The flavor structure of split-UED and implications for the Kaluza-Klein spectrum

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Outline

- UED review / Motivation
- Split UED
  - Flavor violation in sUED
  - Implications for the sUED mass spectrum
- Conclusions and Outlook

UED: The basic setup

- UED models are models with flat, compact extra dimensions in which *all* fields propagate. [Appelquist, Cheng, Dobrescu,(2001)]
- The Standard Model (SM) particles are identified with the lowest-lying modes of the respective Kaluza-Klein (KK) towers.
- Compactification on S^1/Z_2 allows for boundary conditions on the fermion and gauge fields such that
  - half of the fermion zero mode is projected out $\Rightarrow$ chiral fermions
  - $A_5^{(0)}$ is projected out $\Rightarrow$ no additional massless scalar
- The presence of orbifold fixed points breaks 5D translational invariance.

  $\Rightarrow$ KK-number conservation is violated, *but*
  a discrete Z_2 parity (KK-parity) remains.
  $\Rightarrow$ The lightest KK mode (LKP) is stable.

UED basics: The action

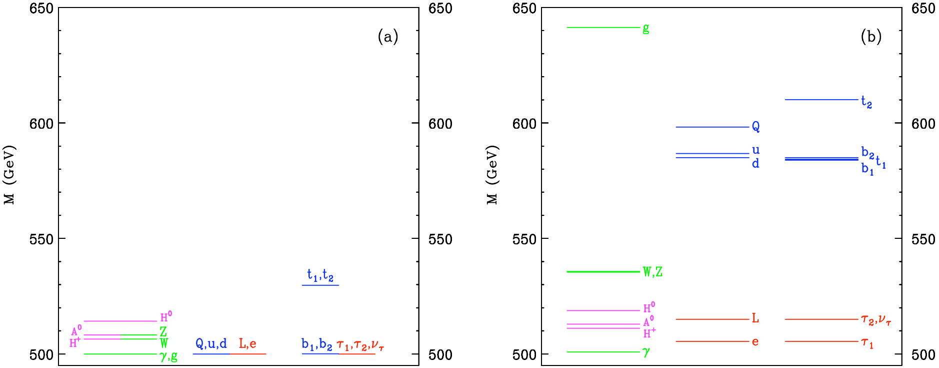
- UED action

$$S_{UED,bulk} = S_g + S_H + S_f,$$

with

$$\begin{aligned}
S_g &= \int d^5x \left\{ -\frac{1}{4\hat{g}_3^2} G^A_{MN} G^{AMN} - \frac{1}{4\hat{g}_2^2} W^I_{MN} W^{IMN} - \frac{1}{4\hat{g}_Y^2} B_{MN} B^{MN} \right\}, \\
S_H &= \int d^5x \left\{ (D_M H)^\dagger (D^M H) + \hat{\mu}^2 H^\dagger H - \hat{\lambda} (H^\dagger H)^2 \right\}, \\
S_f &= \int d^5x \left\{ i\overline{\psi}\gamma^M D_M \psi + \left(\hat{\lambda}_E \overline{L} E H + \hat{\lambda}_U \overline{Q} U \tilde{H} + \hat{\lambda}_D \overline{Q} D H + \text{h.c.} \right) \right\}.
\end{aligned}$$

UED - the spectrum



[Cheng, Matchev, Schmaltz, PRD **66** (2002) 036005, hep-ph/0204342]

UED - Constraints

- Phenomenological constraints on the compactification scale R^{-1}
  - lower bound $\Rightarrow$ $R^{-1} \gtrsim 650\,\mathrm{GeV}$
    - no detection of KK-modes

      [Appelquist *et al.* (2001); Rizzo (2001); Macesanu *et al.* (2002); Lin (2005)]

      $R^{-1} > 280\,\mathrm{GeV}$ at 95% cl.
    - FCNCs [Buras, Weiler *et al.* (2003); Weiler, Haisch (2007)]

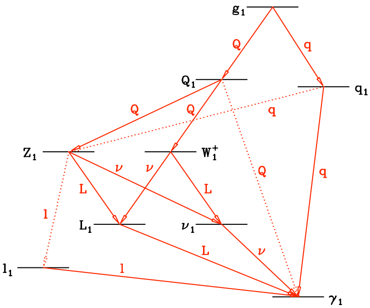
      $R^{-1} > 600(330)\,\mathrm{GeV}$ at 95% (99%) cl.
    - Electroweak Precision Constraints [Appelquist, Yee (2002);Gogoladze, Macesanu (2006)]

      $R^{-1} > 600(300)\,\mathrm{GeV}$ for $m_H = 115(800)\,\mathrm{GeV}$ at 95% cl.
  - upper bound: preventing over closure of the Universe by $B^{(1)}$ dark matter

    $\Rightarrow$ $R^{-1} \lesssim 1.5\mathrm{TeV}$ [Servant, Tait (2002); Matchev, Kong (2005); Burnell, Kribs (2005)]

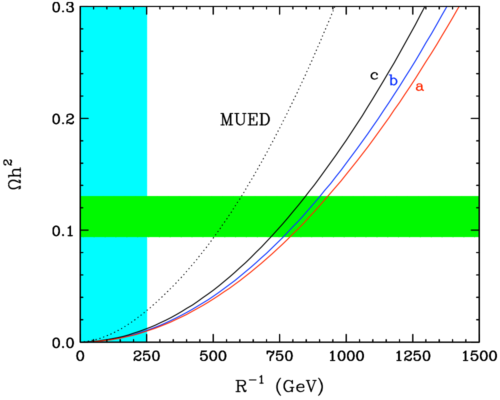
Relevance of the detailed mass spectrum I: LHC phenomenology

The KK mass spectrum determines decay channels and decay rates of KK particles produced at LHC.



[Cheng, Matchev, Schmaltz, PRD**66** (2002) 056006, hep-ph/0205314]

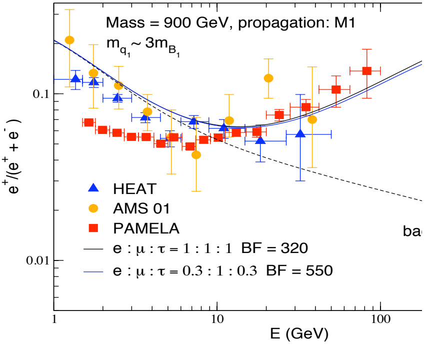
Relevance of the detailed mass spectrum II: Dark Matter relic density



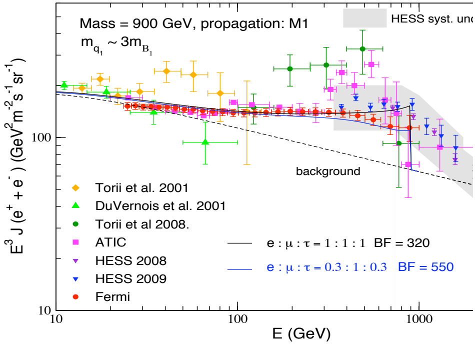
Relic density in the UED model for a) only $\gamma_1 \gamma_1$ annihilation, degenerate mass spectrum, b) including T dependent g^*, c) a) and b) with the MUED spectrum, and dotted for the MUED spectrum including coannihilation processes.

[Kong, Matchev, JHEP 0601:038, hep-ph/0509119]

Relevance of the detailed mass spectrum III: the PAMELA excess



sUED vs PAMELA data



sUED vs. Fermi-LAT and Hess data

[Chen,Nojiri,Park,Shu, arXiv:0908.4317]

The main topic of this talk:

- How can the UED mass spectrum be altered?
- How strongly are these alterations constrained by experiment?

Extensions of UED with minimal field content

Even without extending the field content, the spectrum and/or the interactions of the UED model can by modified by the inclusion of additional operators.
Three classes are

1. Bulk mass terms for fermions (dimension 4 operators),
2. kinetic and mass terms at the orbifold fixed points (dimension 5; radiatively induced in MUED),
3. bulk or boundary localized interactions (dimension 6 or higher)

The former two modify the free field equations and thereby the spectrum and the KK bases $\{f_n^{\psi}(y)\}$.
Today, we focus on fermion bulk mass terms: “split UED”

Bulk mass terms for fermions

[Park, Shu, *et al.* (2009); for earlier work, see Csaki (2003)]

A plain bulk mass term for fermions of the form

$$S \supset \int d^5x \; - M\overline{\Psi}\Psi$$

is forbidden by KK parity, but

it can be allowed if realized by a KK-parity odd background field

$$S \supset \int d^5x - \lambda\Phi\overline{\Psi}\Psi,$$

where $\Phi(-y) = -\Phi(y)$

(Orbifold fixed points are at $\pm L = \pm\pi R/2$)

In the simplest case $M = \mu_5\theta(y)$

(similar to the bulk fermion mass term in Randall-Sundrum models)

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Bulk mass terms for fermions

Structural consequences: [Park, Shu, *et al.* (2009); Kong, Park, Rizzo (2010)]

- The fermion zero modes remain massless, but obtain a non-flat profile

$$f_0^{\psi_{L,R}} = \sqrt{\frac{\pm\mu_5}{1 - e^{\mp 2\mu_5 L}}}\, e^{\mp 2\mu_5 |y|}$$

- The KK mode masses are $m^{(n)} = \sqrt{(\mu_5)^2 + k_n^2}$ with k_n^2 determined from

$$0 = \cot(k_n \pi R/2) \qquad \text{for even-numbered modes}$$

$$(\mu_5)^2 = k_n^2 \cot^2(k_n \pi R/2) \qquad \text{for odd-numbered modes.}$$

- Wave functions of the fermion and gauge KK modes are not orthogonal:

$$g_{002n} = g^{SM}\mathcal{F}^{\psi\psi}_{002m}(\mu_5 L) = g^{SM}\frac{(\mu_5 L)^2(-1 + (-1)^n e^{2\mu_5 L}(\coth(\mu_5 L) - 1)}{\sqrt{2(1+\delta_{0n}}((\mu_5 L)^2 + n^2\pi^2/4)}$$

The sUED fermion action

The most general action for fermions reads

$$S = \int d^5x \mathcal{L}_f + \mathcal{L}_{Yuk}$$

with

$$\begin{aligned}
\mathcal{L}_f &= \sum_{ij} \left\{ \frac{i}{2}\delta_{ij} \left(D_M \overline{\Psi}_i \Gamma^M \Psi_j - \overline{\Psi}_\Gamma^M D_M \Psi_j \right) - M_{ij}^{\Psi}(y) \overline{\Psi}_i \Psi_j \right\}, \\
\mathcal{L}_{Yuk} &= \sum_{ij} \left\{ \lambda_{ij}^U \overline{Q}_i \tilde{H} U_j + \lambda_{ij}^D \overline{Q}_i H D_j + \lambda_{ij}^E \overline{L}_i H E_j \right\} + \text{h.c.}.
\end{aligned}$$

$M^{Q,u,d,L,e}$ are 3×3 hermitian matrices in flavor space,
$\lambda^{U,D,E}$ are 3×3 matrices in flavor space.

Calculating the 4D effective action

Via field redefinitions, the mass matrices M^ψ can be diagonalized, and the fermion zero mode Lagrangian in the zero mode approximation reads

$$
\begin{aligned}
\mathcal{L}_{kin} &= \overline{\psi}^{(0)} i\gamma^\mu \partial_\mu \psi^{(0)} \\
\mathcal{L}_{f,g} &= \sum_{n=0} \left[\overline{\psi}^{(0)} i\gamma^\mu (D_\mu - \partial_\mu)^{(2n)} \psi^{(0)} \mathcal{F}^{\psi,\psi}_{002n} \right] \\
\mathcal{L}_{Yuk} &= \overline{u^q}^{(0)}_{L,i} \frac{\lambda'^U_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{q^i_L, u^j_R}_{000} u^{(0)}_{R,j} + \overline{d^q}^{(0)}_{L,i} \frac{\lambda'^D_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{q^i_L, d^j_R}_{000} d^{(0)}_{R,j} + \overline{e^l}^{(0)}_{L,i} \frac{\lambda'^E_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{l^i_L, e^j_R}_{000} e^{(0)}_{R,j} \\
&\quad + \text{h.c.}
\end{aligned}
$$

The basis in which the M^ψ are diagonal thus signifies the gauge eigenbasis.

Transformation to the quark mass eigenbasis by bi-unitary transformations:

$$u_L = S_u^\dagger (u^q)_L^{(0)}, \quad d_L = S_d^\dagger (d^q)_L^{(0)}, \quad e_L = S_e^\dagger (e^l)_L^{(0)}$$

$$u_R = T_u^\dagger u_R^{(0)}, \quad d_R = T_d^\dagger d_R^{(0)}, \quad e_R = T_e^\dagger e_R^{(0)}.$$

In the fermion mass eigenbasis, the couplings to the gauge bosons read

$$\mathcal{L}_{q,eff} \subset \sum_{n=0} \eta^{\mu\nu} \left[g_3 G_\mu^{A(2n)} J_{q\nu}^{A(2n)} + \left(\frac{g_2}{\sqrt{2}} W_\mu^{+(2n)} J_{q\nu}^{+(2n)} + \text{h.c.} \right) + e A_\mu^{(2n)} J_{q\nu}^{em,(2n)} + \frac{g_2}{\cos(\theta_W)} Z_\mu^{(2n)} J_{q\nu}^{0(2n)} \right],$$

with *e.g.*

$$J_{q\nu}^{A(2n)} = \left(V_{L,ij}^{u(2n)} \overline{u}_{L,i} T^A \gamma_\nu u_{L,j} + V_{L,ij}^{d(2n)} \overline{d}_{L,i} T^A \gamma_\nu d_{L,j} \right) + (L \leftrightarrow R)$$

$$J_{q\nu}^{em(2n)} = \left(\frac{2}{3} V_{L,ij}^{u(2n)} \overline{u}_{L,i} \gamma_\nu u_{L,j} - \frac{1}{3} V_{L,ij}^{d(2n)} \overline{d}_{L,i} \gamma_\nu d_{L,j} \right) + (L \leftrightarrow R)$$

with

$$V_{L,ij}^{u(2n)} = (S_u^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_u)_{ij} \quad , \quad V_{R,ij}^{u(2n)} = (T_u^\dagger \mathcal{F}_{002n}^{u_R,u_R} T_u)_{ij}$$

$$V_{L,ij}^{d(2n)} = (S_d^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_d)_{ij} \quad , \quad V_{R,ij}^{d(2n)} = (T_d^\dagger \mathcal{F}_{002n}^{d_R,d_R} T_d)_{ij}$$

Integrating out all but the zero modes leads the low-energy effective action

$$S_{eff,4q} = -\sum_{n=1} \eta^{\mu\nu} \left[\frac{g_3^2}{2m^2_{G^{(2n)}}} J^{A(2n)}_{q\mu} J^{A(2n)}_{q\nu} + \frac{g_2^2}{2m^2_{W^{(2n)}}} J^{+(2n)}_{q\nu} J^{-(2n)}_{q\nu} + \frac{e^2}{2m^2_{A^{(2n)}}} J^{em(2n)}_{q\mu} J^{em(2n)}_{q\nu} + \frac{g_2^2}{2\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} J^{0(2n)}_{q\mu} J^{0(2n)}_{q\nu} \right],$$

which in particular contain the $\Delta F = 2$ effective operators of sUED, which can be parameterized according to

$$\mathcal{H}^{\Delta F=2}_{int} = \sum_{i=1}^{5} C^i_{q_i q_j} Q_i^{q_i q_j} + \sum_{i=1}^{3} \tilde{C}^i_{q_i q_j} \tilde{Q}_i^{q_i q_j}$$

with

$$\begin{aligned}
Q_1^{q_i,q_j} &= \left(\overline{q}^a_{L,j}\gamma_\mu q^a_{L,i}\right)\left(\overline{q}^b_{L,j}\gamma^\mu q^b_{L,i}\right) \\
Q_2^{q_i,q_j} &= \left(\overline{q}^a_{R,j} q^a_{L,i}\right)\left(\overline{q}^b_{R,j} q^b_{L,i}\right), \quad Q_3^{q_i,q_j} = \left(\overline{q}^a_{R,j} q^b_{L,i}\right)\left(\overline{q}^b_{R,j} q^a_{L,i}\right) \\
Q_4^{q_i,q_j} &= \left(\overline{q}^a_{R,j} q^a_{L,i}\right)\left(\overline{q}^b_{L,j} q^b_{R,i}\right), \quad Q_5^{q_i,q_j} = \left(\overline{q}^a_{R,j} q^b_{L,i}\right)\left(\overline{q}^b_{L,j} q^a_{R,i}\right),
\end{aligned}$$

and $\tilde{Q}_{1,2,3} = Q_{1,2,3}(L \leftrightarrow R)$.

For the Wilson coefficients in sUED, we find

$$C_K^1 = \sum_n \left(\frac{g_3^2}{3m^2_{G^{(2n)}}} + \frac{g_2^2}{2\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} + \frac{e^2}{2m^2_{A^{(2n)}}} \right) V^{d(2n)}_{L,ds} V^{d(2n)}_{L,ds}$$

$$\tilde{C}_K^1 = \sum_n \left(\frac{g_3^2}{3m^2_{G^{(2n)}}} + \frac{g_2^2}{2\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} + \frac{e^2}{2m^2_{A^{(2n)}}} \right) V^{d(2n)}_{R,ds} V^{d(2n)}_{R,ds}$$

$$C_K^4 = \sum_n - \left(\frac{g_3^2}{m^2_{G^{(2n)}}} + \frac{g_2^2}{\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} + \frac{e^2}{m^2_{A^{(2n)}}} \right) V^{d(2n)}_{R,ds} V^{d(2n)}_{L,ds}$$

$$C_K^5 = \sum_n \frac{g_3^2}{3m^2_{G^{(2n)}}} V^{d(2n)}_{R,ds} V^{d(2n)}_{L,ds},$$

where

$$V^{u(2n)}_{L,ij} = (S_u^\dagger \mathcal{F}^{q_L,q_L}_{002n} S_u)_{ij} \quad , \quad V^{u(2n)}_{R,ij} = (T_u^\dagger \mathcal{F}^{u_R,u_R}_{002n} T_u)_{ij}$$

$$V^{d(2n)}_{L,ij} = (S_d^\dagger \mathcal{F}^{q_L,q_L}_{002n} S_d)_{ij} \quad , \quad V^{d(2n)}_{R,ij} = (T_d^\dagger \mathcal{F}^{d_R,d_R}_{002n} T_d)_{ij}$$

The analogous expressions for C^i_{D,B_d,B_s} follow from these by the replacements $(ds) \to (uc), (db), (sb)$.

Experimental constraints: [Bona *et al.* (UTfit Collaboration, 2007)]

Parameter	95% allowed $[TeV^{-2}]$	Parameter	95% allowed $[TeV^{-2}]$
$\mathrm{Re}C_K^1$	$[-9.6, 9.6]\cdot 10^{-7}$	$\mathrm{Im}C_K^1$	$[-4.4, 2.8]\cdot 10^{-9}$
$\mathrm{Re}C_K^2$	$[-1.8, 1.9]\cdot 10^{-8}$	$\mathrm{Im}C_K^2$	$[-5.1, 9.3]\cdot 10^{-11}$
$\mathrm{Re}C_K^3$	$[-6.0, 5.6]\cdot 10^{-8}$	$\mathrm{Im}C_K^3$	$[-3.1, 1.7]\cdot 10^{-10}$
$\mathrm{Re}C_K^4$	$[-3.6, 3.6]\cdot 10^{-9}$	$\mathrm{Im}C_K^4$	$[-1.8, 0.9]\cdot 10^{-11}$
$\mathrm{Re}C_K^5$	$[-1.0, 1.0]\cdot 10^{-8}$	$\mathrm{Im}C_K^5$	$[-5.2, 2.8]\cdot 10^{-11}$
$\lvert C_{B_d}^1\rvert$	$< 2.3\cdot 10^{-5}$	$\lvert C_{B_s}^1\rvert$	$< 1.1\cdot 10^{-3}$
$\lvert C_{B_d}^2\rvert$	$< 7.2\cdot 10^{-7}$	$\lvert C_{B_s}^2\rvert$	$< 5.6\cdot 10^{-5}$
$\lvert C_{B_d}^3\rvert$	$< 2.8\cdot 10^{-6}$	$\lvert C_{B_s}^3\rvert$	$< 2.1\cdot 10^{-4}$
$\lvert C_{B_d}^4\rvert$	$< 2.1\cdot 10^{-7}$	$\lvert C_{B_s}^4\rvert$	$< 1.6\cdot 10^{-5}$
$\lvert C_{B_d}^5\rvert$	$< 6.0\cdot 10^{-7}$	$\lvert C_{B_s}^5\rvert$	$< 4.5\cdot 10^{-5}$
$\lvert C_D^1\rvert$	$< 7.2\cdot 10^{-7}$		
$\lvert C_D^2\rvert$	$< 1.6\cdot 10^{-7}$		
$\lvert C_D^3\rvert$	$< 3.9\cdot 10^{-6}$		
$\lvert C_D^4\rvert$	$< 4.8\cdot 10^{-8}$		
$\lvert C_D^5\rvert$	$< 4.8\cdot 10^{-7}$		

How can FCNCs be avoided in the quark sector?

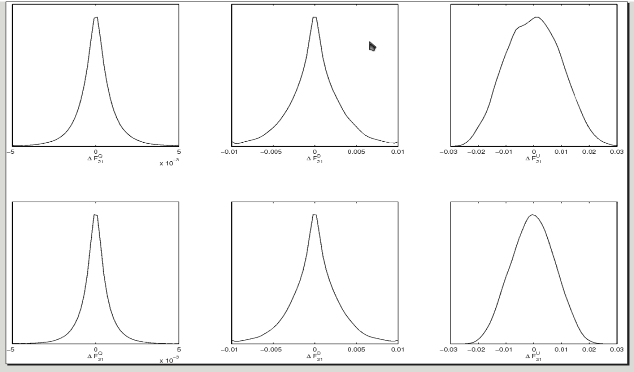
The constraints on the C^i's imply, that the products $\frac{1}{m^2_{G^{(2n)}}} V^{u/d(2n)}_{L/R,ij} V^{u/d(2n)}_{L/R,ij}$ have to be very small. This can be achieved in three ways

1. High compactification scale:
   $R^{-1} \gtrsim 10^5\,\mathrm{TeV}$ satisfies all constraints for $S_{ij}, T_{ij}, \mathcal{F}_{ij}$ of $\mathcal{O}(1)$.
2. Degenerate mass matrices
3. Alignment of the 5D mass matrices with the Yukawa couplings.

Mass degeneracy

$M^{Q,U,D} = \mu_5^{q,u,d}\theta(y)\mathbb{1}$ with $\mu_5^{q,u,d} \in \mathbb{R}$ implies that $\mathcal{F}_{002n} \propto \mathbb{1}$.

Degree of degeneracy of $\mathcal{F}_{002}^{Q,U,D}$ eigenvalues:



PRELIMINARY: Frequencies of ΔF_{ij}^{QUD} obtained by a Markov Chain Monte Carlo.

This setup is not flavor blind. It implies chiral couplings of quarks to KK gluons.

Alignment

- All Wilson coefficients would vanish if we could choose $S_d = T_d = T_u = S_u = \mathbb{1}$, *but* to obtain the SM at the fermion zero mode level, $S_u^\dagger S_d = U_{CKM}$ must be imposed.
- Choosing at least $S_d = T_d = \mathbb{1}$ avoids all bounds from the down-type sector.
- Furthermore choosing $T_u = \mathbb{1}$ avoids all constraints from the up-type sector apart from the C_D^1 constraint.
- The C_D^1 constraint with $T_u = \mathbb{1}$, $S_u = U_{CKM}^\dagger$ implies

$$\left| \left(\mathcal{F}^Q_{002}\right)_{22} - \left(\mathcal{F}^Q_{002}\right)_{11} \right| \times R \lesssim 3 \times 10^{-3}\,\mathrm{TeV}^{-1}$$

with no other constraints on $M^{Q,U,D}$.

Implications for the sUED mass spectrum

The masses of the first KK mode fermions are $m^{(1)} = \sqrt{(\mu_5)^2 + k_1^2} + \delta_{HM}$

where k_1^2 is determined from $(\mu_5)^2 = k_1^2 \cot^2(k_1 \pi R/2)$.

δ_{HM} is a small correction $(m_\psi / m_\psi^{(1)})^2 m^{(1)}$ from mixing via the Yukawas.

The implications of the three solutions to the FCNC problem are therefore:

1. High complactification scale $\Rightarrow$ No new physics at the TeV scale.
2. Degenerate mass matrices $\Rightarrow$ First KK mode quarks come in three mass degenerate sets $\left(u_1^{(1)}, c_1^{(1)}, t_1^{(1)}\right)$, $\left(d_1^{(1)}, s_1^{(1)}, b_1^{(1)}\right)$, $\left(u_2^{(1)}, d_2^{(1)} c_2^{(1)}, s_2^{(1)}, b_2^{(1)}, t_2^{(1)}\right)$.
3. Alignment $\Rightarrow$ The mass degenerate first KK mode sets are $\left(u_2^{(1)}, d_2^{(1)} c_2^{(1)}, s_2^{(1)}\right)$ and $\left(b_2^{(1)}, t_2^{(1)}\right)$, but the remaining first KK quark masses are not constrained by flavor physics at tree level.

Conclusions

- Fermionic bulk mass terms are thought to arise from couplings of the fermions to a KK-odd background field.
  In the absence of a flavor symmetry, there is no reason to assume the 5D mass matrices to be uniform.
- We showed that the absence of FCNCs strongly constrains the allowed Yukawa couplings.
  Solutions in which TeV scale sUED is not ruled out by FCNCs have
  - Mass degenerate sets of KK modes *or*
  - Mass degeneracy between KK partners of the left-handed first and second family quarks *and* a delicate alignment between the Yukawa couplings and the 5D fermion mass terms.

Outlook

- The presented results are about to be published.
- We performed an analogous study for lepton flavor violation which shows that in the lepton sector, FCNCs are generically present, but an aligned solution exists independent of the KK lepton mass spectrum.
- So far, all results are calculated at tree-level, only. A one-loop analysis of flavor and electroweak constraints is needed. (work in progress)
- The analysis presented here can be extended to nUED models with boundary localized kinetic and mass terms (work in progress).
- The requirements of mass degeneracy and/or alignment with the sUED setup calls for a flavor symmetry embedding (work in progress).