qquad g^{\bar{j}i} g_{i\bar{k}} = \delta_{\bar{j}\bar{k}}$$

Supersymmetry is broken if $\langle G_i \rangle \neq 0$

and the Goldstino field is given by $\eta = G_i \Psi^i$

Scalar mass matrix

- Project the scalar mass matrix along the Goldstino direction for any V and obtain

$$\lambda = e^{-G} V_{i\bar{j}} G^i G^{\bar{j}} = -\frac{2}{3} e^{-G} V (e^{-G} V + 3) + \sigma$$

where $$\sigma = \frac{2}{3} (g_{i\bar{j}} G^i G^{\bar{j}})^2 - R_{i\bar{j}n\bar{m}} G^i G^{\bar{j}} G^n G^{\bar{m}}$$

- A necessary condition for metastability is that λ is positive, then if $V > 0$ we need $\sigma > 0$

- Note: the curvature tensor depends only on the Kaehler potential, while the Goldstino direction on the whole G, including W

Simple Kaehler potentials

- Canonical Kaehler potential: $K = \bar{X} X$
  Zero higher derivatives and no curvature !
  For vanishing Λ :
  $$\sigma = \frac{2}{3} \times 9 = 6 > 0$$

- Logarithmic Kaehler: $K = -n \ \ln \left[T + \bar{T} \right]$
  Constant curvature $R \sim 2/n$
  so we have
  $$\sigma = 6 - \frac{18}{n} > 0 \ \rightarrow \ n > 3$$

  Same result also for $K = -n \ \ln \left[T + \bar{T} - \bar{X} X \right]$

- More in general the curvature is not constant...

No-Scale Kaehler

[Cremmer, Ferrara, Kounas & Nanoupoulos 83, ...]

- The no-scale property requires $K_i K^i = 3$ so that the cosmological constant is zero at tree level since the potential vanishes if $W_i = 0$

$$
\begin{aligned}
V &= e^{K(\Phi,\bar{\Phi})}\left[|W_i + K_i W|^2 - 3|W|^2\right] \\
&= e^{K(\Phi,\bar{\Phi})}\left[|W_i|^2 + 2Re[K_i W \bar{W}_i]\right]
\end{aligned}
$$

- For a single field the no-scale Kaehler is simply

$$K = -3\ln[T + \bar{T}]$$

The trouble of no-scale

- The problem is the logarithmic Kaehler potential...

$$K = -3\ln(T+\bar{T}) \qquad G = K + \ln(|W|^2)$$

- For a single modulus in de Sitter one mass is always negative for any superpotential W [Brustein & de Alwis 04]
- In general Minkowski metastable vacua with broken SUSY need the holomorphic sectional curvature for the metric $K_{i\bar{j}}$ to be bounded: $R_{i\bar{j}n\bar{m}}G^iG^{\bar{j}}G^nG^{\bar{m}} < 6$

[Gomez Reino & Scrucca 04]

- This result can be generalised to de Sitter into:

$$\sigma = \frac{2}{3}(g_{i\bar{j}}G^iG^{\bar{j}})^2 - R_{i\bar{j}n\bar{m}}G^iG^{\bar{j}}G^nG^{\bar{m}} > 0$$

- $\sigma = 0$ for $G_i \propto K_i$: NO GO for a single field !

[LC, Gomez Reino, Gross, Luis, Palma & Scrucca I 08]

Two moduli in strings

[LC, Gomez Reino, Gross, Luis, Palma & Scrucca I 08]

Heterotic Calabi-Yau

$$K = -\log(\mathcal{V})$$

$$\mathcal{V} = \frac{4}{3} d_{ijk} v^i v^j v^k$$

$$\Re(T^i) = v^i$$

Type II b orientifolds

$$K = -2\log(\mathcal{V})$$

$$\mathcal{V} = \frac{1}{48} d^{ijk} v_i v_j v_k$$

$$\Re(T^i) = \frac{1}{16} d^{ijk} v_j v_k$$

Then we have simply

$$\sigma \sim -\frac{3}{8} e^{4K} \frac{\Delta}{detg} |C|^4 \qquad \sigma \sim \frac{3}{8} e^{4K} \Delta\, detg\, |C|^4$$

Where Δ is the discriminant of the cubic polynomial

Two moduli in strings

[LC, Gomez Reino, Gross, Luis, Palma & Scrucca I 08]

Heterotic Calabi-Yau

$$K = -\log(\mathcal{V})$$

$$\mathcal{V} = \frac{4}{3} d_{ijk} v^i v^j v^k$$

$$\Re(T^i) = v^i$$

Type II b orientifolds

$$K = -2\log(\mathcal{V})$$

$$\mathcal{V} = \frac{1}{48} d^{ijk} v_i v_j v_k$$

$$\Re(T^i) = \frac{1}{16} d^{ijk} v_j v_k$$

Then we have simply

$$\sigma \sim -\frac{3}{8} e^{4K} \frac{\Delta}{detg} |C|^4 \qquad \sigma \sim \frac{3}{8} e^{4K} \Delta\, detg\, |C|^4$$

Where Δ is the discriminant of the cubic polynomial

Build Tree Level de Sitter

It is possible also for NO-SCALE for more than 2 fields !!!

- Choose intersection numbers with the correct sign of Δ
- Taylor expand the superpotential W around the minimum up to 3rd order and fix the coefficients such that $V \sim 0, V' = 0$ and all masses (apart for the Goldstino partner fields) are positive; W_0 fixes the gravitino mass and the overall scale of the potential.
- Continue the potential away from the minimum using linear and exponential terms (at least 7 parameters needed for two fields with separable W)

[LC, Gomez-Reino, Gross, Palma, Scrucca 09]

Explicit Model(s)

[LC, Gomez-Reino, Gross, Palma, Scrucca 09]

Expand the superpotential around the minimum as

$$W = W_0 + W_i(T_i - T_i^0) + W_{ij}(T_i - T_i^0)(T_j - T_j^0) + W_{ijk}(T_i - T_i^0)(T_j - T_j^0)(T_k - T_k^0) + \dots$$

heterotic: $\Delta < 0$

T_0^1	0.405666
T_0^2	0.749277

W_0	1.00000
W_1	1.64415
W_2	2.60392
W_{11}	−17.4400
W_{22}	3.82418
W_{111}	616.732
W_{222}	2.31275

orientifold: $\Delta > 0$

T_0^1	0.412741
T_0^2	0.714888

W_0	1.000000
W_1	2.021311
W_2	0.931223
W_{11}	0.999657
W_{22}	−0.797685
W_{111}	−0.827204
W_{222}	3.308820

Stringy Model(s)

[LC, Gomez-Reino, Gross, Palma, Scrucca 09]

Match to a string-inspired superpotential like

$$W = \Lambda + A_1 e^{a_1 T_1} + B_1 e^{b_1 T_1} + A_2 e^{a_2 T_2} + B_2 e^{b_2 T_2}$$

heterotic: $\Delta < 0$

Λ	-5.97604×10^{-1}
A_1	-3.62358×10^{5}
B_1	-1.46692×10^{0}
A_2	7.98841×10^{-1}
B_2	7.49672×10^{-1}

a_1	4.36876×10^{1}
b_1	2.66924×10^{0}
a_2	-1.28225×10^{0}
b_2	5.33848×10^{0}

orientifold: $\Delta > 0$

Λ	2.63036×10^{1}
A_1	7.37726×10^{1}
B_1	-9.77287×10^{1}
A_2	-1.50213×10^{0}
B_2	-2.80545×10^{0}

a_1	3.49830×10^{-1}
b_1	2.79764×10^{-1}
a_2	7.30908×10^{0}
b_2	4.19646×10^{-1}

in units of

$m_{3/2}\mathcal{V}_H^{1/2}$ $\quad$ $\mathcal{V}_H^{-1/3}$ $\quad$ $m_{3/2}\mathcal{V}_0$ $\quad$ $\mathcal{V}_0^{-2/3}$

Another way: corrected de Sitter

Subleading corrections can help, if they spoil the no-scale property and change the Kaehler curvature...

$$K = -n \log \left[\mathcal{V} + \hat{\xi} \right]$$

Then we obtain $\sigma \propto \hat{\xi}$ positive for positive $\hat{\xi}$

But then the mass along the Goldstino direction is suppressed compared to the gravitino mass:

$$\frac{\tilde{m}^2}{m_{3/2}^2} \propto \hat{\xi}$$

[LC, Gomez-Reino, Gross, Louis, Palma, Scrucca 08 I]

Part II: Inflation & the gravitino mass

What about inflation ?
A New η problem !

[LC, Gomez Reino, Gross, Luis, Palma & Scrucca II 08]

- In modular inflation eta is constrained:

$$\eta \leq -\frac{2}{3} + \frac{\sigma}{9\gamma\,(1+\gamma)} + \mathcal{O}(\sqrt{\epsilon})$$

where $\gamma = \frac{H_I^2}{m_{3/2}^2}$ for $m_{3/2}^2 = e^G = e^K |W|^2$

- To realise slow roll inflation, i.e. $\epsilon, |\eta| \sim 0$, we need

$$\sigma \geq 6\gamma(1+\gamma)$$

For $\gamma \ll 1$ this reduces to $\sigma > 0$ as for pure de Sitter, while for $\gamma \geq 1$ it is more stringent !
INFLATION at HIGH SCALE is more difficult !

What can we say then ?

- We need more than one field contributing to modular inflation..., possibly one which has a Kaehler potential with zero curvature, e.g.

$$K = -3\ln(T+\bar{T}) + \bar{X}X$$

- We can rely on quantum corrections to modify the curvature and allow de Sitter or inflation, but with some tuning...

- An early inflationary phase, makes present (at least metastable) de Sitter possible...

- Explicit model building still ongoing work !

INFLATION WITH 2 FIELDS ?

It seemed possible for NO-SCALE with 2 or more fields...

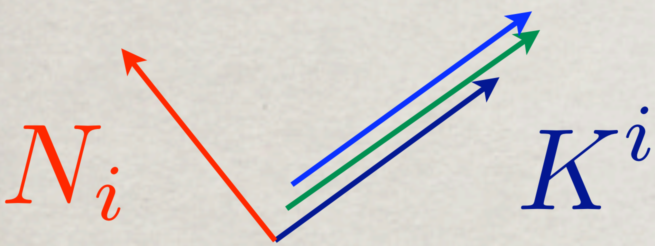
- Choose d_{ijk} with the correct sign of Δ
- Define two orthogonal directions, one along K_i
- If SUSY is broken along K_i , there is one tachyonic state at tree-level and $|\eta|$ is large since $\sigma = 0$
- If SUSY is broken along N_i , inflation cannot proceed along that direction since $V \leq 0$
  It can proceed along K_i , but not for long... Need to keep a restricted phase.
- Mixed case ???

[Burrage, LC, Gross 10]

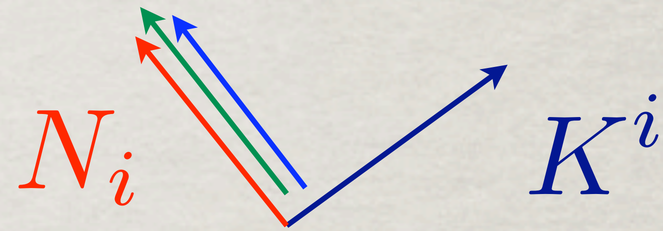
INFLATION WITH 2 FIELDS ?

[Burrage, LC, Gross 10]

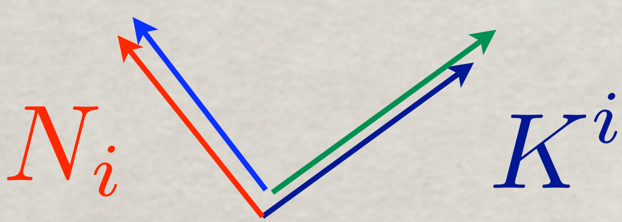
~~SUSY~~ Inflation



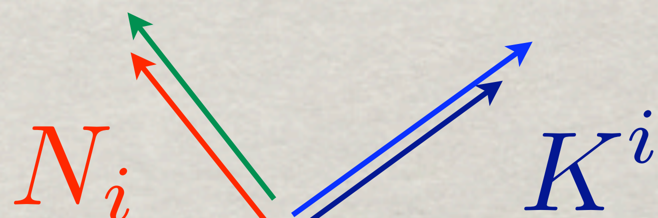
$$\sigma = 0 \to \eta_{||} < -\frac{2}{3}$$



$$G_i K^i = 0 \to V < 0$$



$$\sigma = 0 \to \eta_{\perp} < -\frac{2}{3}$$



$$\eta \sim 0 \to \arg[K^i \nabla_i V] \neq 0$$

General predictions:

- We need more than one modular field to allow for inflation: if it is not possible for realistic W to make all other states heavy, we can expect both isocurvature perturbations and non-gaussianities
- Low scale inflation is preferred !
  Probably no gravity waves signal for modular inflation... apart if the gravitino mass was very large during inflation.

Part III: Hybrid Inflation & quantum corrections

Coupled fields & hybrid inflation

[Baacke, LC, Kevlishvili10]

- Consider a system of coupled fields with non-minimal coupling with gravity and gravity counterterms O(RR):

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\Phi_i\partial^\mu\Phi^i - V(\Phi) - \frac{R}{2}\xi_i\Phi_i^2\right]$$

where $$V(\Phi) = \frac{1}{2}m_i^2\Phi_i^2 + \frac{1}{4}\lambda_{ij}\Phi_i^2\Phi_j^2$$

- Treat the scalar fields as quantum fields in the one loop approx., but keep gravity classical. Regularise the model in n dimensions and renormalise it in a scheme such to make e.o.m. of the fluctuations numerically stable.

Equations of motion

$$\Phi_i = \phi_i(t) + \delta\phi_i(t,x)$$

- Classical field:

$$\xi_n = \frac{n-2}{4(n-1)}$$

$$\phi_i'' + (m_i^2 + (\xi_i - \xi_n)R)a^2\phi_i +$$
$$a^{4-n}\lambda_{ij}\left[(\phi_j^2 - iG_{jj})\phi_i - 2iG_{ij}\phi_j\right] = 0$$

- Quantum fluctuations:

$$f_i''(\tau,k) + k^2 f_i(\tau,k) + \mathcal{M}_{ij} f_j(\tau,k) = 0$$

where

$$\mathcal{M}_{ii} = (m_i^2 + (\xi_i - \xi_n)R)a^2 + a^{4-n}\left(3\lambda_{ii}\phi_i^2 + \lambda_{ij}\phi_j^2\right)$$

$$\mathcal{M}_{ij} = 2a^{4-n}\lambda_{ij}\phi_j^2 \quad i \neq j$$

Fluctuation integral

[Baacke, LC, Kevlishvili10]

- The regularised Greens functions can be written as

$$-iG_{ij} = -\frac{L_\epsilon}{16\pi^2}\mathcal{M}^2_{ij} + \mathcal{F}^{fin}_{ij} + \mathcal{F}^{add}_{ij}$$

where L_ϵ contains the divergent part, $\mathcal{F}^{fin}_{ij}$ the finite parts and the numerical subtracted integral, while $\mathcal{F}^{add}_{ij}$ are finite pieces due to taking GR in n dimensions

- The lagrangian counterterm is simply

$$\delta\mathcal{L} = -a^{n-4}\frac{L_\epsilon}{64\pi^2}\mathcal{M}^2_{kl}\,\mathcal{M}^2_{lk}$$

it contains all the usual renormalisation counterterms for cosmological constant, masses & couplings.

Energy-momentum tensor

[Baacke, LC, Kevlishvili10]

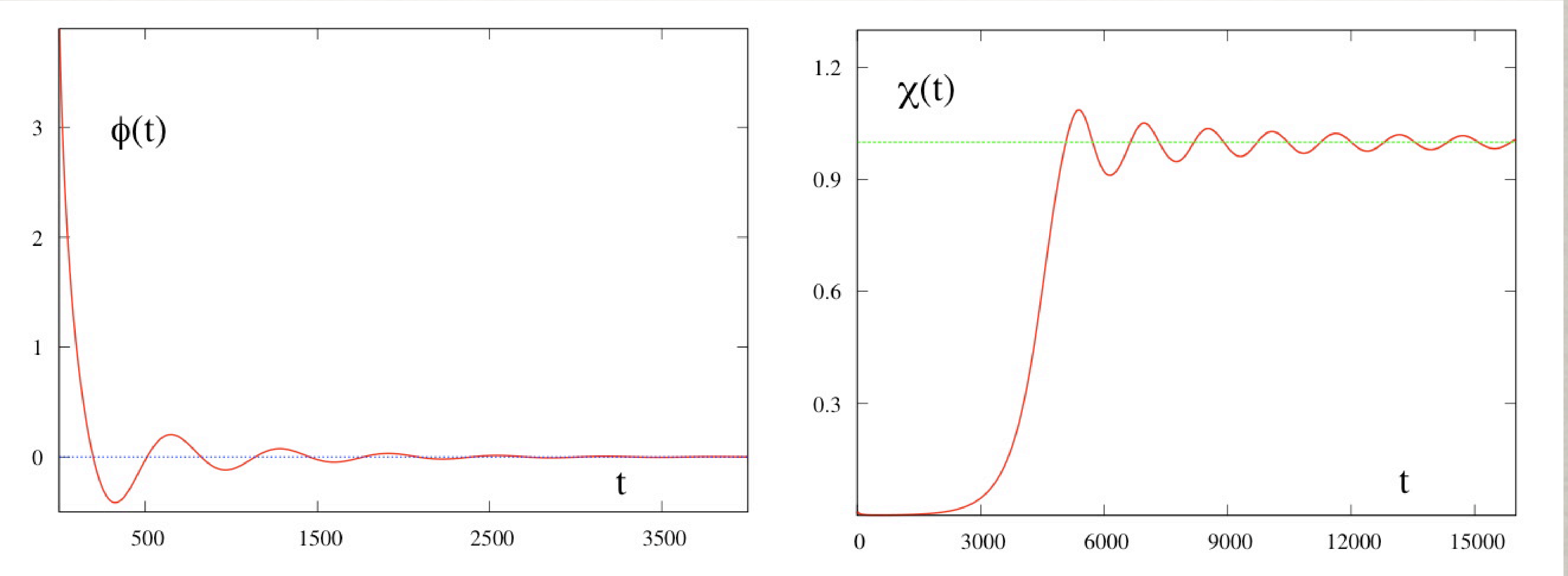
- The renormalised energy-momentum tensor can be written as a function of the renormalised fluctuation integrals and is covariantly conserved.

- There is a contribution from the conformal anomaly, which is finite and has to be added "by hand".

- The renormalisation is independent of time, but some of the analytical finite pieces contain Log(a) terms, which are compensated by the numerical integrals. For a single free field, we can show analytically that the only terms surviving are of the form $Log(m_i/\mu)$

Two fields simulations

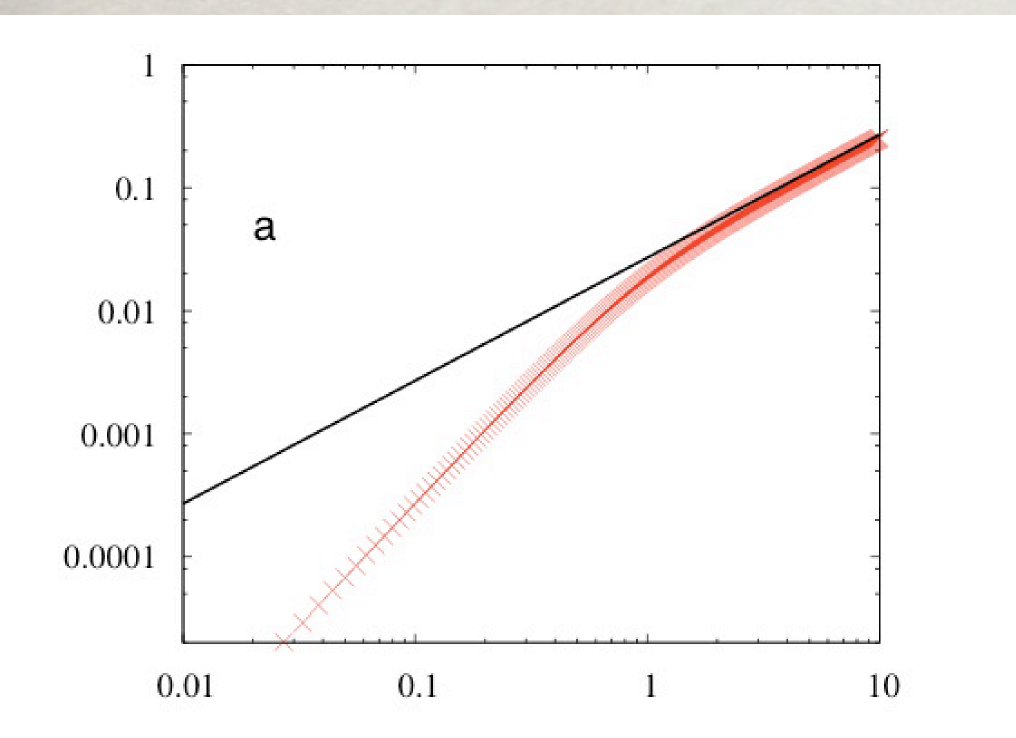
[Baacke, LC, Kevlishvili10]



$$m_\phi^2 = 10^{-2} \qquad \lambda_{ij} = 10^{-5} \qquad m_\chi^2 = 10^{-5}$$

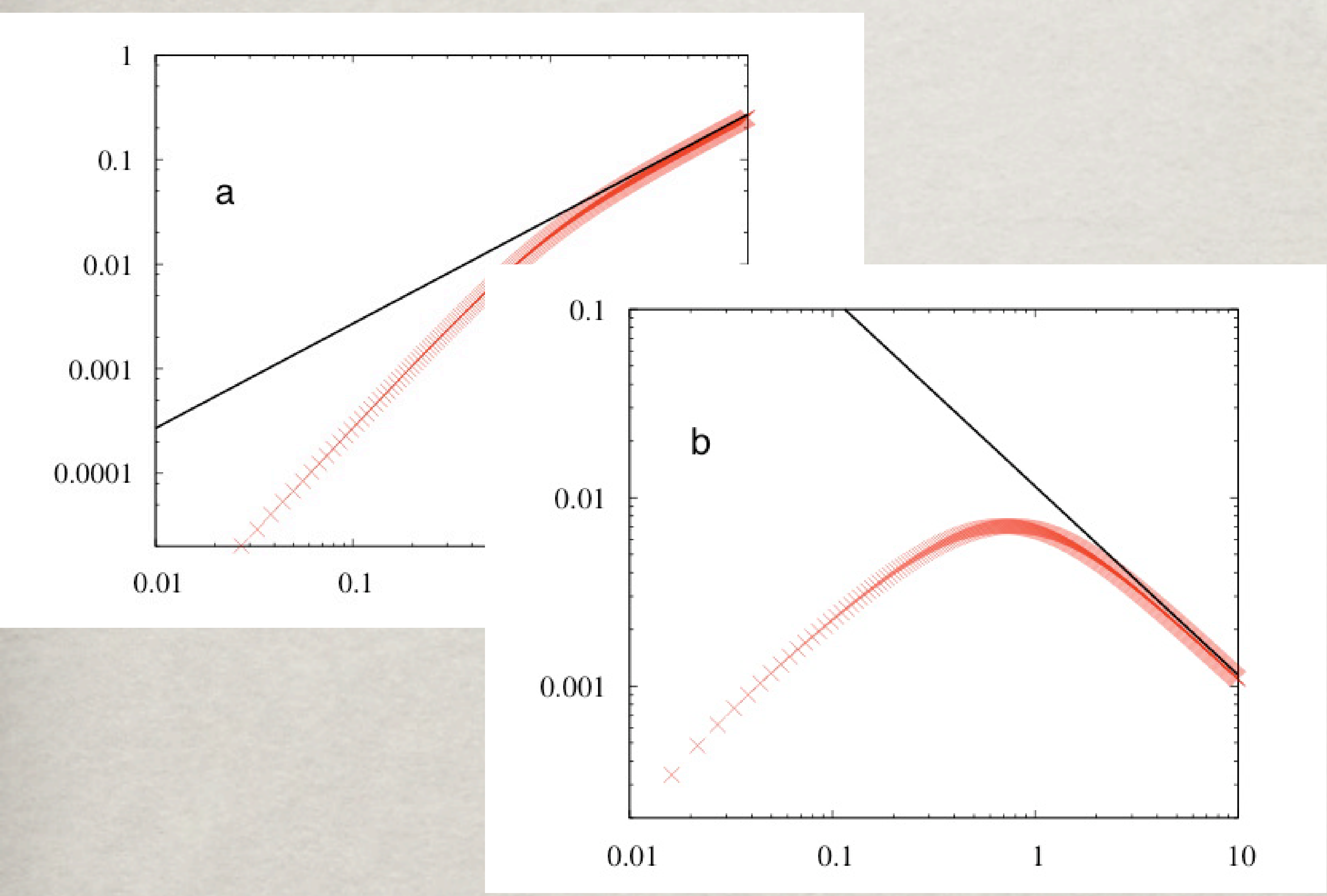
Two fields simulations

[Baacke, LC, Kevlishvili10]



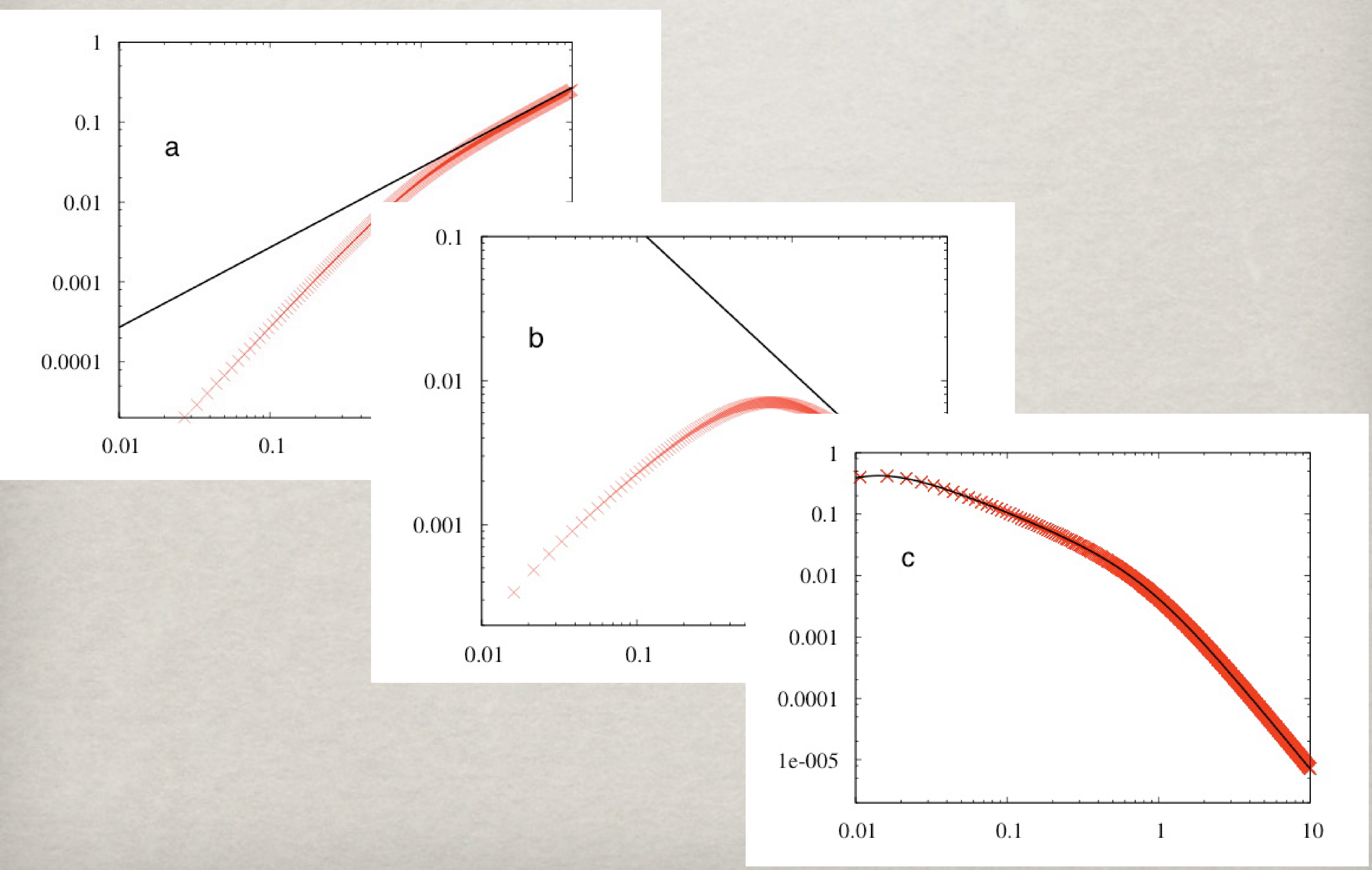
Two fields simulations

[Baacke, LC, Kevlishvili10]



Two fields simulations

[Baacke, LC, Kevlishvili10]



Outlook

Outlook

de Sitter in SUGRA is not so hopeless:

- We were able to build a model with a tree-level metastable de Sitter vacuum, but we need more than one modulus...

- No inflation in this model yet, but we are still exploring new directions:
  - exploit even more scalar fields
  - try to change substantially the gravitino mass during cosmological evolution

- Also some of the fields have a mass not larger than the gravitino mass: moduli problem ???