

Gravitational waves from phase transitions

Ruth Durrer

Department of Theoretical Physics
Geneva University
Switzerland



UNIVERSITÉ
DE GENÈVE

Work in collaboration with: [Chiara Caprini](#), [Elisa Fenu](#), [Tina Kahnashvili](#), [Thomas Konstandin](#),
[Geraldine Servant](#), [Xavier Siemens](#)
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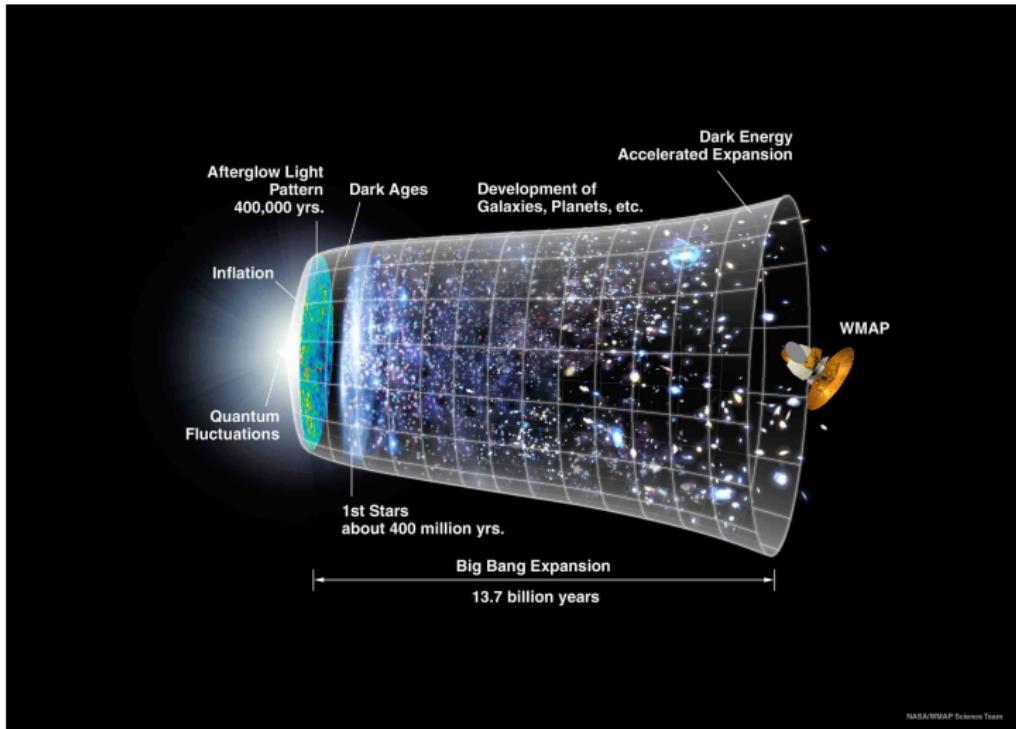
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Outline

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 - Causality
 - Peak position
- 4 The electroweak and QCD phase transitions
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 - The QCD phase transition
 - Limits of primordial magnetic fields from the EW transition
- 5 Conclusions

Introduction

The Universe has expanded and cooled down from a very hot initial state to (presently) 2.7°K. It seems likely that it underwent several phase transitions during adiabatic expansion.



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In this talk I present a semi-analytical evaluation of the GW signal from a first order phase transition in terms of free parameters.

If a phase transition at $T = T_*$ generates a (relativistic) source of gravitational waves with energy density ρ_X , the GW spectrum has the following properties

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- On large scales, $k \ll k_*$ the spectrum is blue,

$$\frac{d\Omega_{GW}(k)}{d \log(k)} \propto k^3, \quad \Omega_{GW} = \int \frac{dk}{k} \frac{d\Omega_{GW}(k)}{d \log(k)}$$

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- The behaviour of the spectrum on small scales, $k \gg k_*$ depends on the details of the source.

Sources of gravitational waves

Gravitational waves are sourced by fluctuations in the energy momentum tensor which have a non-vanishing spin-2 contribution.

$$ds^2 = a^2 \left(d\eta^2 + (\gamma_{ij} + 2h_{ij}) dx^i dx^j \right)$$

where h_{ij} is transverse and traceless. In Fourier space $k^i h_{ij} = h_i^j = 0$.

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$$\left(\partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2 \right) h_{ij} = 8\pi G a^2 \Pi_{ij}$$

Here $\Pi_{ij}(\mathbf{k})$ is the tensors type (spin-2) anisotropic stress and $\mathcal{H} = \frac{a'}{a}$.

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Duration: $\Delta\eta_* < \mathcal{H}_*^{-1}$, bubble size: $R = v_b \Delta\eta_*$, v_b = bubble velocity.
 $k_* = (\Delta\eta_*)^{-1}$ or R^{-1} .

- Because of causality, the correlator $\langle \Pi_{ij}(\eta_1, \mathbf{x}) \Pi_{lm}(\eta_2, \mathbf{y}) \rangle = \mathcal{M}_{jilm}(\eta_1, \eta_2, \mathbf{x} - \mathbf{y})$ is a function of compact support. For distances $|\mathbf{x} - \mathbf{y}| > \max(\eta_1, \eta_2)$, $\mathcal{M} \equiv 0$.

Typical frequencies and the spectrum

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- Therefore, the spatial Fourier transform, $\mathcal{M}_{jilm}(\eta_1, \eta_2, \mathbf{k})$ is analytic in \mathbf{k} .
- We decompose Π_{ij} into two helicity modes which we assume to be statistically equal and uncorrelated (parity),

$$\Pi_{ij}(\eta, \mathbf{k}) = \mathbf{e}_{ij}^+ \Pi_+(\eta, \mathbf{k}) + \mathbf{e}_{ij}^- \Pi_-(\eta, \mathbf{k})$$

$$\begin{aligned}\langle \Pi_+(\eta, \mathbf{k}) \Pi_+^*(\eta', \mathbf{k}') \rangle &= \langle \Pi_-(\eta, \mathbf{k}) \Pi_-^*(\eta', \mathbf{k}') \rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') \rho_X^2 P(\eta, \eta', \mathbf{k}) \\ \langle \Pi_+(\eta, \mathbf{k}) \Pi_-^*(\eta', \mathbf{k}') \rangle &= 0.\end{aligned}$$

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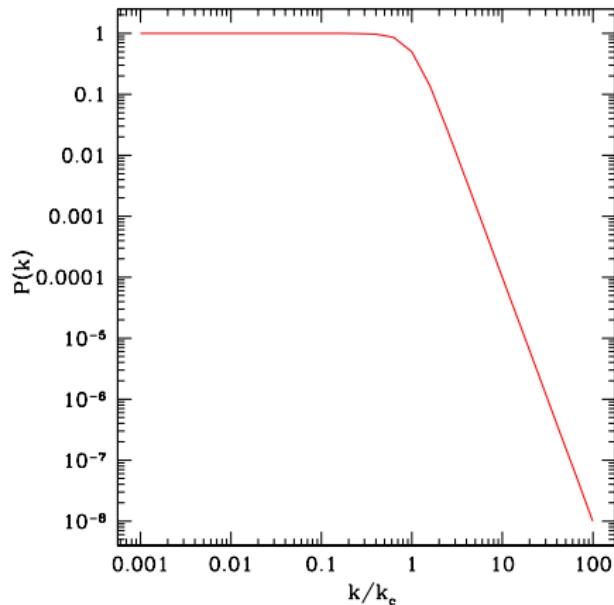
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- Causality implies that the function $P(\eta, \eta', \mathbf{k})$ is analytic in \mathbf{k} . We therefore expect it to start out as white noise and to decay beyond a certain correlation scale $k_*(\eta, \eta') > \min(1/\eta, 1/\eta')$.

The spectrum



The anisotropic stress power spectrum from Kolmogorov turbulence. On small scales $P \propto k^{-11/3}$.

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- The solution with vanishing initial conditions is then

$$\begin{aligned} h(\mathbf{k}, \eta) &= \frac{8i\pi Ga_*^3}{6ak} \left[e^{-ik\eta} \int_{\eta_*}^{\eta_* + \Delta\eta_*} d\eta' e^{ik\eta'} \Pi(\eta', \mathbf{k}) + \right. \\ &\quad \left. e^{ik\eta} \int_{\eta_*}^{\eta_* + \Delta\eta_*} d\eta' e^{-ik\eta'} \Pi(\eta', \mathbf{k}) \right] \\ &= \frac{8i\pi Ga_*^3}{6ak} \left[e^{-ik\eta} \Pi(k, \mathbf{k}) + e^{ik\eta} \Pi(-k, \mathbf{k}) \right] \end{aligned}$$

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- The gravitational wave energy density is given by

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- If the Universe is radiation dominated during the phase when the gravitational waves are generated, this gives on large scales, $k < k_*$

$$\frac{d\Omega_{gw}}{d \ln(k)}(\eta_0) = \frac{12\Omega_{\text{rad}}(\eta_0)}{\pi^2} \left(\frac{\Omega_X(\eta_*)}{\Omega_{\text{rad}}(\eta_*)} \right)^2 \mathcal{H}_*^2 k^3 \text{Re}[P(k, k, k)].$$



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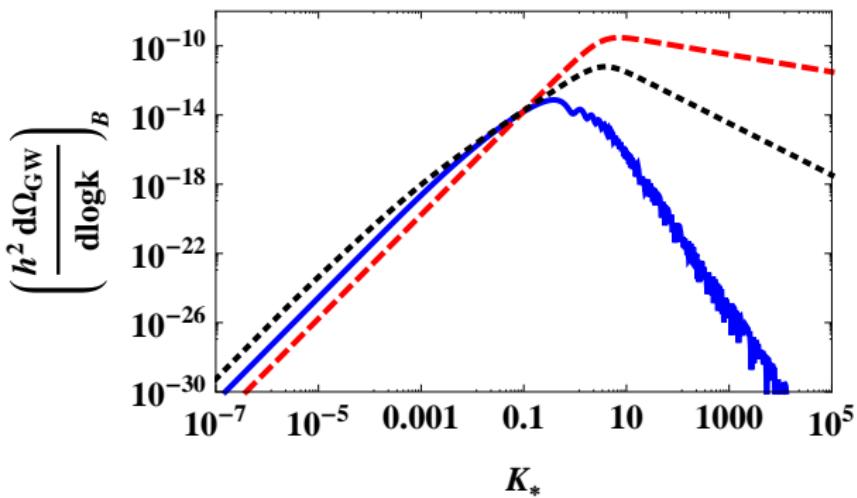
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- The behavior or the spectrum close to the peak and its decay rate on smaller scales depends on the source characteristics, on its temporal behavior (continuity, differentiability) and its power spectrum.

- For a totally incoherent source, $P(\eta, \eta', k) = \delta(\eta - \eta')\Delta\eta_* P(\eta, \eta, k)$ the peak position of the GW spectrum is determined by the peak of the **spatial** Fourier transform of the source.

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- For a coherent source, $P(\eta, \eta', k) = \sqrt{P(\eta', \eta', k)P(\eta, \eta, k)}$, when $P(\eta, \eta, k)$ is continuous in time but not differentiable (bubble collisions) the peak position of the GW spectrum $\propto k^3 P(k, k, k)$ is determined by the peak of the **temporal** Fourier transform of the source.

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- For a source with finite coherence time,
$$P(\eta, \eta', k) = \sqrt{P(\eta', \eta', k)P(\eta, \eta, k)}\Theta(x_c - |\eta - \eta'|k), \quad x_c \sim 1$$
 the GW spectrum is again determined by the peak of the **spatial** Fourier transform of the source.



The GW energy density spectrum in the incoherent (red), tophat (black) and coherent (blue). The parameters are: $T_* = 100$ GeV, $\Delta\eta_* \mathcal{H}_* = 0.01$, $\Omega_X/\Omega_{\text{rad}} = 2/9$ ($\langle v^2 \rangle = 1/3$)

- Caprini, RD and Servant, 2009, arXiv:0909.0622

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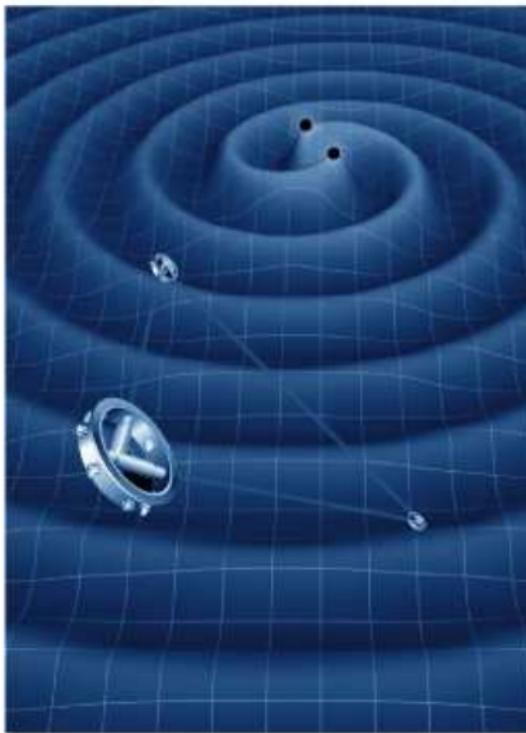
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$k_* = 1/\Delta\eta_* \simeq 100/\eta_* \sim 10^{-3}\text{Hz}$, which is close to the frequency of the peak sensitivity for the space born gravitational wave antenna **LISA**, proposed for launch in 2018, a ESA cosmic vision project.

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The spectrum is such that $k_* = 1/\Delta\eta_* \simeq 100$ Mpc. The sensitivity for the space-based detector LISA, currently planned for launch in 2018, is expected to be



not even second order. This does not lead to the formation of gravitational waves in the Higgs sector or in the electroweak phase transition. However, it can generate gravitational waves in the standard model.

The frequency of the peak is expected to be around 100 Hz, as proposed for the LISA mission.

The electroweak phase transition: GW's from bubble collisions

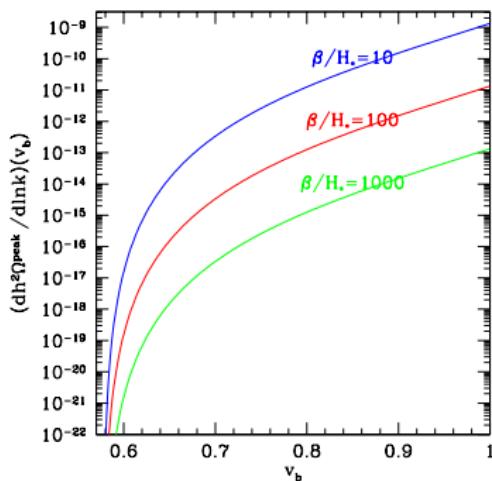
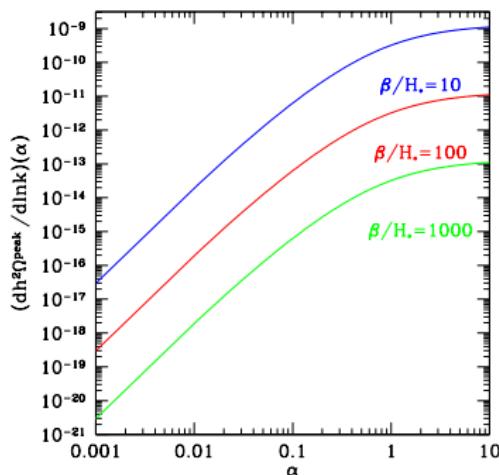
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The peak sensitivity of LISA is supposed to be about $h^2 \left. \frac{d\Omega_{\text{GW}}}{d \ln(k)} \right|_{k=k_p} \simeq 10^{-12}$,
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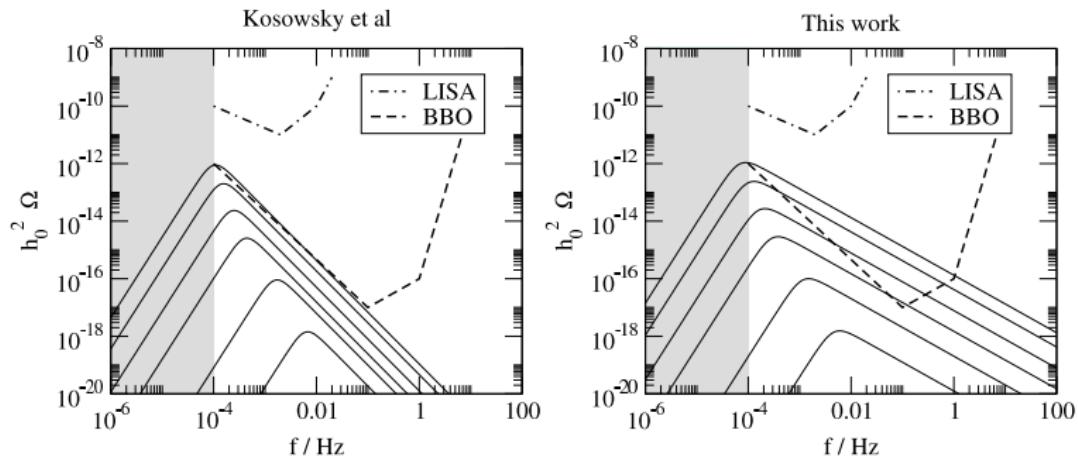
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Caprini, RD, Konstandin, Servant, 2009 arXiv:0901.1661 ($\beta = (\Delta\eta_*)^{-1}$)

The electroweak phase transition: GW's from bubble collisions



Huber & Konstandin 2008

Ω_{GW} from colliding bubbles, numerical results, $\Omega_X/\Omega_{\text{rad}} = 0.03$.

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- For $k > k_c$ we expect a **Kolmogorov spectrum** for the vorticity field, $P_v \propto k^{-11/3}$ and an **Iroshnikov–Kraichnan spectrum** for the magnetic field, $P_B \propto k^{-7/2}$.

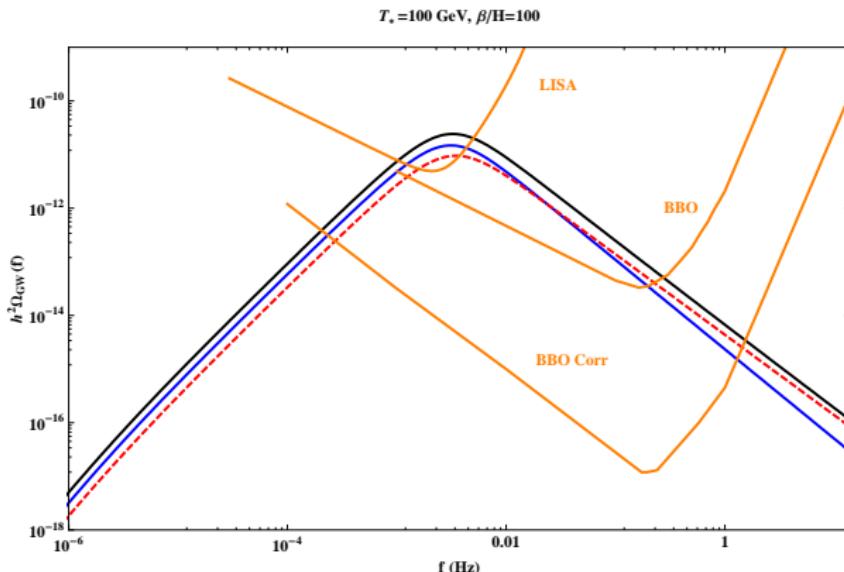
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- For the induced GW spectrum this yields

$$\frac{d\Omega_{GW\bullet}(k, \eta_0)}{d \ln(k)} \simeq \epsilon \Omega_{\text{rad}}(\eta_0) \left(\frac{\Omega_{\bullet}(\eta_*)}{\Omega_{\text{rad}}(\eta_*)} \right)^2 \times \begin{cases} (k/k_c)^3 & \text{for } k < k_c \\ (k/k_c)^{-\alpha} & \text{for } k > k_c \end{cases}$$

For $\bullet = v$ we have $\alpha = 11/3 - 1 = 8/3$ and for $\bullet = B$ we have $\alpha = 7/2 - 1 = 5/2$.
(See [Caprini & RD, 2006, astro-ph/0603476](#))

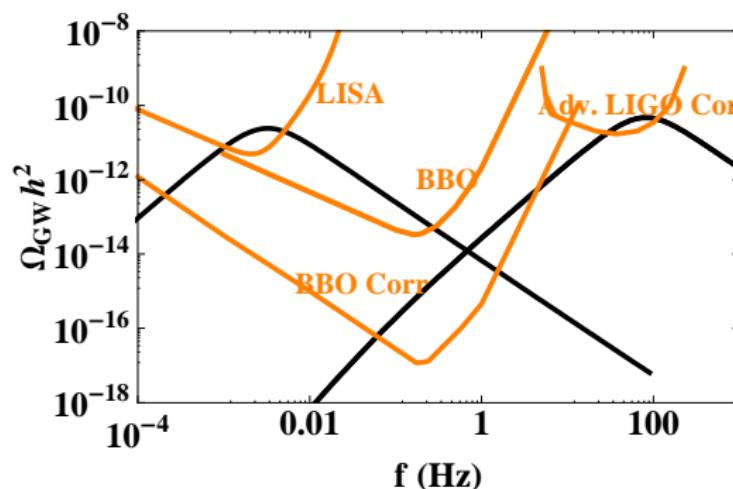
The electroweak phase transition: GW's from turbulence and magnetic fields



Caprini, RD, Servant, [arXiv:0909.0622](https://arxiv.org/abs/0909.0622)

Ω_{GW} from magnetic fields (red) and turbulence (blue), total (black). Modelling the time-decorrelation of the source (Kraichnan decorrelation) by a 'top-hat' in Fourier space. Sensitivity curves from A. Buonanno 2003.

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We also consider a phase transition at $T = 5 \times 10^6$ GeV with $\Delta\eta_* \mathcal{H}_* = 0.02$.

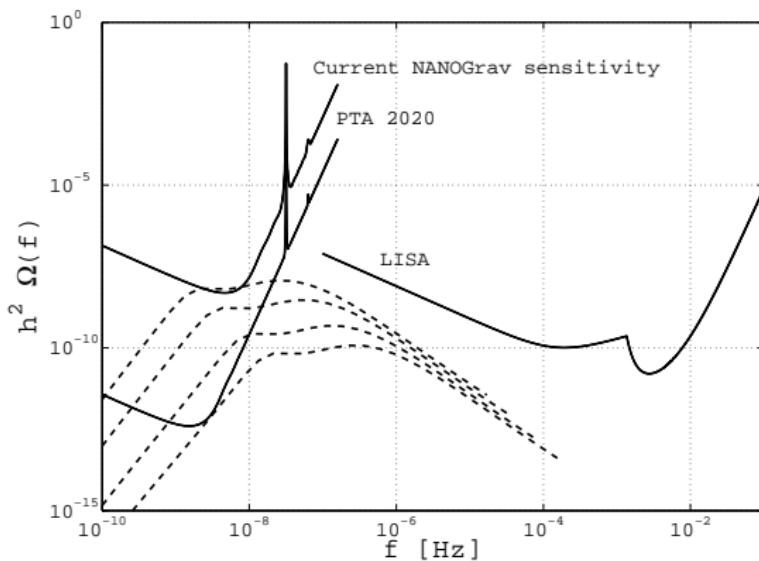
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- In this case the QCD phase transition proceeds via bubble nucleation and leads to turbulence and probably the generation of magnetic fields (very large kinetic and magnetic Reynolds numbers).

GW's from the QCD phase transition



Caprini, RD, Siemens, arXiv:1007.1218

GW's from the QCD phase transition at $T = 100$ MeV with $\Omega_X = 0.1$ and $\Delta\eta_*\mathcal{H}_* = 1, 0.5, 0.2, 0.1$ (from top to bottom).

It is difficult to estimate $\Omega_B(\eta_*)$ or $\Omega_V(\eta_*)$ accurately, but since causality requires the spectra to be so blue, $\frac{d\Omega_B(k, \eta_*)}{d \ln(k)} \propto k^5$, the limit on gravitational waves (which comes from small scales $k \simeq k_*$) yields very strong limits on primordial magnetic fields on large scales.

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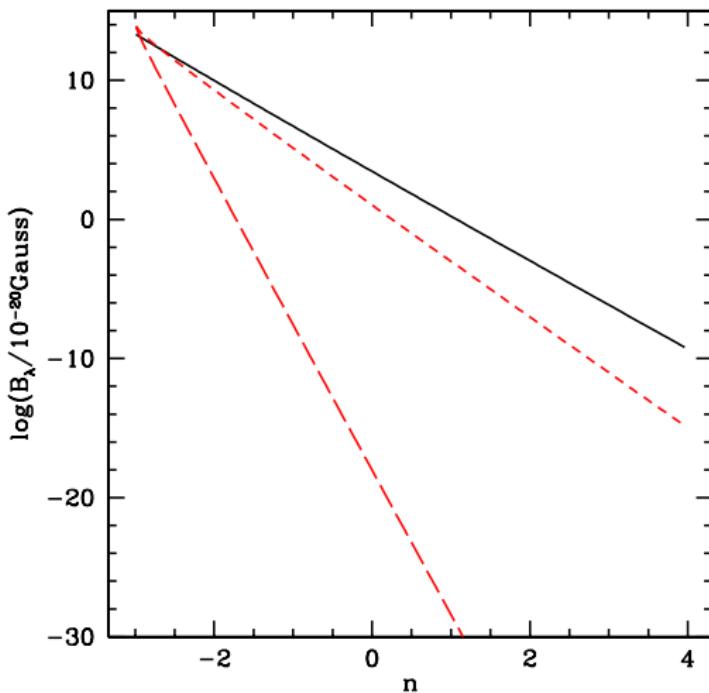
E.g. for $k_1 = (0.1\text{Mpc})^{-1}$ we obtain $k_1^{3/2}B(k_1) < 10^{-30}\text{Gauss}$.

This is purely a consequence of the redness of the magnetic field spectrum. The simple requirement that $\rho_B(t_*) < \rho_c(t_*)$ yields

$$k_1^{3/2}B(k_1) < 10^{-29}\text{Gauss} \left(\frac{k_1 \cdot 0.1\text{Mpc}}{k_* \cdot 10^3\text{sec}} \right)^{5/2}.$$

$$(10^3\text{sec} = 10^{-11}\text{Mpc})$$

Limits of primordial magnetic fields



$(\lambda = 0.1 \text{Mpc})$ Caprini & RD., 2001, astro-ph/0106244

The electroweak phase transition: helical magnetic fields and parity violation

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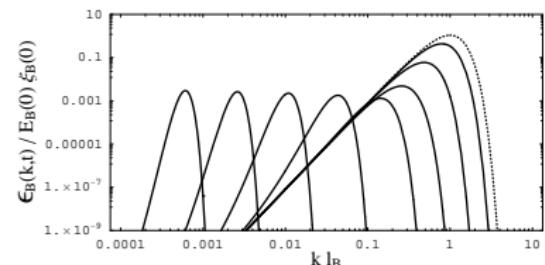
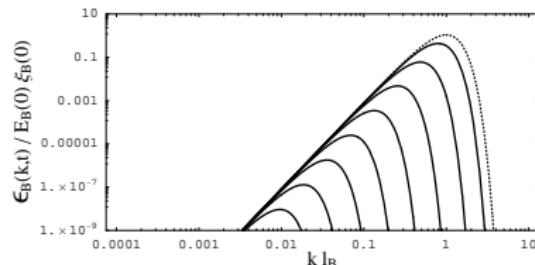
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- This relates the baryon number to the magnetic helicity.
- Such helical magnetic fields lead to T-B and E-B correlations in the CMB, and they also generate gravitational waves with non-vanishing helicity ([Caprini, Kahnashvili, RD. 2004, astro-ph/0304556](#)).

The electroweak phase transition: helical magnetic fields and parity violation

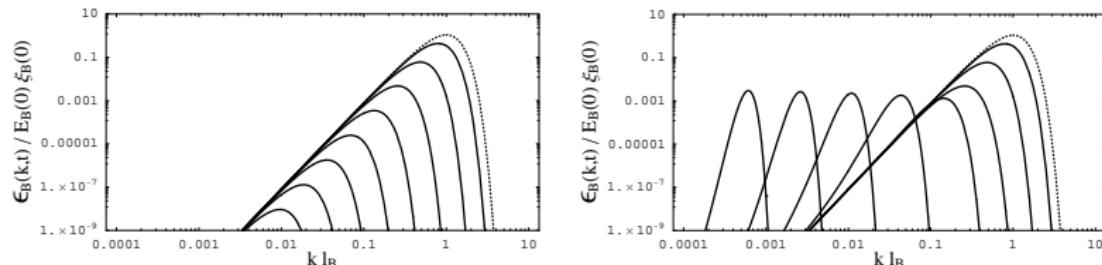
- Contrary to a non-helical magnetic field, helicity conservation for a helical field does lead to an inverse cascade in the evolution of the magnetic field:



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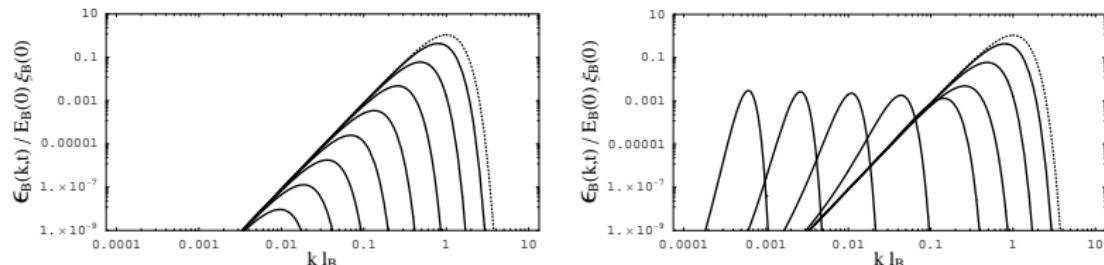


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- In this case, the GW background would not be parity symmetric. There would be more GW's of one helicity than of the other.

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- In this case we also expect a parity violating gravitational wave background, $|h_+(k)|^2 \neq |h_-(k)|^2$.