EVOLUTION OF MAGNETIC FIELDS THROUGH COSMOLOGICAL PERTURBATIONS

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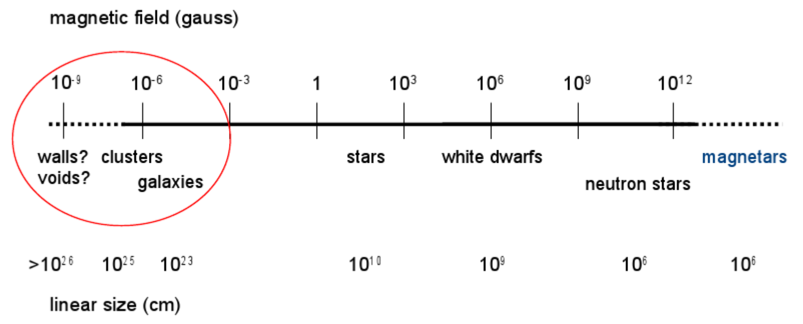
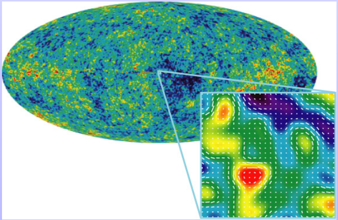
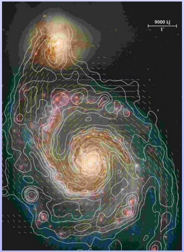
Abstract

- The explanation for the origin of galactic and extra-galactic magnetic fields continues being an unsolved problem in the modern cosmology.
- A possible explanation comes from the fact that these fields emerged from a small field, "a seed”, produced in the early universe (phase transitions or after of inflation) and these evolved with time.
- In this talk I will present the perturbed Einstein Equations, with sources that contain a background of perfect fluid and an electromagnetic field.
- Through the evolution equations, the propagation of these fields in the cosmological history it’s shown.

INTRODUCTION







Origin of Magnetic Field

- **PRIMORDIAL Magnetic Field**
  - Phase Transitions
  - Amplification of Perturbations in electromagnetic Field during Inflation
  - Perturbation to 2° order in coupled $\gamma - p - e^-$.

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- **ASTROPHYSIC MAGNETIC FIELD**
  - Biermann Battery
  - Harrison´s Mechanism.
  - Biermann mechanism in the supernova explosions of first stars.

COSMOLOGICAL PERTURBATIONS

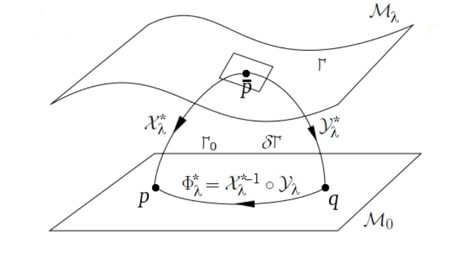
General covariance states that there is no preferred coordinate system in nature and it introduce a *gauge* in perturbation theory. This *gauge* is an unphysical degree of freedom and we have to fix the *gauge* or to extract some invariant quantities to have physical results.

$$\Gamma(\bar{p}) = \Gamma_0(p) + \delta\Gamma(p) \qquad (1)$$

Eq.(2) implies that we are using a gauge choice.

A tensor field Γ is gauge-invariant to order $n \geq 1$, iff: $L_\xi \delta^k \Gamma = 0$, for any vector field ξ on $\mathscr{M}$ and $\forall k < n$[a].

[a] Bruni M., Matarrense S., Mollerach S. & Sonego S., gr-qc: 9609040v2, (1997).



Given a tensor field Γ, the relations between the 1° and 2° perturbations of Γ in two different gauges are:

$$\delta\Gamma^{\mathfrak{X}} - \delta\Gamma^{\mathfrak{Y}} = L_{\xi_1}\Gamma_0 \qquad (2)$$

$$\delta^2\Gamma^{\mathfrak{Y}} - \delta^2\Gamma^{\mathfrak{X}} = 2L_{\xi_1}\left(\delta\Gamma^{\mathfrak{X}}\right) + \left(L^2_{\xi_1} + L_{\xi_2}\right)\Gamma_0 \qquad (3)$$

Cosmological Perturbations to first order

Matter Perturbations

We do the perturbative expansion of these quantities:

$$\mu = \mu_{(0)} + \sum_{r=1}^{\infty} \frac{1}{r!} \mu_{(r)}$$

$$P = P_{(0)} + \sum_{r=1}^{\infty} \frac{1}{r!} P_{(r)}$$

$$B^i = \frac{1}{a(\tau)^2} \left(B^i_{(0)} + \sum_{r=1}^{\infty} \frac{1}{r!} B^i_{(r)} \right)$$

$$E^i = \frac{1}{a(\tau)^2} \left(E^i_{(0)} + \sum_{r=1}^{\infty} \frac{1}{r!} E^i_{(r)} \right)$$

$$u^\mu = \frac{1}{a(\tau)} \left(\delta^\mu_0 + \sum_{r=1}^{\infty} \frac{1}{r!} v^\mu_{(r)} \right) \qquad u^\mu u_\mu = -1$$

$$j^\mu = \frac{1}{a(\tau)} \left(j^\mu_0 + \sum_{r=1}^{\infty} \frac{1}{r!} j^\mu_{(r)} \right), \qquad j^\mu = (\rho, J^i)$$

We consider first and second order perturbations about a FRW background, so that the metric tensor is:

$$g_{00} = -a(\tau)^2 \left(1 + 2\sum_{r=1}^{\infty} \frac{1}{r!}\psi^{(r)}\right)$$

$$g_{oi} = a(\tau)^2 \sum_{r=1}^{\infty} \frac{1}{r!}\omega_i^{(r)}$$

$$g_{ij} = a(\tau)^2 \left[\left(1 - 2\sum_{r=1}^{\infty} \frac{1}{r!}\phi^{(r)}\right)\delta_{ij} + \sum_{r=1}^{\infty} \frac{1}{r!}\chi_{ij}^{(r)}\right]$$

The perturbations are splitting into scalar, transverse vector part, and transverse trace-free tensor:

$$\omega_i^{(r)} = \partial_i \omega^{(r)\parallel} + \omega_i^{(r)\perp} \tag{4}$$

With $\partial^i \omega_i^{(r)\perp} = 0$.

$$\chi_{ij}^{(r)} = D_{ij}\chi^{(r)\parallel} + \partial_i \chi_j^{(r)\perp} + \partial_j \chi_i^{(r)\perp} + \chi_{ij}^{(r)\top} \tag{5}$$

With $\partial^i \chi_{ij}^{(r)\top} = 0$ and $D_{ij} \equiv \partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2$.

$$\xi_{(r)}^0 = \alpha^{(r)}$$

$$\xi_{(r)}^i = \partial^i \beta^{(r)} + d^{(r)i} \tag{6}$$

With $\partial_i d^{(r)i} = 0$.

Gauge Invariants Quantities

Scalar Modes

$$\Psi^{(1)} \equiv \psi^{(1)} + \frac{1}{a}\left(\mathscr{S}^{||}_{(1)} a\right)' \tag{7}$$

$$\Phi^{(1)} \equiv \phi^{(1)} + \frac{1}{6}\nabla^2 \chi^{(1)} - H\mathscr{S}^{||}_{(1)} \tag{8}$$

$$\Delta^{(1)} \equiv \mu_{(1)} + \left(\mu_{(0)}\right)' \mathscr{S}^{||}_{(1)} \tag{9}$$

$$\upsilon^{(1)} \equiv v^{(1)} + \left(\frac{1}{2}\chi^{||(1)}\right)' \tag{10}$$

With $\mathscr{S}^{||}_{(1)} \equiv \left(\omega^{||(1)} - \frac{\left(\chi^{||(1)}\right)'}{2}\right)$ and $H = \frac{a'}{a}$.

Gauge Invariants Quantities

Vector modes

$$\upsilon^i_{(1)} \equiv v^i_{(1)} + \left(\chi^i_{\perp(1)}\right)' \tag{11}$$

$$\vartheta_i^{(1)} \equiv \omega_i^{(1)} - \left(\chi_i^{\perp(1)}\right)' \tag{12}$$

$$\mathscr{V}^i_{(1)} \equiv \omega^i_{(1)} + v^i_{(1)} \tag{13}$$

$$\Pi_{ij}^{(1)t} \equiv \Pi_{ij}^{(1)fl} + \Pi_{ij}^{(1)EM} \tag{14}$$

Tensor and null quantities are G.I to first order by definition

Field Equation to First Order

$$\delta G_0^0 = \frac{2}{a^2}\left(3H\left(H\Psi_{(1)} + \Phi'_{(1)}\right) - \partial_k\partial^k\Phi^{(1)} - 3H\left(H^2 - H'\right)\mathscr{S}^{||}_{(1)}\right) \tag{15}$$

$$\begin{aligned}\delta G_0^i = \frac{2}{a^2}&\left(\partial^i\left(H\Psi_{(1)} + \Phi'_{(1)} + \left(H^2 - H'\right)\frac{1}{2}\chi^{||\prime}_{(1)}\right)\right.\\ &\left.-\left(H^2 - H' - \frac{1}{4}\partial_k\partial^k\right)\vartheta^i_{(1)} - \left(H^2 - H'\right)\chi^{i\prime}_{(1)}\right)\end{aligned} \tag{16}$$

Field Equation to First Order

$$\delta G^0_i = \frac{2}{a^2}\left(\partial_i\left(-H\Psi_{(1)} - \Phi'_{(1)} + \left(H^2 - H'\right)\mathscr{S}^{||}_{(1)}\right) + \frac{1}{4}\partial_k\partial^k\vartheta^{(1)}_i\right) \tag{17}$$

$$\delta G^i_j = \frac{2}{a^2}\left[\left[\left(H^2 + 2H'\right)\Psi_{(1)} + H\left(\Psi'_{(1)} + 2\Phi'_{(1)}\right) - \frac{1}{2}\partial_k\partial^k\left(\Phi_{(1)} - \Psi_{(1)}\right)\right.\right.$$

$$+\Phi''_{(1)} + \left(H'' - HH' - H^3\right)\mathscr{S}^{||}_{(1)}\Big]\delta^i_j + \frac{1}{2}\partial_j\partial^i\left(\Phi_{(1)} - \Psi_{(1)}\right) - \frac{H}{2}\left(\partial^i\vartheta^{(1)}_j + \partial_j\vartheta^i_{(1)}\right)$$

$$\left. -\frac{1}{4}\left(\partial^i\vartheta^{(1)}_j + \partial_j\vartheta^i_{(1)}\right)' + \frac{H}{2}\left(\chi^{i\top}_{j(1)}\right)' + \frac{1}{4}\left(\chi^{i\top}_{j(1)}\right)'' - \frac{1}{4}\partial^k\partial_k\chi^{i\top}_{j(1)}\right] \tag{18}$$

We use the spatial part of Ohm's law, which is the current proyected in the slices:

$$\left(g_{\mu i}+u_{\mu}u_{i}\right)j^{\mu}=\sigma g_{\lambda i}g_{\alpha\mu}F^{\lambda\alpha}u^{\mu}$$

Without perturbations:

$$J_i^{(0)}=\sigma E_i^{(0)} \tag{19}$$

To first order, we find:

$$J_i^{(1)}+J^j_{(0)}\chi_{ij}^{(1)}-2\Phi^{(1)}J_i^{(0)}+2\left(\frac{1}{6}\nabla^2\chi^{(1)}+H\mathscr{S}^{||}_{(1)}\right)J_i^{(0)}-\rho^{(0)}\left(\upsilon_i^{(1)}-\left(\chi_i^{\perp(1)}\right)'\right)$$

$$=\sigma\left[E_i^{(1)}+\left(\mathscr{V}^{(1)}\times B^{(0)}\right)_i\right.$$

$$\left.-2E_i^{(0)}\left(\Phi^{(1)}-\frac{1}{2}\Psi^{(1)}-\frac{1}{6}\nabla^2\chi^{(1)}-\frac{H}{2}\mathscr{S}^{||}_{(1)}+\frac{1}{2}\left(\mathscr{S}^{||}_{(1)}\right)'\right)+E^j_{(0)}\chi_{ij}^{(1)}\right] \tag{20}$$

The conductivity σ is large during most of the thermodynamic history of the universe, since it is proportional to the temperature when the temperature is much larger than the mass of the corresponding species:

$$\sigma\rightarrow\infty\Rightarrow E_i^{(0)}\sim J_i^{(0)}\sim\rho^{(0)}=0,\qquad\left\langle B_i^{(0)}\right\rangle=0,\ \left\langle B_i^{(0)}B^i_{(0)}\right\rangle\neq 0$$

E.M Energy Momentum Tensor

$$T^{\alpha\beta}_{(em)} = \frac{1}{4\pi}\left[F^{\alpha\mu}F^{\beta\nu}g_{\mu\nu} - \frac{1}{4}g^{\alpha\beta}\left(\mathfrak{F}^2\right)\right], \qquad \mathfrak{F}^2 \equiv F^{\delta\omega}F^{\sigma\varepsilon}g_{\delta\sigma}g_{\omega\varepsilon} \tag{21}$$

$$T^0_0 = -\frac{1}{8\pi}\left(E^2_{(0)} + B^2_{(0)}\right) = -\frac{1}{8\pi}B^2_{(0)} \tag{22}$$

$$T^0_i = \frac{1}{4\pi}\left(E_{(0)} \times B_{(0)}\right)_i \sim T^i_0 = \frac{1}{4\pi}\left(E_{(0)} \times B_{(0)}\right)^i = 0 \tag{23}$$

$$T^i_l = \frac{1}{24\pi}\left(E^2_{(0)} + B^2_{(0)}\right)\delta^i_l + \Pi^{i(0)}_{l(em)} = \frac{1}{24\pi}B^2_{(0)}\delta^i_l \tag{24}$$

Where $\Pi^{i(0)}_{l(em)}$ is the Maxwell stress tensor:

$$\Pi^{i(0)}_{l(em)} = \frac{1}{4\pi}\left(-B^i_{(0)}B^{(0)}_l - E^i_{(0)}E^{(0)}_l + \frac{1}{3}\left(E^2_{(0)} + B^2_{(0)}\right)\delta^i_l\right)$$

COMPONENT $T^{\alpha(1)}_{(em)\,\beta}$:

$$\mathfrak{F}^2_{(1)} = -2\left[E^i_{(0)}E^l_{(0)}\left(-2\phi_{(1)}\delta_{li} + \chi^{(1)}_{li} + 2\psi_{(1)}\delta_{li}\right)\right.$$

$$-\frac{1}{2}B^k_{(0)}B^s_{(0)}\varepsilon^{lmk}\varepsilon^{ijs}\left(-4\phi^{(1)}\delta_{li}\delta_{mj} + \delta_{mj}\chi^{(1)}_{li} + \delta_{li}\chi^{(1)}_{mj}\right)$$

$$\left.+\left(2E^i_{(0)}E_{i(1)}\right) - \frac{1}{2}\varepsilon^{lmk}\varepsilon^{ijs}\left(B^s_{(0)}B^k_{(1)} + B^k_{(0)}B^s_{(1)}\right)\delta_{li}\delta_{mj} + 2\varepsilon^{ik}_{l}E^l_{(0)}B^k_{(0)}\omega^{(1)}_i\right]$$

$$\Downarrow$$

$$\mathfrak{F}^2_{(1)} = 2\Delta^{(1)}_{(mag)} - 8\Phi^{(1)}B^2_{(0)} - 2\left(B^2_{(0)}\right)'\mathscr{S}^{||}_{(1)} + \frac{4}{3}\nabla^2\chi^{(1)}B^2_{(0)} - 8H\mathscr{S}^{||}_{(1)}B^2_{(0)}$$

Component $T^{\alpha(1)}_{(em)\,\beta}$:

$$T^{0}_{(em)\,0} = -\frac{1}{16\pi}\mathfrak{F}^{2}_{(1)} \tag{25}$$

$$T^{i}_{(em)0} = \frac{1}{4\pi}\left[B^{2}_{(0)}\vartheta^{i}_{(1)} - \varepsilon^{ikm}E^{(1)}_{k}B^{m}_{(0)} + B^{2}_{(0)}\left(\chi^{i}_{\perp(1)}\right)'\right] \tag{26}$$

$$T^{0}_{(em)i} = \frac{1}{4\pi}\left[\varepsilon^{km}_{i}E^{(1)}_{k}B^{m}_{(0)}\right] \tag{27}$$

$$T^{i}_{(em)\,l} = \frac{1}{4\pi}\left[\frac{1}{12}\mathfrak{F}^{2}_{(1)}\delta^{i}_{l} + \Pi^{i(1)}_{l(em)}\right] \tag{28}$$

${}^{(1)}T^{\alpha}_{(fluid)\beta}$

We expand the matter Energy Momentum Tensor around a homogeneous density $\mu_{(0)}$ and pressure $P_{(0)}$ (Perturbation of a perfect fluid $T^{\mu}_{\nu} = (\rho + P)\, u^{\mu} u_{\nu} + P\delta^{\mu}_{\nu}$), one can find:

$$T^0_0 = -\Delta^{(1)} + \left(\mu_{(0)}\right)' \mathscr{S}^{||}_{(1)} \tag{29}$$

$$T^i_0 = (\mu_0 + P_0)\left(\mathscr{V}^i_{(1)} - \vartheta^i_{(1)} - \left(\chi^i_{\perp(1)}\right)'\right) \tag{30}$$

$$T^0_i = -(\mu_0 + P_0)\, \mathscr{V}_i^{(1)} \tag{31}$$

$$T^i_j = \left(\Delta^{(1)}_P - \left(P_{(0)}\right)' \mathscr{S}^{||}_{(1)}\right)\delta^i_j + \Pi^{i(1)}_{j(f)} \tag{32}$$

Where $\Pi^{i(1)}_{j(f)}$ is the anisotropic stress tensor.

CONSERVATION EQUATIONS.

$$\mathscr{T}^{\alpha}_{\beta;\alpha} = T^{\alpha(f)}_{\beta;\alpha} + T^{\alpha(E.M)}_{\beta;\alpha} = 0 \tag{33}$$

where:

$$T^{\alpha(E.M)}_{\beta;\alpha} = F_{\beta\alpha} j^{\alpha} \tag{34}$$

The continuity equation is given by $\mathscr{T}^{\alpha}_{0;\alpha} = 0$:

$$\left(\Delta^{(1)}\right)' + 3H\left(\Delta^{(1)}_{P} + \Delta^{(1)}\right) - 3\left(\Phi^{(1)}\right)'(P_0+\mu_0) + (P_0+\mu_0)\nabla^2 \upsilon^{(1)}$$

$$-\left(\left(\mu_{(0)}\right)' \mathscr{S}^{||}_{(1)}\right)' - 3H\left(P_{(0)}+\mu_{(0)}\right)' \mathscr{S}^{||}_{(1)} + (P_0+\mu_0)\left(-\frac{1}{2}\nabla^2\chi^{(1)} + 3H\mathscr{S}^{||}_{(1)}\right)'$$

$$-(P_0+\mu_0)\nabla^2\left(\frac{1}{2}\chi^{||(1)}\right)' = -a^4\left(E^{(1)}_{i} J^{i}_{(0)} + E^{(0)}_{i} J^{i}_{(1)}\right) \tag{35}$$

The Navier-Stokes equation $\mathscr{T}^{\alpha}_{i;\alpha} = 0$:

$$\left(\mathscr{V}_i^{(1)}\right)' + \frac{\left(\mu_{(0)} + P_{(0)}\right)'}{\left(\mu_{(0)} + P_{(0)}\right)} \mathscr{V}_i^{(1)} - 4H\mathscr{V}_i^{(1)} + \partial_i \Psi^{(1)} - \frac{\partial_i\left(\Delta_P^{(1)} - \left(P_{(0)}\right)' \mathscr{S}^{||}_{(1)}\right) + \partial_l \Pi_i^{(1)l}}{\left(\mu_{(0)} + P_{(0)}\right)}$$
$$-\partial_i \frac{1}{a}\left(\mathscr{S}^{||}_{(1)} a\right)' = \frac{-a^4}{\left(\mu_{(0)} + P_{(0)}\right)}\left(E_i^{(1)}\rho_{(0)} + \varepsilon_{ijk} J^j_{(0)} B_k^{(1)} + E_i^{(0)}\rho_{(1)} + \varepsilon_{ijk} J^j_{(1)} B_k^{(0)}\right) \tag{36}$$

Maxwell Equations zero order

$$\partial_i E^i_{(0)} = a\rho_{(0)} \tag{37}$$

$$\varepsilon^{ilk}\partial_l B^k_{(0)} - \left(E^i_{(0)}\right)' - 2HE^i_{(0)} = aJ^i_{(0)} \tag{38}$$

$$\partial_i B^{(0)}_i + \partial_j B^{(0)}_j + \partial_k B^{(0)}_k = 0 \tag{39}$$

$$B'_{k(0)} + 2HB^{(0)}_k + \varepsilon^{ij}_k \partial_i E^{(0)}_j = 0 \tag{40}$$

$$\Downarrow$$

$$B^2_{(0)} \sim \frac{1}{a^2(\tau)}$$

MAXWELL EQUATIONS FIRST ORDER

We find the first non-homogeneous Maxwell equation, given by:

$$\partial_i E^i_{(1)} + E^i_{(0)}\partial_i\left(\Psi^{(1)} - \frac{1}{a}\left(\mathscr{S}^{||}_{(1)}a\right)' - 3\left(\Phi^{(1)} - \frac{1}{6}\nabla^2\chi^{(1)} + H\mathscr{S}^{||}_{(1)}\right)\right) = a\rho_{(1)} \quad (41)$$

The first homogeneous Maxwell equation is given by:

$$B'_{k(1)} + 2HB^{(1)}_k + \varepsilon^{ij}_k\partial_i E^{(1)}_j = 0 \quad (42)$$

Maxwell Equations first order

We find the second non-homogeneous Maxwell equation, given by:

$$\varepsilon^{ilk}\partial_l B^k_{(1)} - \left(E^i_{(1)}\right)' - E^i_{(0)}\left(\Psi^{(1)} - \frac{1}{a}\left(\mathscr{S}^{||}_{(1)}a\right)' - 3\left(\Phi^{(1)} - \frac{1}{6}\nabla^2\chi^{(1)} + H\mathscr{S}^{||}_{(1)}\right)\right)'$$

$$-2HE^i_{(1)} + \varepsilon^{ilk}B^k_{(0)}\partial_l\left(\Psi^{(1)} - \frac{1}{a}\left(\mathscr{S}^{||}_{(1)}a\right)' - 3\left(\Phi^{(1)} - \frac{1}{6}\nabla^2\chi^{(1)} + H\mathscr{S}^{||}_{(1)}\right)\right) = aJ^i_{(1)} \tag{43}$$

The divergence free of magnetic field given by:

$$\partial_i B^{(1)}_i + \partial_j B^{(1)}_j + \partial_k B^{(1)}_k = 0 \tag{44}$$

“Cosmological Dynamo” to First Order

$$\left(B_k^{(1)}\right)' + 2HB_k^{(1)} + \eta\left[\nabla\times\left(\nabla\times B^{(1)} - \left(E^{(1)}\right)' - 2HE^{(1)}\right)\right]_k$$
$$+\left(\nabla\times B_{(0)}\times\mathscr{V}_{(1)}\right)_k + O^{(1)} = 0 \qquad (45)$$

Where:

$$O^{(1)} = \eta\varepsilon_k^{lj}\partial_l\left(-E_j^{(0)}\left(\Psi^{(1)} - 3\Phi^{(1)}\right)' + \varepsilon_{jlm}\partial_l B_{(0)}^m\left(\Psi^{(1)} - 3\Phi^{(1)}\right)\right.$$

$$+\varepsilon^{ilm}\partial_l B_{(0)}^m\chi_{ij}^{(1)} + \chi_{ij}^{(1)}\left(E_{(0)}^i\right)' - 2HE_{(0)}^i\chi_{ij}^{(1)} - 2H\mathscr{S}_{(1)}^{||}\left(\varepsilon^{ilm}\partial_l B_{(0)}^m + \left(E_{(0)}^i\right)' - 2HE_{(0)}^i\right)$$

$$\left.-E_j^{(0)}\left(-2\Phi^{(1)} + \frac{1}{3}\nabla^2\chi^{(1)}\right)\right) + \varepsilon_k^{lj}\partial_l\left(2E_j^{(0)}\left(\Psi^{(1)} - \frac{1}{2}\Phi^{(1)}\right) - \chi_{ij}^{(1)}E_{(0)}^i\right)$$

With $\eta\sim\frac{1}{\sigma}$ the diffusion coefficient.

SECOND ORDER

To second-order variable perturbation $\delta^2 \daleth$ is transformed as:

$$\delta^2 \daleth^{\mathfrak{Y}} - \delta^2 \daleth^{\mathfrak{X}} = 2L_{\xi_1}\left(\delta \daleth^{\mathfrak{X}}\right) + \left(L^2_{\xi_1} + L_{\xi_2}\right)\daleth_0$$

Inspecting the gauge transformation, is introduced the variable $\delta^2 T$ defined by[1]:

$$\delta^2 T \doteq \daleth - 2L_{\mathfrak{X}}\left(\delta \daleth^{\mathfrak{X}}\right) + L^2_{\mathfrak{X}}\daleth_0$$

Then, the variable $\delta^2 T$ is transformed as:

$$\delta^2 T^{\mathfrak{Y}} - \delta^2 T^{\mathfrak{X}} = L_\sigma \daleth_0 \quad \textit{with}:\ \sigma \doteq \xi_2 + [\xi_1, \xi] \tag{46}$$

ξ is variant part of the linear-order perturbations. The gauge transformation rule (46), is identical to that for a linear perturbation.

This property is not general but happens in the case of cosmological perturbation around FRW metric, it's the key to extend this theory to second order.

$$L\left[\delta^2 T\right] = S\left[\delta T, \delta T\right]$$

[1] K. Nakamura, Prog. Theor. Phys. 110 (2003)

Gauge Invariants Quantities up Second Order

Scalar Modes

$$\Psi^{(2)} \equiv \psi^{(2)} + \frac{1}{a}\left(\mathscr{S}^{||}_{(2)} a\right)' + \mathscr{T}(\Psi^{(1)}\mathscr{O}^{(1)}) \tag{47}$$

$$\Phi^{(2)} \equiv \phi^{(2)} + \frac{1}{6}\nabla^2\chi^{(2)} - H\mathscr{S}^{||}_{(2)} + \mathscr{T}(\Phi^{(1)}\mathscr{O}^{(1)}) \tag{48}$$

$$\Delta^{(2)} \equiv \mu_{(2)} + (\mu_{(0)})'\mathscr{S}^{||}_{(2)} + \mathscr{T}(\Delta^{(1)}\mathscr{O}^{(1)}) \tag{49}$$

$$\upsilon^{(2)} \equiv v^{(2)} + \left(\frac{1}{2}\chi^{||(2)}\right)' + \mathscr{T}(\upsilon^{(2)}\mathscr{O}^{(1)}) \tag{50}$$

With $\mathscr{S}^{||}_{(2)} \equiv \left(\omega^{||(2)} - \frac{(\chi^{||(2)})'}{2}\right) + \mathscr{T}(\mathscr{S}^{||}_{(1)}\mathscr{O}^{(1)})$.

Gauge Invariants Quantities up Second Order

Vector and Tensor modes

$$\upsilon^i_{(2)} \equiv v^i_{(2)} + \left(\chi^i_{\perp(2)}\right)' + \mathscr{T}(\upsilon^i_{(1)}\mathscr{O}^{(1)}) \tag{51}$$

$$\vartheta^{(2)}_i \equiv \omega^{(2)}_i - \left(\chi^{\perp(2)}_i\right)' + \mathscr{T}(\vartheta^{(1)}_i\mathscr{O}^{(1)}) \tag{52}$$

$$\mathscr{V}^i_{(2)} \equiv \omega^i_{(2)} + v^i_{(2)} + \mathscr{T}(\mathscr{V}^i_{(1)}\mathscr{O}^{(1)}) \tag{53}$$

$$\mathscr{B}^i_{(2)} \equiv B^i_{(2)} + \frac{2}{a^2}B^j_{(1)}\partial_j\chi^i_{(1)\perp} - \frac{2}{a^2}\partial_l B^i_{(1)}\chi^l_{(1)\perp} \tag{54}$$

$$\mathscr{E}^i_{(2)} \equiv E^i_{(2)} + \frac{2}{a^2}E^j_{(1)}\partial_j\chi^i_{(1)\perp} - \frac{2}{a^2}\partial_l E^i_{(1)}\chi^l_{(1)\perp} \tag{55}$$

$$\triangle^{(2)}_\rho \equiv \rho^{(2)} + \mathscr{T}(\rho^{(1)}\mathscr{O}^{(1)}) \tag{56}$$

$$\mathscr{J}^i_{(2)} \equiv J^i_{(2)} + \mathscr{T}(J^i_{(1)}\mathscr{O}^{(1)}) \tag{57}$$

$$\Pi^{(2)t*}_{ij} \equiv \Pi^{(2)f}_{ij} + \Pi^{(2)EM}_{ij} + \mathscr{T}(\Pi^{(1)t}_{ij}\mathscr{O}^{(1)}) \tag{58}$$

Field Equation to Second Order

$$\delta^2 G^0_0 = \frac{2}{a^2}\left(3H\left(H\Psi_{(2)} + \Phi'_{(2)}\right) - \partial_k\partial^k\Phi^{(2)} - 3H\left(H^2 - H'\right)\mathscr{S}^{||}_{(2)} + S\left(\delta G^0_0\right)\right) \tag{59}$$

$$\begin{aligned}\delta^2 G^i_0 = \frac{2}{a^2}&\left(\partial^i\left(H\Psi_{(2)} + \Phi'_{(2)} + \left(H^2 - H'\right)\frac{1}{2}\chi^{||\prime}_{(2)}\right)\right.\\ &\left.-\left(H^2 - H' - \frac{1}{4}\partial_k\partial^k\right)\vartheta^i_{(2)} - \left(H^2 - H'\right)\chi^{i\prime}_{(2)} + S\left(\delta G^i_0\right)\right)\end{aligned} \tag{60}$$

Sources $S\left(\delta G_0^0\right)$ and $S\left(\delta G_0^i\right)$

The couples of first orden in $\delta^2 G_0^0$ are given by:

$$S\left(\delta G_0^0\right)=\frac{-1}{a^2}\left(6\partial_i\Psi^{(1)}\partial^i\Psi^{(1)}+6\left(\Psi'_{(1)}\right)^2+16\Psi^{(1)}\nabla^2\Psi^{(1)}\right.$$
$$\left.+12H^2\left(\Psi_{(1)}\right)^2-6H\left(H^2-H'\right)\mathscr{T}(\mathscr{S}^{||}_{(1)}\mathscr{O}^{(1)})\right)$$

The couples of first orden in $\delta^2 G_0^i$ are given by:

$$S\left(\delta G_0^i\right)=\frac{4}{a^2}\left(\Psi'_{(1)}\partial^i\Psi_{(1)}+4\Psi_{(1)}\partial^i\Psi'_{(1)}-\left(H^2-H'-\frac{1}{4}\partial_k\partial^k\right)\mathscr{T}(\vartheta_i^{(1)}\mathscr{O}^{(1)})\right)$$

Field Equation to Second Order

$$\delta^2 G^0_i = \frac{2}{a^2}\left(\partial_i\left(-H\Psi_{(2)} - \Phi'_{(2)} + \left(H^2 - H'\right)\mathscr{S}^{||}_{(2)}\right) + \frac{1}{4}\partial_k\partial^k\vartheta_i^{(2)} + S\left(\delta G^0_i\right)\right) \tag{61}$$

$$\delta^2 G^i_j = \frac{2}{a^2}\Bigg[\Bigg[\left(H^2 + 2H'\right)\Psi_{(2)} + H\left(\Psi'_{(2)} + 2\Phi'_{(2)}\right) - \frac{1}{2}\partial_k\partial^k\left(\Phi_{(2)} - \Psi_{(2)}\right) + \Phi''_{(2)}$$

$$+\left(H'' - HH' - H^3\right)\mathscr{S}^{||}_{(2)} + S\left(\delta G \cdot \delta^i_j\right)\Bigg]\delta^i_j + \frac{1}{2}\partial_j\partial^i\left(\Phi_{(2)} - \Psi_{(2)}\right) - \frac{H}{2}\left(\partial^i\vartheta_j^{(2)} + \partial_j\vartheta^i_{(2)}\right)$$

$$-\frac{1}{4}\left(\partial^i\vartheta_j^{(2)} + \partial_j\vartheta^i_{(2)}\right)' + \frac{H}{2}\left(\chi^{i\top}_{j(2)}\right)' + \frac{1}{4}\left(\chi^{i\top}_{j(2)}\right)'' - \frac{1}{4}\partial^k\partial_k\chi^{i\top}_{j(2)} + S\left(\delta G^i_j\right)\Bigg] \tag{62}$$

${}^{(2)}T^{\alpha}_{(fluid)\beta}$

$$T^0_0 = -\frac{\Delta^{(2)}}{2} - (\mu_0 + P_0)\left(\upsilon^{(1)}_l \upsilon^l_{(1)} + \vartheta^{(1)}_l \upsilon^l_{(1)}\right) + S\left(\mathscr{S}^{||}_{(1)}, \left(\chi^i_{\perp(1)}\right)', \mathscr{T}(\Delta^{(1)}, \vartheta^{(1)}_l, \upsilon^l_{(1)})\right) \tag{63}$$

$$T^i_0 = -(\mu_0 + P_0)\left(\frac{\mathscr{V}^i_{(2)} - \vartheta^i_{(2)}}{2} + \Psi^{(1)}\upsilon^i_{(1)}\right) - \left(\Delta^{(1)} + \Delta^{(1)}_P\right)\upsilon^i_{(1)} + S\left(\left(\chi^i_{\perp(1)}\right)', \mathscr{T}(\mathscr{O}^{(1)})\right) \tag{64}$$

$$\begin{aligned} T^0_i = &-(\mu_0 + P_0)\left(\frac{\mathscr{V}^{(2)}_i}{2} - 2\vartheta^{(2)}_i \Psi^{(1)} - 2\upsilon^{(1)}_i \Phi^{(1)} + \upsilon^j_{(1)}\chi^{(1)}_{ij} - \upsilon^{(1)}_i \Psi^{(1)}\right) \\ &- \left(\Delta^{(1)} + \Delta^{(1)}_P\right)\mathscr{V}^{(1)}_i + S\left(\left(\chi^i_{\perp(1)}\right)', \mathscr{T}(\Psi^{(1)}, \vartheta^{(1)}_l, \upsilon^l_{(1)})\right) \end{aligned} \tag{65}$$

$$T^i_j = \frac{1}{2}\Delta^{(2)}_P \delta^i_j + \frac{1}{2}\Pi^{i(2)*}_j + (\mu_0 + P_0)\left(\upsilon^{(1)}_j \upsilon^i_{(1)} + \vartheta^{(1)}_j \upsilon^i_{(1)}\right) + S\left(\left(\chi^i_{\perp(1)}\right)', \mathscr{T}(\vartheta^{(1)}_l, \upsilon^l_{(1)})\right) \tag{66}$$

COMPONENT $T^{\alpha(2)}_{(em)\,\beta}$:

$$T^{0}_{(em)\,0} = -\frac{1}{16\pi}\left(\mathfrak{F}^2_{(2)} - \left(B^m_{(0)}E^{(1)}_k\right)\varepsilon^{ikm}\vartheta^{(1)}_i + S^1(\mathscr{O}^{(1)})\right) \tag{67}$$

$$T^{i}_{(em)0} = \frac{1}{4\pi}\left[B^2_{(0)}\vartheta^i_{(2)} - \varepsilon^{ikm}\mathscr{E}^{(2)}_k B^m_{(0)} - \varepsilon^{ikm}E^{(1)}_k B^m_{(1)} + S^i_2(\mathscr{O}^{(1)})\right] \tag{68}$$

$$T^{0}_{(em)i} = \frac{1}{4\pi}\left[\varepsilon^{km}_i\mathscr{E}^{(2)}_k B^m_{(0)} + \varepsilon^{km}_i E^{(1)}_k B^m_{(1)} + S_{3i}(\mathscr{O}^{(1)})\right] \tag{69}$$

$$T^{i}_{(em)\,l} = \frac{1}{4\pi}\left[\frac{1}{12}\left(\mathfrak{F}^2_{(2)} - \left(B^m_{(0)}E^{(1)}_k\right)\varepsilon^{ikm}\vartheta^{(1)}_i + S^i_{4l}(\mathscr{O}^{(1)})\right)\delta^i_l + \Pi^{i(2)*}_{l(em)} + S_5(\mathscr{O}^{(1)})\right] \tag{70}$$

Conservation Equations to Second Order.

The continuity equation is given by $\mathscr{T}^{\alpha}_{0;\alpha}=0$:

$$\left(\Delta^{(2)}\right)'+3H\left(\Delta_P^{(2)}+\Delta^{(2)}\right)-3\left(\Phi^{(2)}\right)'(P_0+\mu_0)+(P_0+\mu_0)\nabla^2\upsilon^{(2)}$$

$$-S\left(\mathscr{S}^{||}_{(1)},\nabla^2\chi^{(1)},\mathscr{T}(\Delta^{(1)},\vartheta_l^{(1)},\upsilon^l_{(1)})\right)=-a^4\left(\frac{1}{2}E_i^{(2)}J^i_{(0)}+\frac{1}{2}E_i^{(0)}J^i_{(2)}+2E_i^{(1)}J^i_{(1)}\right) \tag{71}$$

The Navier-Stokes equation $\mathscr{T}^{\alpha}_{i;\alpha} = 0$:

$$\left(\mathscr{V}_i^{(2)}\right)' + \frac{\left(\mu_{(0)} + P_{(0)}\right)'}{\left(\mu_{(0)} + P_{(0)}\right)}\mathscr{V}_i^{(2)} - 4H\mathscr{V}_i^{(2)} + \partial_i \Psi^{(2)} - \frac{\partial_i \Delta_P^{(2)} + \partial_l \Pi_i^{(2)l}*}{\left(\mu_{(0)} + P_{(0)}\right)}$$

$$= \frac{a^4}{\left(\mu_{(0)} + P_{(0)}\right)}\left(2E_i^{(1)}\rho_{(1)} + 2\varepsilon_{ijk}J^j_{(1)}B_k^{(1)} + E_i^{(2)}\rho_{(0)} + \varepsilon_{ijk}J^j_{(0)}B_k^{(2)} + E_i^{(0)}\rho_{(2)} + \varepsilon_{ijk}J^i_{(2)}B_k^0\right) \tag{72}$$

Ohm's Law to Second Order

$$
J^i_{(2)} - 4J_i^{(1)}\phi^{(1)} + 2J^j_{(0)}\left(\omega_i^{(1)}\omega_j^{(1)} + v_i^{(1)}v_j^{(1)} + 2\omega_{(i}^{(1)}\omega_{j)}^{(1)} - \phi^{(2)}\delta_{ij}\right)
$$

$$
+2J^j_{(1)}\chi_{ij}^{(1)} - \rho^{(1)}v_i^{(1)} + 2\chi_{ij}^{(2)}J^j_{(0)} - 2\rho^{(0)}\left(\frac{v_i^{(2)}}{2} + v_i^{(1)}\left(2\phi^{(1)} - \psi^{(1)}\right) - v^j_{(1)}\chi_{ij}^{(1)}\right)
$$

$$
= 2\sigma\left(\frac{1}{2}\left(v^{(2)}\times B^{(0)}\right)_i + \frac{1}{2}\left(\omega^{(2)}\times B^{(0)}\right)_i + \left(v^{(1)}\times B^{(1)}\right)_i + \left(\omega^{(1)}\times B^{(1)}\right)_i\right.
$$

$$
-2E_i^{(1)}\left(\phi^{(1)} - \frac{1}{2}\psi^{(1)}\right) + \frac{1}{2}E_i^{(2)} + E^j_{(1)}\chi_{ij}^{(1)} - \left(2\phi^{(1)} + \psi^{(1)}\right)\left(\left(v^{(1)} + \omega^{(1)}\right)\times B^{(0)}\right)_i
$$

$$
+E_i^{(0)}\left(\frac{1}{2}\psi^{(2)} - \phi^{(2)}\right) + \frac{1}{2}E^j_{(0)}\chi_{ij}^{(2)} - 2\phi^{(1)}\psi^{(1)}E_i^{(0)} + E^j_{(0)}\chi_{ij}^{(1)}\psi^{(1)} + \varepsilon^{ljk}\omega_j^{(1)}B^k_{(0)}\chi_{li}^{(1)}
$$

$$
\left.+\varepsilon^{mlk}v_l^{(1)}B^k_{(0)}\chi_{mi}^{(1)} + \varepsilon_i^{\,lk}v^j_{(1)}B^k_{(0)}\chi_{lj}^{(1)} - \frac{1}{2}\left(\psi^{(1)}\right)^2E_i^{(0)} + E_i^{(0)}\frac{1}{2}v_k^{(1)}v^k_{(1)} + E_j^{(0)}v^j_{(1)}\omega_i^{(1)}\right) \tag{73}
$$

OHM'S LAW TO SECOND ORDER

$$\mathscr{J}_i^{(2)} - 4J_i^{(1)}\Phi^{(1)} - \rho^{(1)}\upsilon_i^{(1)} + S_i^1(J_i^{(1)}, \chi_{ij}^{(1)}, \rho^{(1)})$$

$$= 2\sigma\left(\frac{1}{2}\left(\mathscr{V}_{(2)} \times B^{(0)}\right)_i + \left(\mathscr{V}_{(1)} \times B^{(1)}\right)_i - 2E_i^{(1)}\left(\Phi^{(1)} - \frac{1}{2}\Psi^{(1)}\right)\right.$$

$$\left. + \frac{1}{2}E_i^{(2)} - \left(2\Phi^{(1)} + \Psi^{(1)}\right)\left(\mathscr{V}_{(1)} \times B^{(0)}\right)_i + S_i^2(E_{(1)}^j, J_i^{(1)}, B_i^{(1)}, \chi_{ij}^{(1)}, \rho^{(1)})\right) \tag{74}$$

Maxwell Equations to Second Order

$$\partial_i E^i_{(2)} + E^i_{(0)} \left(\partial_i \psi^{(2)} - 3\partial_i \phi^{(2)} - 4\psi^{(1)} \partial_i \psi^{(1)} - 12\phi^{(1)} \partial_i \phi^{(1)} + 2\omega^l_{(1)} \partial_l \omega^{(1)}_i \right.$$

$$\left. - \chi^{lm}_{(1)} \partial_i \chi^{(1)}_{lm} \right) + E^i_{(1)} \left(4\partial_i \psi^{(1)} - 12\partial_i \phi^{(1)} \right) = a\rho_{(2)} \tag{75}$$

$$\Downarrow$$

$$\partial_i \mathscr{E}^i_{(2)} + 4E^i_{(1)} \partial_i \left(\Psi^{(1)} - 3\Phi^{(1)} \right) + \mathscr{T}_{G1} = a\Delta^{(2)}_\rho \tag{76}$$

With:

$$\mathscr{T}_{G1} = -\partial_i \left(\chi^i_{\perp(2)}\right)' - E^i_{(0)} \left(\partial_i \left(\frac{1}{a}\left(\mathscr{S}^{||}_{(2)} a\right)' + \left(-\frac{1}{2}\nabla^2 \chi^{(2)} + 3H\mathscr{S}^{||}_{(2)}\right)\right)\right)$$

$$-4\left(\frac{1}{a}\left(\mathscr{S}^{||}_{(1)} a\right)'\right)\partial_i\left(\frac{1}{a}\left(\mathscr{S}^{||}_{(1)} a\right)'\right) - 12\left(-\frac{1}{6}\nabla^2\chi^{(1)} + H\mathscr{S}^{||}_{(1)}\right)\partial_i\left(-\frac{1}{6}\nabla^2\chi^{(1)} + H\mathscr{S}^{||}_{(1)}\right)$$

$$+2\left(\chi^l_{\perp(1)}\right)'\partial_l\left(\chi_i^{\perp(1)}\right)' + a\left(\rho_{(0)}\right)'\mathscr{S}^{||}_{(2)} - \chi^{lm}_{(1)}\partial_i\chi^{(1)}_{lm}$$

$$-4E^i_{(1)}\partial_i\left(\frac{1}{a}\left(\mathscr{S}^{||}_{(1)} a\right)' + \left(-\frac{1}{2}\nabla^2\chi^{(1)} + 3H\mathscr{S}^{||}_{(1)}\right)\right)$$

Maxwell Equation to Second Order

$$\varepsilon^{ilk}\partial_l B^k_{(2)} - \left(E^i_{(2)}\right)' - 2HE^i_{(2)} - 2E^i_{(0)}\left(\frac{1}{2}\left(\psi^{(2)}\right)' - \frac{3}{2}\left(\phi^{(2)}\right)' - 2\psi^{(1)}\left(\psi^{(1)}\right)'\right.$$

$$\left. -6\phi^{(1)}\left(\phi^{(1)}\right)' + \omega^k_{(1)}\left(\omega^{(1)}_k\right)' - \frac{1}{2}\chi^{lk}_{(1)}\left(\chi^{(1)}_{lk}\right)'\right) + 2\varepsilon^{ilk}B^k_{(0)}\left(\frac{1}{2}\partial_l\psi^{(2)}\right.$$

$$\left. -\frac{3}{2}\partial_l\phi^{(2)} - 2\psi^{(1)}\partial_l\psi^{(1)} - 6\phi^{(1)}\partial_l\phi^{(1)} + \omega^m_{(1)}\partial_m\omega^{(1)}_l - \frac{1}{2}\chi^{lm}_{(1)}\partial_i\chi^{(1)}_{lm}\right)$$

$$-2E^i_{(1)}\left(2\left(\psi^{(1)}\right)' - 6\left(\phi^{(1)}\right)'\right) + 2\varepsilon^{ilk}B^k_{(1)}\partial_l\left(2\psi^{(1)} - 6\phi^{(1)}\right) = aj^i_{(2)} \qquad (77)$$

$$\Downarrow$$

$$\left(\nabla\times\mathscr{B}^{(2)}_k\right)^i - \left(\mathscr{E}^i_{(2)}\right)' - 2H\mathscr{E}^i_{(2)} - 2E^i_{(1)}\left(2\left(\Psi^{(1)}\right)' - 6\left(\Phi^{(1)}\right)'\right)$$

$$-2\left(\nabla\times B_{(1)}\left(2\Psi^{(1)} - 6\Phi^{(1)}\right)\right)^i = a\mathscr{J}^i_{(2)} + S^{i3}(\mathscr{S}^{||}_{(2)}, J^i_{(1)}, \mathscr{S}^{||}_{(1)}, E^i_{(1)}, B^i_{(1)}) \qquad (78)$$

Maxwell Equation to Second Order

We found the first homogeneous Maxwell equation given by:

$$\partial_i B_i^{(2)} + \partial_j B_j^{(2)} + \partial_k B_k^{(2)} = 0 \tag{79}$$

$$\Downarrow$$

$$\partial_i \mathscr{B}_i^{(2)} + \partial_j \mathscr{B}_j^{(2)} + \partial_k \mathscr{B}_k^{(2)} = -S(\left(\chi^i_{\perp(2)}\right)', B_j^{(1)}) \tag{80}$$

The second homogeneous Maxwell equation is given by:

$$B'_{k(2)} + 2HB_k^{(2)} + \varepsilon_k^{ij} \partial_i E_j^{(2)} = 0$$

$$\Downarrow$$

$$\left(\mathscr{B}_k^{(2)}\right)' + 2H\left(\mathscr{B}_k^{(2)}\right) + \left(\nabla \times \mathscr{E}_{j(2)}\right)_k = -S_k^4(\left(\chi_{k\perp(2)}\right)', E_k^{(1)}, B_k^{(1)}) \tag{81}$$

"Cosmological Dynamo" to Second Order

Using the above Maxwell equations, we obtain the follow relation for the evolution of magnetic fields to second order:

$$\begin{aligned}
&\left(\mathscr{B}_k^{(2)}\right)' + 2H\left(\mathscr{B}_k^{(2)}\right) + \eta\Bigg[\nabla\times\Bigg(\frac{1}{a}\Bigg(\left(\nabla\times\mathscr{B}^{(2)}\right) - 2E_{(1)}\left(2\left(\Psi^{(1)}\right)' - 6\left(\Phi^{(1)}\right)'\right) \\
&-\left(\mathscr{E}_{(2)}\right)' - 2H\mathscr{E}_{(2)} - 2\left(\nabla\times B_{(1)}\left(2\Psi^{(1)} - 6\Phi^{(1)}\right)\right) - S^3(\mathscr{S}^{||}_{(2)}, J^i_{(1)}, \mathscr{S}^{||}_{(1)}, E^i_{(1)}, B^i_{(1)})\Bigg) \\
&-\rho^{(1)}\upsilon^{(1)} + S^1\left(J_i^{(1)}, \chi_{ij}^{(1)}, \rho^{(1)}\right)\Bigg)\Bigg]_k + \Bigg(\nabla\times\Big[-4\left(E^{(1)} + \mathscr{V}_{(1)}\times B^{(0)}\right)\Phi^{(1)} - \left(\mathscr{V}_{(2)}\times B^{(0)}\right) \\
&-2\left(\mathscr{V}_{(1)}\times B^{(1)}\right) + \left(2\Phi^{(1)} + \Psi^{(1)}\right)\left(\mathscr{V}_{(1)}\times B^{(0)}\right) + 4E^{(1)}\left(\Phi^{(1)} - \frac{1}{2}\Psi^{(1)}\right) \\
&-2S^2(E^j_{(1)}, J_i^{(1)}, B_i^{(1)}, \chi_{ij}^{(1)}, \rho^{(1)})\Big]\Bigg)_k = -S^4_k\left(\left(\chi^i_{\perp(2)}\right)', E_j^{(1)}, B_j^{(1)}\right) \qquad (82)
\end{aligned}$$

We find that perturbations in the space-time plays an important role in the evolution of primordial magnetic fields

Summary

- We find an expression similar to the dynamo equation up to second order, and clearly perturbations in the metric affect the evolution of the primordial magnetic fields.

Summary

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Summary

- We find an expression similar to the dynamo equation up to second order, and clearly perturbations in the metric affect the evolution of the primordial magnetic fields.
- Due to existence of geometrical perturbations in the dynamo equation, we need to resolve simultaneously the Maxwell equation and Einstein Field Equations because they are coupled.
- The Einstein and Maxwell Equations were found up to second order in terms of Gauge Invariant Quantities.

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