Non-trivial quantum corrections to ΛCDM

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C. Dappiaggi, TH, J. Möller, N. Pinamonti, arXiv:1007.5009
C. Dappiaggi, TH, N. Pinamonti, arXiv:1009.5179
TH, arXiv:1008.1776

Quantum Field Theory in Curved Spacetimes

- Quantum field theory on curved spacetimes (QFTCST): matter is a quantum field, the background spacetime is classical
- more *fundamental* than Minkowskian QFT $\rightarrow$ broader perspective is necessary to capture all features
- The influence of quantum fields on the background is described by the semiclassical Einstein equation (SEE)

$$G_{\mu\nu} = 8\pi G \langle :T_{\mu\nu}: \rangle_\Omega$$

The role of QFTCST in Cosmology

- Particle physics $\simeq$ QFT & Universe $\simeq$ CST $\rightarrow$ QFTCST is the natural framework for studying Cosmology!
- Quantized inflationary perturbations (of a classical field) lead to structure formation.
- But the first inflationary model of Starobinsky (1980) has considered only quantum fields!
- It should be possible to understand all (sub-Planckian) features of Cosmology within QFTCST, and we can expect to learn more by this more fundamental treatment.

Three questions

- This talk: analysis of cosmological implications of QFTCST, taking also previously ignored contributions to $\langle :T_{\mu\nu}: \rangle_\Omega$ into account.

1. Is it possible to model the late cosmological evolution completely within QFTCST?

   Yes.

2. Are there non-trivial QFTCST contributions not present in ΛCDM?

   Yes!

3. Are these contributions significantly visible in experimental data?

   Not yet, but ...

Outline of the remaining talk

1. Fundamentals of $\langle :T_{\mu\nu}: \rangle_\Omega$ computations
2. Analysis of solutions to $G_{\mu\nu} = 8\pi G \langle :T_{\mu\nu}: \rangle_\Omega$ in flat FRW spacetimes
3. Beyond flat FRW: modelling Dark Matter halos

Fundamentals of $\langle :T_{\mu\nu}: \rangle_\Omega$ computations

Basic requirements for $\langle :T_{\mu\nu}: \rangle_\Omega$

- $\langle :T_{\mu\nu}: \rangle_\Omega$ should have finite fluctuations ($G_{\mu\nu}$ has none) $\to$ Ω has to be a *Hadamard state*, i.e. it has to share the UV properties of the Minkowski vacuum.
- There are many Hadamard states (e.g. Bunch-Davies in dS) $\to$ there is no unique "vacuum" in QFTCST!
- Generalisation of Lorentz-invariance: $:T_{\mu\nu}:$ has to be *local* and *covariant* $\to$ we are only allowed to subtract local and geometric terms.
- $\nabla^\mu :T_{\mu\nu}: = 0$

The finite regularisation freedom of $\langle :T_{\mu\nu}: \rangle_\Omega$

- There are many viable prescriptions (point-splitting, ζ-function, dimensional regularisation, ...) but there is no unique one! *[Wald 1978, Hollands & Wald 2005]*

- $\langle :T_{\mu\nu}: \rangle_\Omega$ is determined up to finite, local, conserved curvature tensors.

$$\langle :T_{\mu\nu}(x):' \rangle_\Omega = \langle :T_{\mu\nu}(x): \rangle_\Omega +$$

$$+ \alpha_1 m^4 g_{\mu\nu}(x) + \alpha_2 m^2 G_{\mu\nu}(x) + \alpha_3 I_{\mu\nu}(x) + \alpha_4 J_{\mu\nu}(x)$$

$$I_{\mu\nu} \doteq \frac{1}{\sqrt{|\det g|}} \frac{\delta}{\delta g_{\mu\nu}} \int_M dx \sqrt{|\det g|}\; R^2$$

$$J_{\mu\nu} \doteq \frac{1}{\sqrt{|\det g|}} \frac{\delta}{\delta g_{\mu\nu}} \int_M dx \sqrt{|\det g|}\; R_{\alpha\beta} R^{\alpha\beta}$$

A few words on the Cosmological Constant problem

- Finite, *a priori* undetermined regularisation freedom of $\langle :T_{\mu\nu}: \rangle_\Omega \rightarrow$ QFT(CST) does not predict *any* value of the Cosmological Constant Λ.

- $\Lambda \sim M^4_{\text{Planck}}$ is *not* even a natural value, UV-divergence of unrenormalised $\varrho_Q = \langle T_{00} \rangle_\Omega$ is *not* due to unknown trans-Planckian physics, but due to well-known bad mathematics.

The trace anomaly

- Universal property of $\langle :T_{\mu\nu}: \rangle_\Omega$: the *trace anomaly* *[Duff 1977, Christensen 1978, Wald 1978, ...]*
- The classical stress-energy tensor $T_{\mu\nu}$ fulfils $g^{\mu\nu} T_{\mu\nu} = 0$ if $m = 0$,
- but $g^{\mu\nu} \langle :T_{\mu\nu}: \rangle_\Omega \neq 0$ for $m = 0$!
- This (among other things) will lead to higher order curvature corrections to ΛCDM, but QFTCST is *not* equivalent to $f(R)$-gravity:
  1. the higher order curvature terms are *derived* and not imposed.
  2. they correspond to a *non-local* rather than to a local $f(R)$ action.

Analysis of solutions to $G_{\mu\nu} = 8\pi G \langle :T_{\mu\nu}: \rangle_\Omega$ in flat FRW spacetimes

Basic setup

- Flat FRW spacetimes

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2) \qquad H \doteq \frac{\dot{a}}{a}$$

- We consider only free fields, massive/massless, scalar/Dirac/vector, in order to model everything but the baryons.

- $\varrho_Q \doteq \langle :T_{00}: \rangle_\Omega$ receives contributions from:

  1. the trace anomaly (geometric, known)
  2. the finite regularisation freedom (geometric, known)
  3. the state Ω (geometric and non-geometric, unknown)

The state dependence

- We fix the state Ω to be an approximate thermal equilibrium state with $\beta = T^{-1}$ at $a = a_F$ *[Dappiaggi, TH, Pinamonti 2010, TH 2010]*, e.g.

$$\langle \phi(\tau_x, \vec{x}) \phi(\tau_y, \vec{y}) \rangle_\Omega = \int_{\mathbb{R}^3} d\vec{k} \left(\frac{\overline{\phi_k(\tau_x)} \phi_k(\tau_y)}{1 - e^{-\beta \sqrt{k^2 + a_F^2 m^2}}} + \frac{\overline{\phi_k(\tau_y)} \phi_k(\tau_x)}{e^{\beta \sqrt{k^2 + a_F^2 m^2}} - 1} \right) e^{i\vec{k}(\vec{x} - \vec{y})}$$

$$\left(\partial_\tau^2 + k^2 + a^2 m^2 \right) a\, \phi_k(\tau) = 0 \qquad a\, \phi_k(\tau) \sim \frac{e^{-ik\tau}}{\sqrt{2\pi}^3 \sqrt{2k}} \quad \text{for} \quad a \to 0$$

- This state is Hadamard and models the quantum state of (dark) matter after the *freeze-out* of the (dark) matter interactions at $a = a_F$.

A few necessary approximations

- Exact analytical computations of the Ω-contributions and the solutions of the SEE are impossible.
- We employ the following approximations to investigate SEE solutions describing the late cosmological evolution:
  1. $H \ll m$ $[H_0 = O(10^{-33} eV)] \to$ discard $O(H^5)$ terms in ϱ_Q
  2. $\dot{H} \ll H^2$ [observations] $\to$ discard $O(\dot{H}, \ddot{H}, ...)$ terms in ϱ_Q
  3. $T/a \ll m$ [dark matter is currently non-relativistic]

The solutions

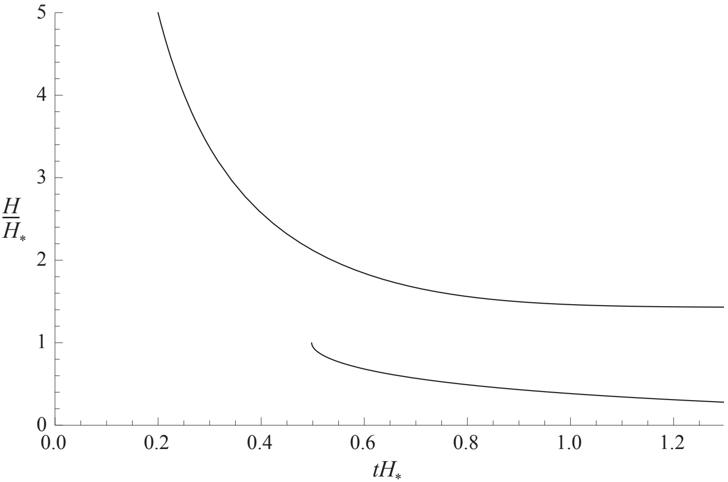
$$\rho_Q = c_1 H^4 + c_2 \frac{mT^3}{a^3} + c_3 \frac{T^5}{ma^5} + c_4 \frac{T^4}{a^4} + c_5 m^4 + c_6 H^2 + O\left(\frac{T^7}{m^3 a^7}\right)$$

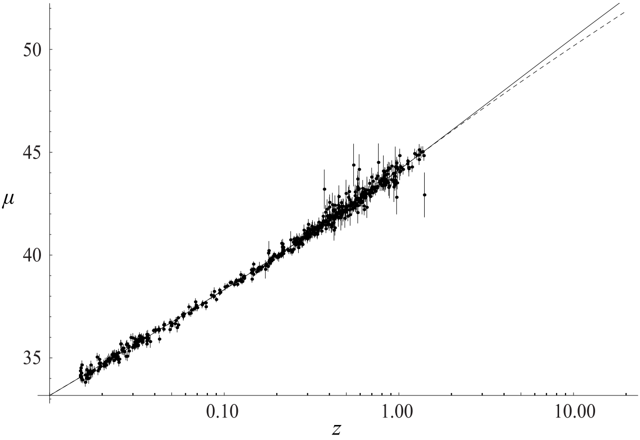
$$H_\pm^2(a) = H_*^2 \pm \sqrt{H_*^4 - C_2 \frac{mT^3}{a^3} - C_3 \frac{T^5}{ma^5} - C_4 \frac{T^4}{a^4} - C_5 m^4}$$

H_*: field content & fin. reg. freedom C_2, C_3: massive field content & $\dfrac{m}{T/a_F}$

C_4: massless field content C_5: fin. reg. freedom

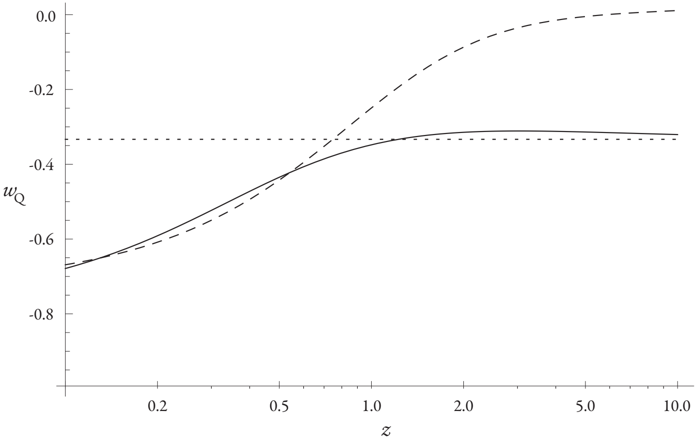
Plot of generic solutions with $\dot{H} < 0$





Union2 supernova compilation *[Amanullah et al. 2010]*

Both solution branches can fit supernova data equally well.



But there is no significant matter-dominated phase on the upper branch.
($w_Q = p_Q/\rho_Q$)

Non-trivial quantum corrections to ΛCDM

- For the lower branch, the supernova fits generally yield values for which $H_-^2(a)$ can be expanded in H_*^{-1}.

$$H_-^2(a) = H_*^2 - \sqrt{H_*^4 - C_2 \frac{mT^3}{a^3} - C_3 \frac{T^5}{ma^5} - \frac{C_4}{a^4} - C_5 m^4}$$
$$= K_0 + \frac{K_3}{a^3} + \frac{K_4}{a^4} + \frac{K_5}{a^5} + O\left(\frac{1}{a^6}\right)$$

- We find standard ΛCDM values for K_0 and K_3, whereas the current data is compatible with and insensitive to quantum corrections to ΛCDM as large as $K_5 = O(10^{-3} K_0)$
- But, if part of the dark matter is warm (T^2/m^2 not too small), then the K_5-corrections could be detected with better data!

Conclusions & beyond

Bringing in the harvest

We have seen ...

- ... that it is possible to reproduce the phenomenology of ΛCDM from first QFTCST principles.
- Non-trivial quantum corrections to ΛCDM arise, which entail that *dark matter* has a different scaling behaviour w.r.t. a and that *dark energy* is dynamical!
- The currently available data is insensitive to these quantum corrections, but we can expect an exponential increase in (supernova) data in the next years.

First steps towards modelling dark matter halos

- We estimate ρ_Q in the freeze-out state, in a spherically symmetric static solution of the SEE.
- $a \cong \sqrt{|g_{00}|} \to \rho_Q \propto \frac{mT^3}{a^3} \;\cong\; \rho_Q \propto \frac{mT^3}{\sqrt{|g_{00}|}^3}$
- In solutions with approximately constant energy density, $|g_{00}| = c_1 + c_2 r^2 + O(r^3)$.
- The resulting $\rho_Q(r) \propto \frac{mT^3}{\sqrt{|g_{00}|}^3}$ has the shape of a ("core") density profile of dark matter dominated, dwarf spheroidal galaxies! *[Burkert 1995, Gilmore & al. 2007, ...]*

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