

Gravitational waves from phase transitions

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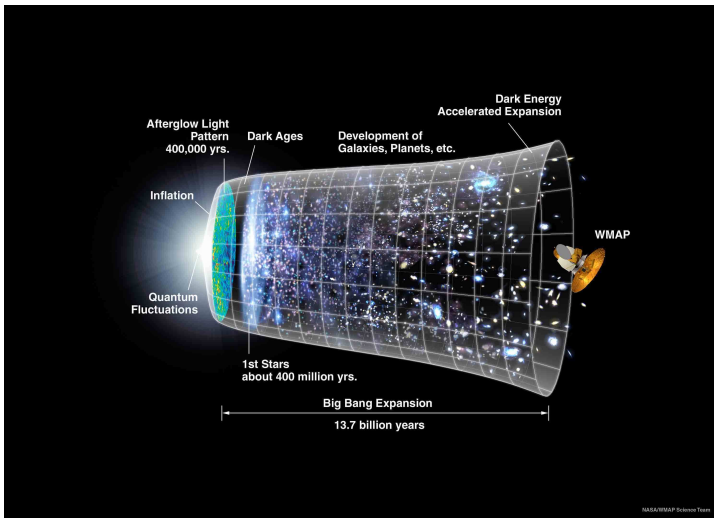
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Introduction

The Universe has expanded and cooled down from a very hot initial state to (presently) 2.7°K . It seems likely that it underwent several phase transitions during adiabatic expansion.



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In this talk I present a semi-analytical evaluation of the GW signal from a first order phase transition in terms of free parameters.

If a phase transition at $T = T_*$ generates a (relativistic) source of gravitational waves with energy density ρ_X , the GW spectrum has the following properties

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- On large scales, $k \ll k_*$ the spectrum is blue,

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- The behaviour of the spectrum on small scales, $k \gg k_*$ depends on the details of the source.

Sources of gravitational waves

Gravitational waves are sourced by fluctuations in the energy momentum tensor which have a non-vanishing spin-2 contribution.

$$ds^2 = a^2 \left(d\eta^2 + (\gamma_{ij} + 2h_{ij}) dx^i dx^j \right)$$

where h_{ij} is transverse and traceless. In Fourier space $k^i h_{ij} = h_i^i = 0$.

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$$\left(\partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2 \right) h_{ij} = 8\pi G a^2 \Pi_{ij}$$

Here $\Pi_{ij}(\mathbf{k})$ is the tensors type (spin-2) anisotropic stress and $\mathcal{H} = \frac{a'}{a}$.

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Duration: $\Delta\eta_* < \mathcal{H}_*^{-1}$, bubble size: $R = v_b \Delta\eta_*$, v_b = bubble velocity.

$k_* = (\Delta\eta_*)^{-1}$ or R^{-1} .

- Because of causality, the correlator $\langle \Pi_{ij}(\eta_1, \mathbf{x}) \Pi_{lm}(\eta_2, \mathbf{y}) \rangle = \mathcal{M}_{ijlm}(\eta_1, \eta_2, \mathbf{x} - \mathbf{y})$ is a function of compact support. For distances $|\mathbf{x} - \mathbf{y}| > \max(\eta_1, \eta_2)$, $\mathcal{M} \equiv 0$.

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- We decompose Π_{ij} into two helicity modes which we assume to be statistically equal and uncorrelated (parity),

$$\Pi_{ij}(\eta, \mathbf{k}) = \mathbf{e}_{ij}^+ \Pi_+(\eta, k) + \mathbf{e}_{ij}^- \Pi_-(\eta, k)$$

$$\begin{aligned} \langle \Pi_+(\eta, k) \Pi_+^*(\eta', k') \rangle &= \langle \Pi_-(\eta, k) \Pi_-^*(\eta', k') \rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') \rho_X^2 P(\eta, \eta', k) \\ \langle \Pi_+(\eta, k) \Pi_-^*(\eta', k') \rangle &= 0. \end{aligned}$$

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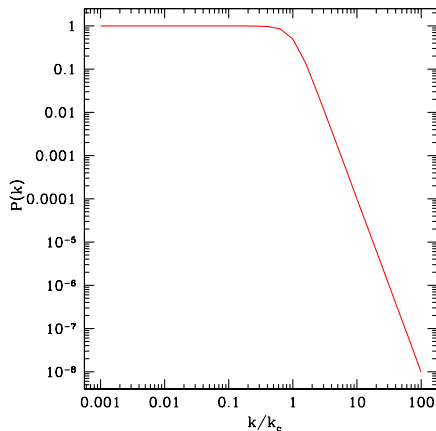
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- Causality implies that the function $P(\eta, \eta', k)$ is analytic in k . We therefore expect it to start out as white noise and to decay beyond a certain correlation scale $k_*(\eta, \eta') > \min(1/\eta, 1/\eta')$.

The spectrum



The anisotropic stress power spectrum from Kolmogorov turbulence. On small scales $P \propto k^{-11/3}$.

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$$\begin{aligned} h(\mathbf{k}, \eta) &= \frac{8i\pi G a_*^3}{6ak} \left[e^{-ik\eta} \int_{\eta_*}^{\eta_* + \Delta\eta_*} d\eta' e^{ik\eta'} \Pi(\eta', \mathbf{k}) + \right. \\ &\quad \left. e^{ik\eta} \int_{\eta_*}^{\eta_* + \Delta\eta_*} d\eta' e^{-ik\eta'} \Pi(\eta', \mathbf{k}) \right] \\ &= \frac{8i\pi G a_*^3}{6ak} \left[e^{-ik\eta} \Pi(k, \mathbf{k}) + e^{ik\eta} \Pi(-k, \mathbf{k}) \right] \end{aligned}$$

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- The gravitational wave energy density is given by

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- If the Universe is radiation dominated during the phase when the gravitational waves are generated, this gives on large scales, $k < k_*$

$$\frac{d\Omega_{gw}}{d\ln(k)}(\eta_0) = \frac{12\Omega_{\text{rad}}(\eta_0)}{\pi^2} \left(\frac{\Omega_X(\eta_*)}{\Omega_{\text{rad}}(\eta_*)} \right)^2 \mathcal{H}_*^2 k^3 \text{Re}[P(k, k, k)].$$

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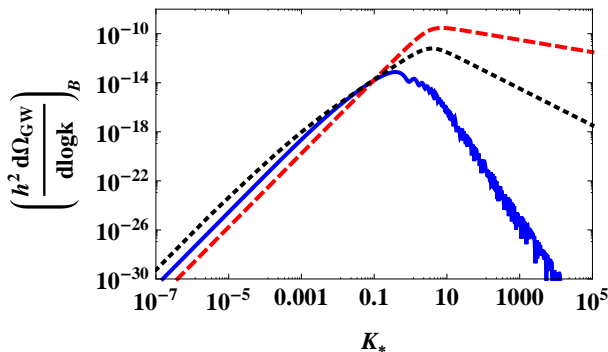
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- The behavior of the spectrum close to the peak and its decay rate on smaller scales depends on the source characteristics, on its temporal behavior (continuity, differentiability) and its power spectrum.

- For a totally incoherent source, $P(\eta, \eta', k) = \delta(\eta - \eta') \Delta\eta_* P(\eta, \eta, k)$ the peak position of the GW spectrum is determined by the peak of the **spatial** Fourier transform of the source.

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- For a coherent source, $P(\eta, \eta', k) = \sqrt{P(\eta', \eta', k) P(\eta, \eta, k)}$, when $P(\eta, \eta, k)$ is continuous in time but not differentiable (bubble collisions) the peak position of the GW spectrum $\propto k^3 P(k, k, k)$ is determined by the peak of the **temporal** Fourier transform of the source.

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- For a source with finite coherence time,
 $P(\eta, \eta', k) = \sqrt{P(\eta', \eta', k)P(\eta, \eta, k)} \Theta(x_c - |\eta - \eta'|k)$, $x_c \sim 1$ the GW spectrum is again determined by the peak of the **spatial** Fourier transform of the source.



The GW energy density spectrum in the incoherent (red), tophat (black) and coherent (blue). The parameters are: $T_* = 100$ GeV, $\Delta\eta_*\mathcal{H}_* = 0.01$,
 $\Omega_X/\Omega_{\text{rad}} = 2/9$ ($\langle v^2 \rangle = 1/3$)
 . Caprini, RD and Servant, 2009, arXiv:0909.0622

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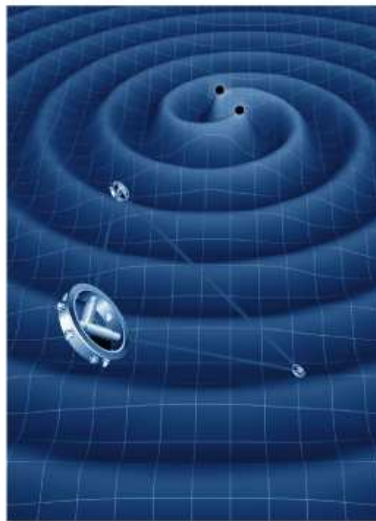
$k_* = 1/\Delta\eta_* \simeq 100/\eta_* \sim 10^{-3}\text{Hz}$, which is close to the frequency of the peak sensitivity for the space born gravitational wave antenna **LISA**, proposed for launch in 2018, a ESA cosmic vision project.

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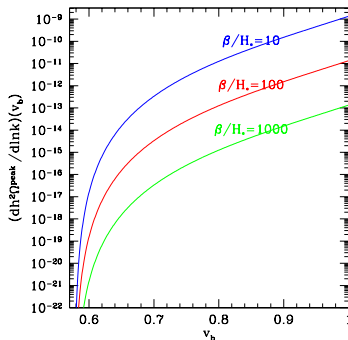
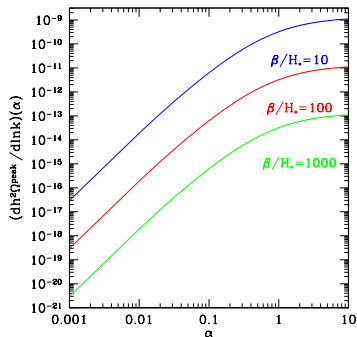
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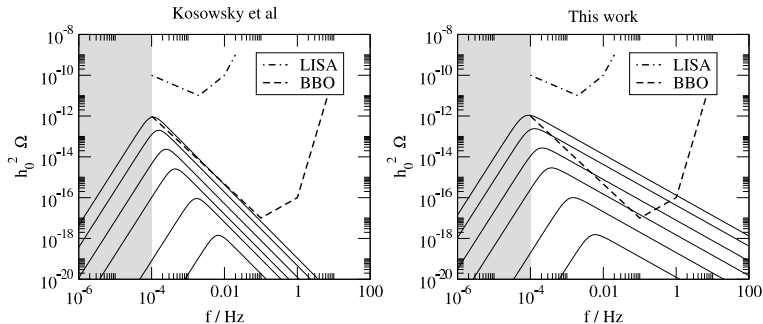
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Caprini, RD, Konstandin, Servant, 2009 arXiv:0901.1661 ($\beta = (\Delta\eta_*)^{-1}$)

The electroweak phase transition: GW's from bubble collisions



Huber & Konstandin 2008

Ω_{GW} from colliding bubbles, numerical results, $\Omega_{\text{X}}/\Omega_{\text{rad}} = 0.03$.

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- For $k > k_c$ we expect a **Kolmogorov spectrum** for the vorticity field, $P_v \propto k^{-11/3}$ and an **Iroshnikov–Kraichnan spectrum** for the magnetic field, $P_B \propto k^{-7/2}$.

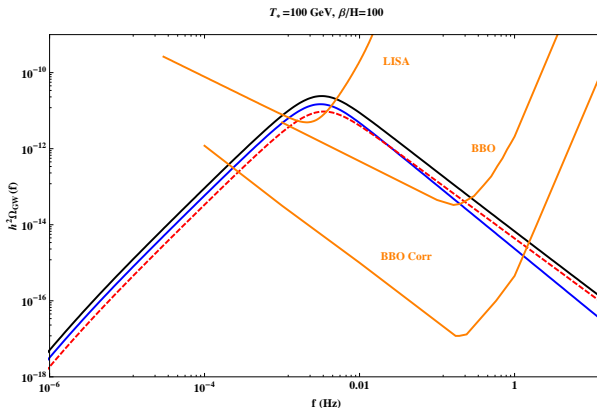
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- For the induced GW spectrum this yields

$$\frac{d\Omega_{GW\bullet}(k, \eta_0)}{d\ln(k)} \simeq \epsilon \Omega_{\text{rad}}(\eta_0) \left(\frac{\Omega_\bullet(\eta_*)}{\Omega_{\text{rad}}(\eta_*)} \right)^2 \times \begin{cases} (k/k_c)^3 & \text{for } k < k_c \\ (k/k_c)^{-\alpha} & \text{for } k > k_c \end{cases}$$

For $\bullet = v$ we have $\alpha = 11/3 - 1 = 8/3$ and for $\bullet = B$ we have $\alpha = 7/2 - 1 = 5/2$.
(See [Caprini & RD, 2006, astro-ph/0603476](#))

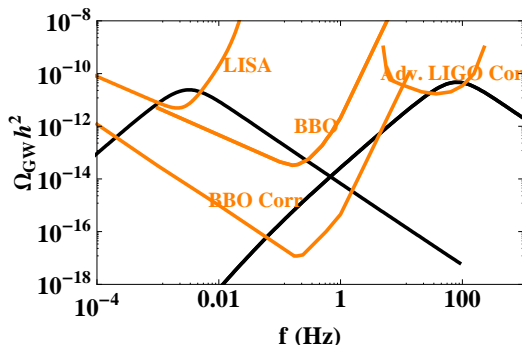
The electroweak phase transition: GW's from turbulence and magnetic fields



Caprini, RD, Servant, [arXiv:0909.0622](https://arxiv.org/abs/0909.0622)

Ω_{GW} from magnetic fields (red) and turbulence (blue), total (black). Modelling the time-decorrelation of the source (Kraichnan decorrelation) by a 'top-hat' in Fourier space. Sensitivity curves from [A. Buonanno 2003](#).

The electroweak phase transition: GW's from turbulence and magnetic fields



Caprini, RD, Servant, arXiv:0909.0622

We also consider a phase transition at $T = 5 \times 10^6$ GeV with $\Delta\eta_* \mathcal{H}_* = 0.02$.

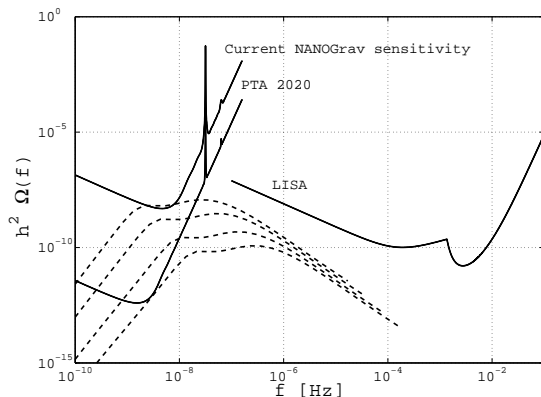
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- In this case the QCD phase transition proceeds via bubble nucleation and leads to turbulence and probably the generation of magnetic fields (very large kinetic and magnetic Reynolds numbers).

GW's from the QCD phase transition



Caprini, RD, Siemens, arXiv:1007.1218

GW's from the QCD phase transition at $T = 100$ MeV with $\Omega_X = 0.1$ and $\Delta\eta_* \mathcal{H}_* = 1, 0.5, 0.2, 0.1$ (from top to bottom).

It is difficult to estimate $\Omega_B(\eta_*)$ or $\Omega_V(\eta_*)$ accurately, but since causality requires the spectra to be so blue, $\frac{d\Omega_B(k, \eta_*)}{d\ln(k)} \propto k^5$, the limit on gravitational waves (which comes from small scales $k \simeq k_*$) yields very strong limits on primordial magnetic fields on large scales.

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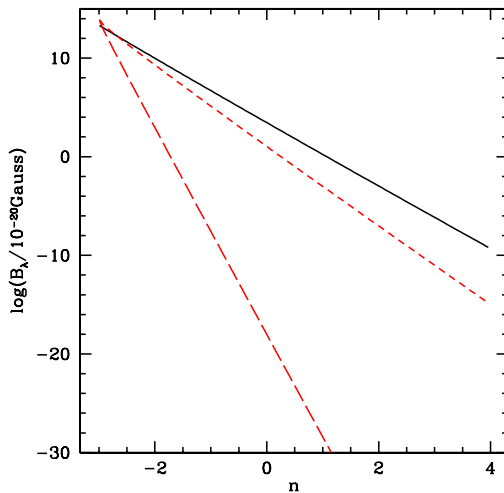
E.g. for $k_1 = (0.1 \text{Mpc})^{-1}$ we obtain $k_1^{3/2} B(k_1) < 10^{-30} \text{Gauss}$.

This is purely a consequence of the redness of the magnetic field spectrum. The simple requirement that $\rho_B(t_*) < \rho_c(t_*)$ yields

$$k_1^{3/2} B(k_1) < 10^{-29} \text{Gauss} \left(\frac{k_1 \cdot 0.1 \text{Mpc}}{k_* \cdot 10^3 \text{sec}} \right)^{5/2}.$$

$$(10^3 \text{sec} = 10^{-11} \text{Mpc})$$

Limits of primordial magnetic fields



($\lambda = 0.1 \text{ Mpc}$) [Caprini & RD., 2001, astro-ph/0106244](#)

The electroweak phase transition: helical magnetic fields and parity violation

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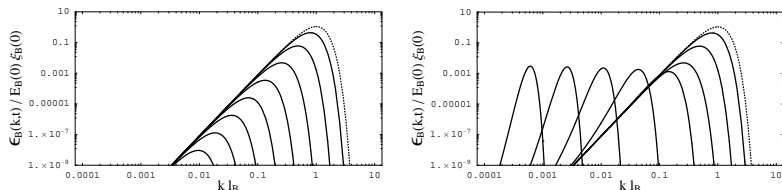
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- Such helical magnetic fields lead to T-B and E-B correlations in the CMB, and they also generate gravitational waves with non-vanishing helicity (Caprini, Kahnishvili, RD. 2004, astro-ph/0304556).

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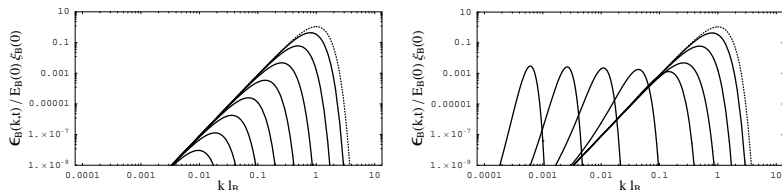
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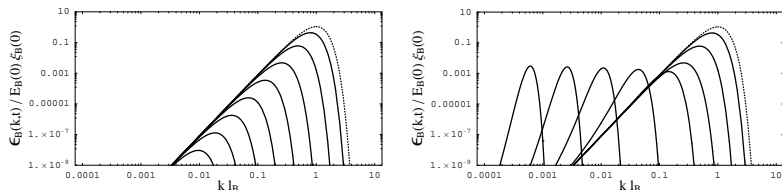


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- In this case, the GW background would not be parity symmetric. There would be more GW's of one helicity than of the other.

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- In this case we also expect a parity violating gravitational wave background, $|h_+(k)|^2 \neq |h_-(k)|^2$.