Halo biasing with time renormalization group

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$$\delta(\mathbf{x},t) = \frac{\rho(\mathbf{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)} \qquad \langle \delta(\mathbf{k})\,\delta(\mathbf{k}')\rangle \equiv \delta_D(\mathbf{k}+\mathbf{k}')P(k)$$

The power spectrum directly measures the fractional density contributions on different scales.

$$\begin{array}{ccc} \text{Observations} & \rightarrow & \text{galaxy distribution} \\ & & \neq \\ \text{Theory} & \rightarrow & \text{mass distribution} \end{array}$$

The clustering of galaxies is different from that of the underlying matter distribution: $P_g(k) \neq P_m(k) \rightsquigarrow$ bias.
Since the galaxies form inside dark matter haloes, as a starting point we construct a model for dark haloes power spectrum P_h .

The large-scale structure observed today in the universe is believed to be the result of gravitational amplification of primordial fluctuations, caused by the interaction among cold dark matter particles.

Continuity equation + Euler equation + Poisson equation

$$\frac{\partial\,\delta_m}{\partial\,\tau} + \nabla\cdot[(1+\delta_m)\mathbf{v}] = 0$$

$$\frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\,\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\,\mathcal{H}^2\,\Omega_m\,\delta_m$$

Non-linear equations!

The distribution of dark matter haloes.

$$\frac{\partial \delta_h}{\partial \tau} + \nabla \cdot [(1 + \delta_h)\mathbf{v}_h] = 0$$

How?

1)

A proto-halo is the Lagrangian region of space that will collapse to form a halo at a certain redshift.

Unlike real haloes that undergo merging, by construction proto-haloes always preserve their identity.

2)

On scales much larger that the characteristic size of (and separation between) the proto-haloes, the density fluctuations traced by the centre-of-mass trajectories can be described with a continuous overdensity field $\delta_{\rm h}$.

But proto-haloes are discrete objects! $\rightarrow$ shot noise ($\propto 1/N$)

$$\langle \delta_m(\mathbf{k})\, \delta_h(\mathbf{k}') \rangle \equiv \delta_D(\mathbf{k} + \mathbf{k}') P_{mh}(k)$$

Time renormalization group (TRG) (Pietroni 2008)

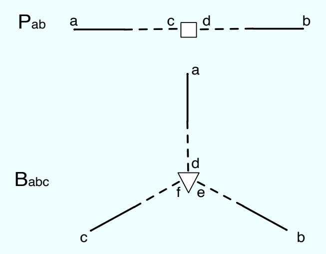
Applying the “equations of motion” straightforwardly to the computation of the power spectrum, a hierarchy of equations is generated (like BBGKY), including higher orders (bispectrum $\langle\delta\,\delta\,\delta\rangle$...).
We close this system with a truncation at a definite order (but corrections will always be subdominant).
We have a system of differential equations which allows us to obtain the power spectrum at any time.

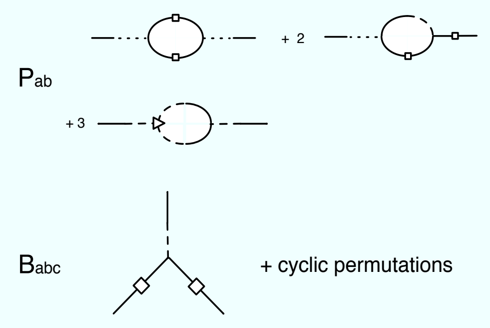
$$P(k)(\text{at early times} \Leftrightarrow \text{ linear}) \mapsto \text{TRG} \mapsto P(k)(\text{today } \Leftrightarrow \text{ non} - \text{linear})$$

In particle physics the RG equation describes the running of the coupling at different energy scales.

$\Updownarrow$

Time RG describes the running of the power spectrum at different time scales.





A non-local model for the halo bias at the initial conditions (adapted from Matsubara 1999)

The haloes are supposed to form from density peaks.

$$\delta_h(k) = (b_1 + b_2 k^2)\delta_m(k)\,, \qquad \mathbf{v}_h(k) = \mathbf{v}(k)$$

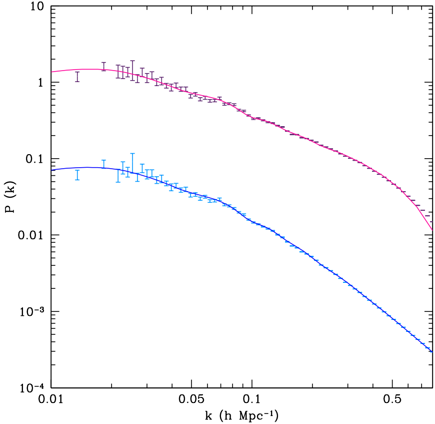
$$P_{mh}(k) = (b_1 + b_2 k^2) P_m(k) \mathrm{e}^{-k^2 R^2/2}$$

- the exponential function accounts for the exclusion effects of the haloes, due to their finite size;
- P_m is from linear theory;
- we can also compute the bispectrum for the haloes, which is important because the haloes density field is not gaussian!

$$P_{mh}(k) = (b_1 + b_2 k^2) P_m(k) \mathrm{e}^{-k^2 R^2/2}$$

We fit the parameters of the model from P_{mh} measured at a redshift z=50 from the N-body simulations by Pillepich et al. 2010, for haloes with a mass $M > 10^{13} M_\odot$:

- $b_1 = 19.2 \pm 0.3$
- $b_2 = 467 \pm 66\,\mathrm{Mpc}^2\, h^{-2}$
- $R = 2.6 \pm 0.4\,\mathrm{Mpc}/h$



AIM: a power spectrum which accounts
for non-linear evolution and halo biasing

DM particles equations

$+$

DM haloes equation

Bias model (IC)

$\searrow \quad \swarrow$

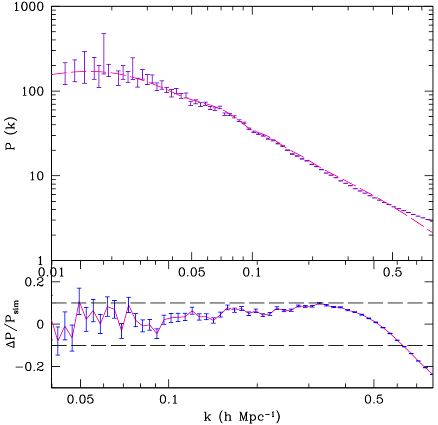
TRG

$\Downarrow$

Power spectrum P_{mh}

Power spectrum today ($z = 0$)

TRG result versus N-body simulations (box size: 1200 h^{-1} Mpc)



Difference within 10% : $k \sim 0.6\, h/\mathrm{Mpc}$

Conclusions

- The TRG method allows us to obtain a cross PS with good precision up to non-linear scales.
- The adopted model for the initial bias has proven successful.
- An analytic formalism is also available.
- Future work: redshift space distortion.

TRG equations

$$\partial_\eta P_{ab}(k) = -\Omega_{ac}P_{cb}(k) - \Omega_{bc}P_{ac}(k) + e^\eta \frac{4\pi}{k}\left[I_{acd,bcd}(k) + I_{bcd,acd}(k)\right] \,,$$

$$\partial_\eta I_{acd,bef}(k) = -\Omega_{bg}I_{acd,gef}(k) - \Omega_{eg}I_{acd,bgf}(k) - \Omega_{fg}I_{acd,beg}(k) + 2e^\eta A_{acd,bef}(k)\,.$$

$$I_{acd,bef}(k) \equiv \int_{k/2}^{\infty} dq\, q \int_{|q-k|}^{q} dp\, p\, \frac{1}{2}\left[\tilde{\gamma}_{acd}(k,q,p)\,\tilde{B}_{bef}(k,q,p) + (q \leftrightarrow p)\right].$$

$$A_{acd,bef}(k) \equiv \int_{k/2}^{\infty} dq\, q \int_{|q-k|}^{q} dp\, p\, \frac{1}{2}\{\tilde{\gamma}_{acd}(k,q,p)\,[\tilde{\gamma}_{bgh}(k,q,p)P_{ge}(q)P_{hf}(p) + \tilde{\gamma}_{egh}(q,p,k)P_{gf}(p)P_{hb}(k) + \tilde{\gamma}_{fgh}(p,k,q)P_{gb}(k)P_{he}(q)] + (q \leftrightarrow p)\}\,.$$

Bispectrum expressions

$$B_{hhh}(k_1, k_2, k_3) = (b_1 + b_2 k_1^2)(b_1 + b_2 k_2^2)(b_1 + b_2 k_3^2) B_m(k_1, k_2, k_3)\,,$$

$$B_{mhh}(k_1, k_2, k_3) = (b_1 + b_2 k_2^2)(b_1 + b_2 k_3^2) B_m(k_1, k_2, k_3)\,,$$

$$B_{mmh}(k_1, k_2, k_3) = (b_1 + b_2 k_3^2) B_m(k_1, k_2, k_3)\,,$$

The matter bispectrum at tree level is:

$$B_m(k_1, k_2, k_3) = 2\left[\frac{1}{2} + \frac{1}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right)\frac{\vec{k}_1 \cdot \vec{k}_2}{k_1 k_2} + \frac{1}{2}\left(\frac{\vec{k}_1 \cdot \vec{k}_2}{k_1 k_2}\right)^2\right] P(k_1, z_{in}) P(k_2, z_{in}) + cycl.$$

Shot-noise problem

Selected mass range: $M > 1.24 \cdot 10^{13} M_\odot$
(haloes with more than 100 particles)

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Number of haloes: 613343

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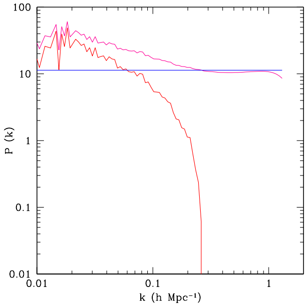
Shot noise: 2817.35 $(\mathrm{Mpc}/h)^3$



N-body simulation (Pillepich, Porciani, Hahn 2009)

Specifics:

- ♯ number of particles: 1024^3;
- ♯ box size: $1200\,h^{-1}$ Mpc;
- ♯ resolution: 512;
- ♯ initial redshift: $z = 50$;
- ♯ cosmology: $h = 0.701\,, \sigma_8 = 0.817\,, n_s = 0.96\,,$ $\Omega_m = 0.279\,, \Omega_b = 0.0462\,, \Omega_\Lambda = 0.721$.