Running-mass Inflation Model and Primordial Black Holes

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Based on work with Manuel Drees

Outline

1. Primordial Black Holes (PBHs)
   - PBHs properties
   - PBHs formation
2. Running-mass inflation model
3. Conclusion

PBHs

Definition

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Lifetime: $\tau_{BH} \approx 10^{64} \left(\frac{M}{M_\odot}\right)^{3}$ yr

Why PBHs are useful?

- PBHs as a probe of the early Universe $\left(M < 10^{15}\ \text{g}\right)$
- PBHs as a probe of gravitational collapse $\left(M > 10^{15}\ \text{g}\right)$ ✓
  DM candidates $\Omega^0_{\text{PBH}} \lesssim \Omega^0_{\text{CDM}} (= 0.25)$
- PBHs as a probe of High Energy Physics $\left(M \sim 10^{15}\ \text{g}\right)$
- PBHs as a probe of quantum gravity $\left(M \sim 10^{-5}\ \text{g}\right)$
  (DM candidates)

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$$f(>M) = 2\int_{\delta_{th}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_\delta(R)} \exp\left(-\frac{\delta^2(R)}{2\sigma_\delta^2(R)}\right) \mathrm{d}\delta(M)$$

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$$M_{\mathrm{PBH}} = \gamma M_{\mathrm{PH}} \xrightarrow{\gamma = w^{3/2}} \frac{R}{1\ \mathrm{Mpc}} = 5.54\times 10^{-24}\gamma^{-\frac{1}{2}}\left(\frac{M_{\mathrm{PBH}}}{1\ \mathrm{g}}\right)^{1/2}\left(\frac{g_*}{3.36}\right)^{1/6}$$

Power Spectrum

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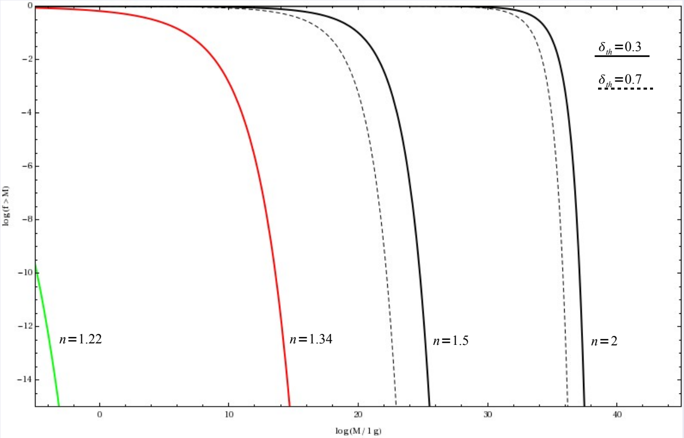
$f(>M)$ diagram for the mass range $10^{-5} - 10^{50}$ g



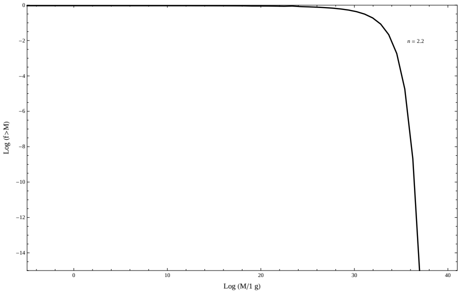
Scale dependent spectral index

$$\mathcal{P}_{\mathcal{R}_c}(k) = \mathcal{P}_{\mathcal{R}_c}(k_0)\left(\frac{k}{k_0}\right)^{n(k)-1}$$

$$n(k) = n_s + \frac{1}{2!}\,\alpha_s \ln\left(\frac{k}{k_0}\right) + \ldots$$

$$n_s \equiv \frac{d\ln\mathcal{P}_{\mathcal{R}_c}}{d\ln k}\,\Big|_{k=k_0}$$

$$\alpha_s \equiv \frac{dn}{d\ln k}\,\Big|_{k=k_0}$$



WMAP 7+BAO+H_0 [arXiv:1001.4538]

$n_s = 1.008 \pm 0.042$ $k_0 = 0.002$ Mpc^{-1}

$\alpha_s = -0.022 \pm 0.020$

Running-mass Inflation Model

The inflation potential is dominated by the soft SUSY breaking mass term generated by V_0 and its radiative corrections

$$V = V_0 + \frac{1}{2} m_\phi^2(\phi)\phi^2 + \ldots$$

RGE $\quad \dfrac{dm^2}{d\ln\phi} \equiv \beta_m \qquad$ with $\quad \beta_m = -\dfrac{2C}{\pi}\alpha\tilde{m}^2 + \dfrac{D}{16\pi^2}|\lambda_Y|^2 m_{\rm loop}^2$

Over a sufficiently small range of ϕ, or small inflaton coupling, we can do the Taylor expansion:

$$V = V_0 + \frac{1}{2} m^2(\phi_0)\phi^2 + \frac{1}{2}\frac{dm^2}{d\ln\phi}|_{\phi=\phi_0} \ln\left(\frac{\phi}{\phi_0}\right) + \frac{1}{4}\,\frac{d^2 m^2}{d(\ln\phi)^2}|_{\phi=\phi_0} \ln^2\left(\frac{\phi}{\phi_0}\right)$$

where ϕ_0 is the inflaton value at the epoch of the horizon exit for the pivot scale k_0.

Slow-roll parameters

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta \equiv M_P^2 \frac{V''}{V}$$

$$\xi^2 \equiv M_P^4 \frac{V' V'''}{V^2}$$

$$\sigma^3 \equiv M_P^6 \frac{V'^2 V''''}{V^3}$$

Spectral index and its running

$$n_s = 1 - 6\epsilon + 2\eta$$

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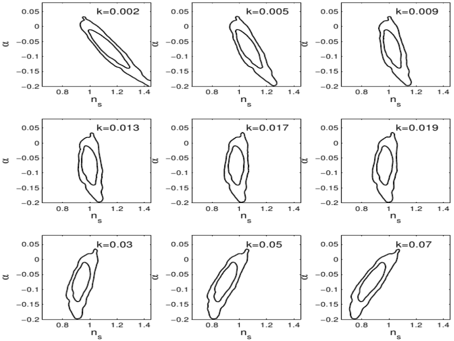
Value of running index

$$\alpha_s \sim \mathcal{O}\left(10^{-2}\right)$$

Correlation of spectral index and its running [astro-ph/0702170]



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Results

$n_s = 1.008,$ $\qquad \alpha_s = 6 \times 10^{-3},$ $\qquad \beta_s = 4 \times 10^{-4}$

$n(k) = 1.24$ $\quad$ for $M_{\mathrm{PBH}} > 10^{15}$ g

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Conclusions

- The fluctuation which arise at inflation are the most likely source of PBHs.
- One needs the fluctuation amplitude to increase with decreasing scale in order to produce PBHs.
- The value of spectral index should be larger than 1.34 in the scale corresponding to PBHs with mass larger than 10^{15} g.
- The COBE normalization on Lyman-α range puts an upper bound on the running of running spectral index, $\beta_s < 0.042$.
- With WMAP 7-year data, the running-mass inflation model is **not** a good candidate for DM PBHs formation.


PBHs formed at the beginning of time may have had masses similar to that of Mount Everest.
Thanks for your attention