FERMIONS IN WARPED SPACETIME AND THE SMALLNESS OF THE COSMOLOGICAL CONSTANT

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Based on: *SD et. al. JHEP* **05** *(2008), 042*

RK et. al. Phys. Rev. **D 79** *(R), 041902 (2009)*

OUTLINE

- Warped spacetime :
  - Braneworlds : Randall-Sundrum (RS) model
  - Solution of gauge hierarchy problem
  - Generalised RS
- Standard model fields in warped spacetime
  - Fermions on brane
  - The mechanism of confinement
  - Localization in generalised RS
- Smallness of cosmological constant: linked with fermion confinement
- Summary

WARPED SPACETIME : BRANEWORLDS

- The basis of Einstein's theory of General Relativity
- Most familiar example : FRW metric

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2) \quad (1)$$

- Example of a five dimensional warped spacetime :

$$ds^2 = d\sigma^2 + e^{2f(\sigma)}(-dt^2 + dx^2 + dy^2 + dz^2) \quad (2)$$

$\Rightarrow$ The 4D spacetime is warped by the factor $f(\sigma)$

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Gives a novel solution of the Gauge Hierarchy problem

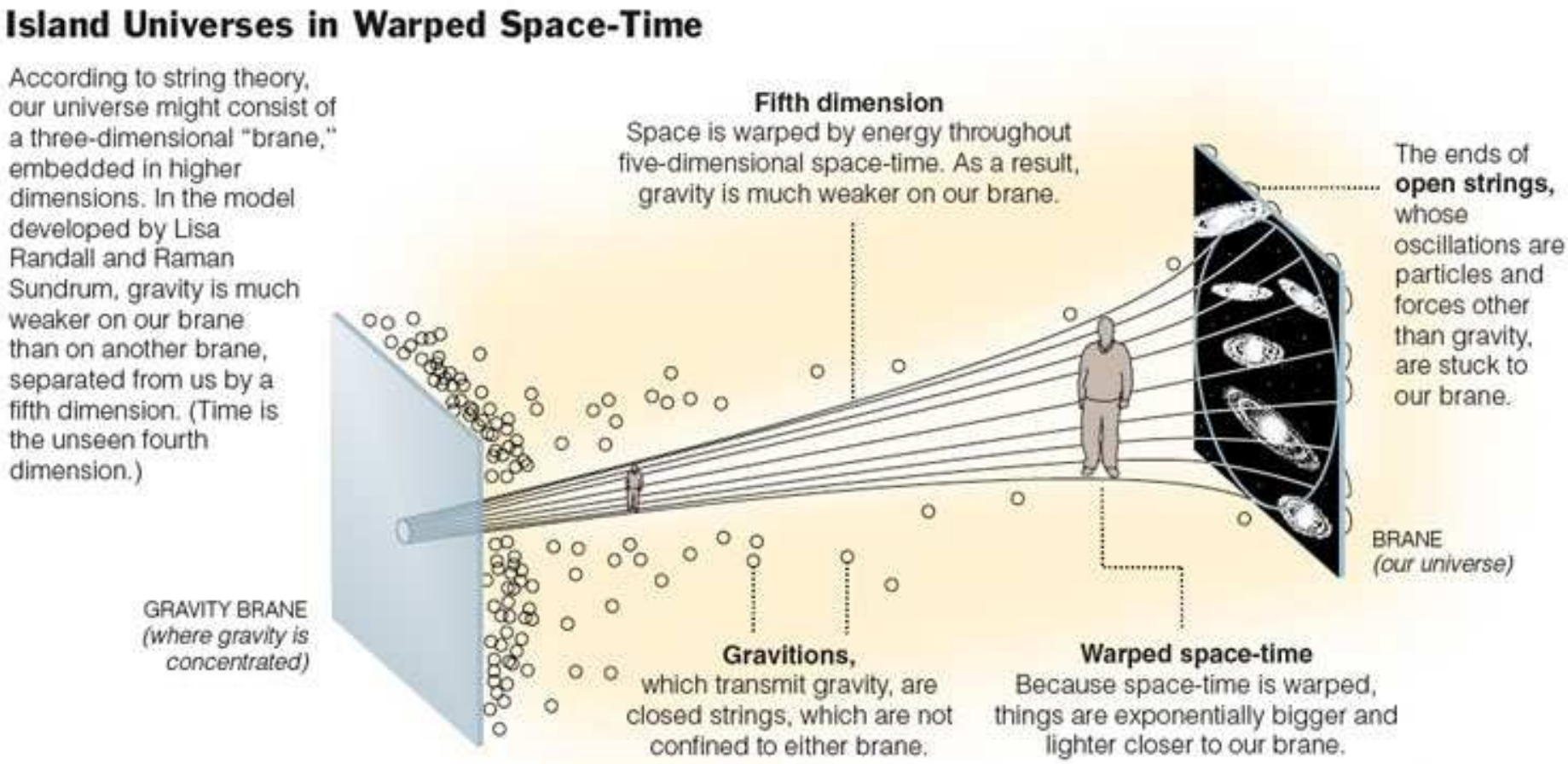

Figure courtsey : cosmicvariance by S. M. Carroll

RS MODEL : SOLUTION OF GAUGE HIERARCHY PROBLEM

Some intriguing question about our physical universe

- Why does it appear to have (3 +1) spacetime dimensions ? Are there additional unobserved dimensions?
- Why the ratio of electroweak scale/Higgs mass (m) to Planck mass (m_0) so tiny ($\simeq 10^{-16}$) ? $\Rightarrow$ *Gauge Hierarchy Problem*

♦ Randall-Sundrum model : 5D spacetime

$$ds^2 = r_c^2 d\sigma^2 + e^{-2kr_c|\sigma|}(-dt^2 + dx^2 + dy^2 + dz^2) \qquad (3)$$

♦ RS prescription : masses get scaled on the visible brane by the warp factor as

$$m = e^{-kr_c\pi} m_0 \quad where \quad kr_c \simeq 11.84$$

GENERALISED RS MODEL

RS $\rightarrow$ visible brane is flat $\rightarrow$ inconsistent with present value

Generalised RS model :

◊ visible brane $\rightarrow$ non-flat

◊ visible brane tension $\rightarrow$ can be both positive/negative

◊ The 5D spacetime metric :

$$ds^2 = e^{-2A(y)} g_{\mu\nu} dx^\mu dx^\nu + r^2 dy^2 \tag{4}$$

◊ The warp factor $e^{-2A(\sigma)}$ obtained by extremising the action :

$$S = \int d^5x \sqrt{-G}(M^3\mathcal{R} - \Lambda) + \int d^4x \sqrt{-g_i}\mathcal{V}_i \tag{5}$$

$\Lambda \rightarrow$ bulk cosmological constant, $\mathcal{R} \rightarrow$ bulk Ricci scalar and $\mathcal{V}_i \rightarrow$ tension of the i^{th} brane.

◊ For AdS bulk ($\Lambda < 0$) & -ve induced curvature Ω on visible brane

$$e^{-A} = \omega \cosh\left(\ln \frac{\omega}{c_1} + ky\right) \tag{6}$$

where $k = \sqrt{\frac{-\Lambda}{12M^3}}$, $\omega^2 \equiv -\Omega/3k^2$ and $c_1 = 1 + \sqrt{1-\omega^2}$

◊ The hierarchy problem has been solved for

$k\pi r_1 \simeq 36.84 + 10^{-93}$, $k\pi r_2 = 250.07$

◊ visible brane tension $\rightarrow$ + ve for second solution.

◊ For AdS bulk ($\Lambda < 0$) & +ve induced curvature Ω on visible brane

$$e^{-A} = \omega \sinh\left(\ln \frac{c_2}{\omega} - ky\right) \tag{7}$$

where $\omega^2 \equiv \Omega/3k^2$ and $c_2 = 1 + \sqrt{1+\omega^2}$.

Curvature on the brane is parametrized by ω^2

Brane cosmological constant, can be of arbitrary magnitude.

STANDARD MODEL FIELDS IN WARPED SPACETIME

STUDY LOCALIZATION : why?

Extra dimensions are hidden from us

$\Rightarrow$ Kaluza-Klein compactification

Assumption in warped braneworld models:

- Gravity can access the whole bulk spacetime
- But SM fields are localized on the brane

  The idea is borrowed from string theory (D-branes)

  Localization of fields $\Rightarrow$ alternative to compactification

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Are the fields really localized in different braneworld models ?

THE MECHANISM : FOR FERMIONS

- Consider that the field can access the whole bulk

  Consider fermion : $\Psi(x^\mu, y) = \psi(x^\mu)\xi(y)$

- Express the action of the field in canonical form

  $$\int \sqrt{-g}\mathcal{L}_{Dirac} d^5x = \int \sqrt{-g}\,(i\bar{\Psi}\Gamma^a \mathcal{D}_a \Psi) d^5x$$

- Extract out the standard four dimensional part

  $$i\gamma^\mu \partial_\mu \psi(x^\mu) = m\psi(x^\mu)$$

- Rest of the integral gives the localization condition

  $$\int e^{-3A(y)} \xi_m \xi_n dy = \delta_{mn}$$

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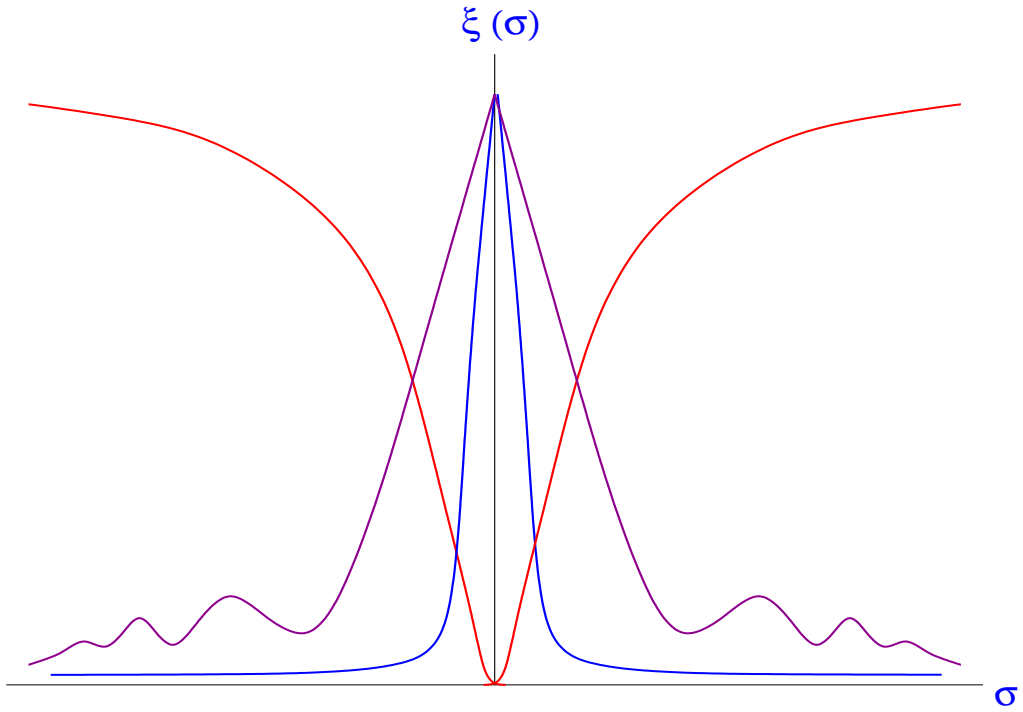
Whether this function, $\xi(y)$, satisfies the localization condition ?

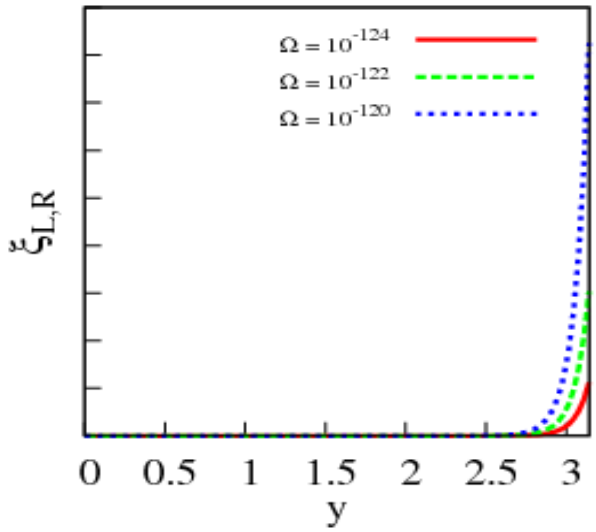
Functional dependence & localization scenario

- Function sharply peaked at the brane $\Rightarrow$ Localized
- Function not sharply peaked at the brane $\Rightarrow$ Quasi localized
- Function grows along the extra dimension $\Rightarrow$ Not localized



FERMION LOCALISATION IN GENERALISED RS MODEL

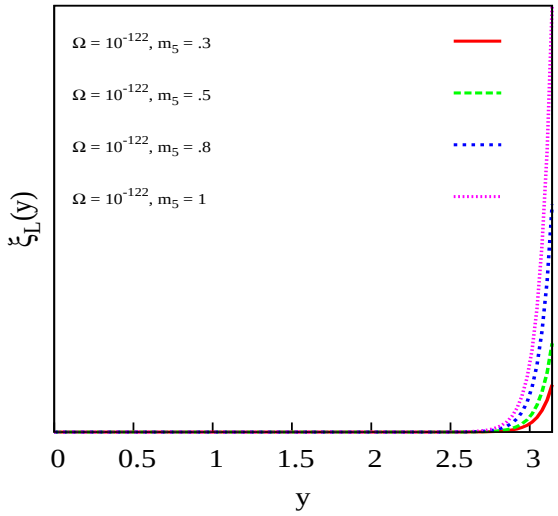
- Massless bulk fermion in AdS 3-brane model: left and right chiral massless modes turn out to be $\xi_{L,R}(y) = N_1 sech^2\left(\ln\frac{\omega}{c_1} + ky\right)$



$\Rightarrow$ As magnitude of Ω becomes close to 10^{-124} the zero modes get sharply localized on the visible brane with -ve brane tension.

But, for positive tension brane fermions are clearly localized deep inside the bulk space-time

♦ Massive bulk fermion in AdS 3-brane model: left and right chiral massless modes : $\xi_{L,R}(y) = N_3 \; sech^2 \left(\ln \frac{\omega}{c_1} + ky \right) e^{\pm m_5 y}$

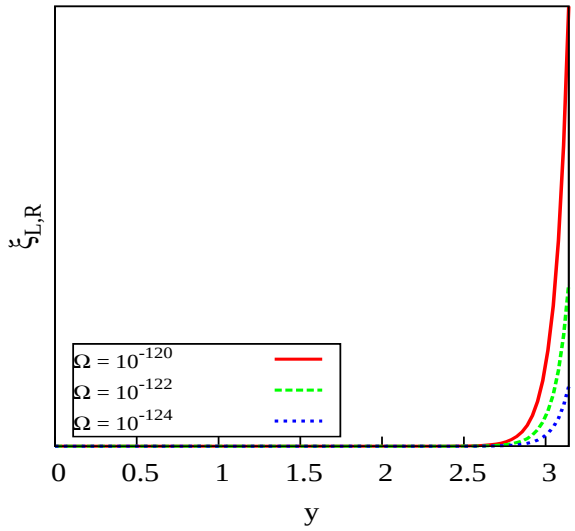


▲ Degeneracy between two chiral modes lifted by the mass term.

▲ For a small value of Ω as bulk fermion becomes more and more massive left chiral mode has higher peak values on visible brane

▲ The right chiral mode shows exactly the reverse nature.

♦ Massless bulk fermion in dS 3-brane model: left and right chiral massless modes : $\xi_{L,R}(y) = N_2 \; cosech^2 \left(\ln \frac{c_2}{\omega} + ky \right)$



▲ As the value of Ω becomes smaller the fermions become more sharply localized on the visible brane.

▲ Thus the fact that our universe has a very small and positive curvature $\Rightarrow$ the bulk fermions are likely to get localized near the visible brane just as predicted in string theory.

SUMMARY

- Studied crucial features of a brane with non-zero brane curvature
- Profiles of massless and massive modes of bulk fermions studied in generalised RS model
- Fermions get localized near the visible brane for a very small positive value of the brane curvature.
- This in an important finding for the explanation of the smallness of the cosmological constant in connection with the brane localization of the fermion in phenomenological braneworld model.