A unique $\mathbb{Z}_4^R$ symmetry for the MSSM

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Based on:

- M. Blaszczyk, S. Groot Nibbelink, F. Ruehle, M.R., M. Trapletti & P. Vaudrevange, Phys. Lett. B 683, 340-348 (2010)
- F. Brümmer, R. Kappl, M.R. & K. Schmidt-Hoberg, JHEP 1004:006 (2010)
- H.M. Lee, S. Raby, G. Ross, M.R., R. Schieren & P. Vaudrevange, http://arxiv.org/abs/1009.0905 & to appear
- R. Kappl, B. Petersen, M.R., R. Schieren & P. Vaudrevange, to appear

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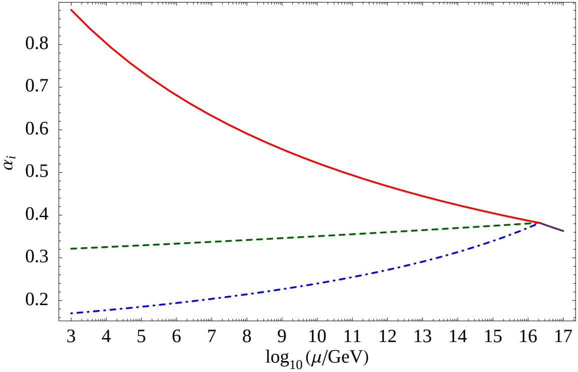

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↪ Supersymmetry alone seems not to be enough

Outline

1. Introduction & Motivation ✓
2. A simple $\mathbb{Z}_4^R$ symmetry can explain
   - suppressed μ term
   - proton stability
3. String theory realization
4. Summary

Proton decay operators

- Gauge invariant superpotential terms up to order 4 include

$$
\begin{aligned}
\mathscr{W} \;=\; & \mu \overline{H} H + \kappa_i L_i H \\
& + Y_e^{ij} L_i \overline{H} \overline{E}_j + Y_d^{ij} Q_i \overline{H} \overline{D}_j + Y_u^{ij} Q_i H \overline{U}_j \\
& + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ij}^{(0)} H L_i H L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell
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forbidden by matter parity

Farrar & Fayet (1978); Dimopoulos, Raby & Wilczek (1981)

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- Proton hexality = matter parity + baryon triality

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☞ Proton hexality P_6 = matter parity $\mathbb{Z}_2^\mathcal{M}$ × baryon triality B_3

	Q	U	D	L	E	H	$\overline{H}$	$\overline{\nu}$
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B_3	0	−1	1	−1	2	1	−1	0
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    . . . with the common notion of anomaly freedom

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- ☹ not consistent with unification for matter (i.e. inconsistent with universal discrete charges for all matter fields)

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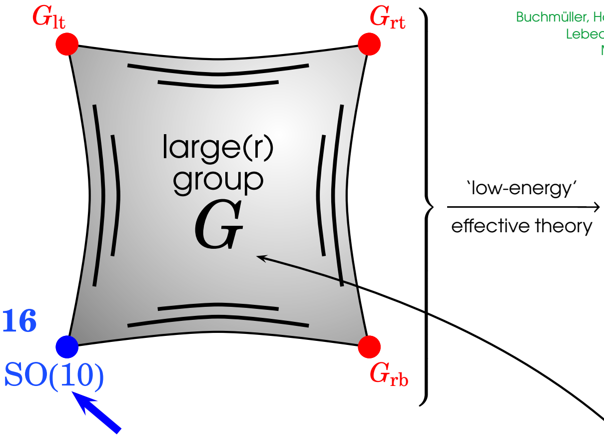
  - ☹ embedding into string theory not yet fully convincing

Förste, Nilles, Ramos-Sánchez, Vaudrevange (2010)

Local grand unification (using **small** extra dimensions)

Buchmüller, Hamaguchi, Lebedev, M.R. (2004-2006)
Lebedev, Nilles, Raby, Ramos-Sánchez, M.R., Vaudrevange, Wingerter (2006)



standard model as an intersection of G_{rb}, G_{rt}, G_{lt} & $\mathrm{SO}(10)$ in G

SM generation(s):

localized in region with $\mathrm{SO}(10)$ symmetry

Higgs doublets:

live in the 'bulk'

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need to be strongly suppressed

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↪ Two prejudices from string model building:

1. Local Grand Unification
2. 'anomalous' discrete symmetries whose anomalies are canceled the Green-Schwarz mechanism

From anomaly freedom to anomaly universality

Dine & Graesser (2004); Araki, Kobayashi, Kubo, Ramos-Sánchez, M.R. & Vaudrevange (2008)

- ☞ Important lesson from explicit string-derived (MSSM) models

'anomalous' discrete symmetries:

Anomalies of discrete symmetries canceled by Green-Schwarz mechanism

- ☞ The `anomalies' in the discrete symmetries do **not** arise from a mixing with the `anomalous' $\mathrm{U}(1)$

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$$A_{\text{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

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sum over all representations of G

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sum over all fermions

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Dynkin index

discrete charges

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$$\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

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⬇ ⬇ ⬇ ⬇ ⬇

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Green-Schwarz anomaly cancellation

- ☞ Under 'anomalous' $\mathrm{U}(1)$ symmetry the path integral measure exhibits non-trivial transformation Fujikawa (1979)

$\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi} \;\to\; J(\alpha)\,\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi}$ with non-trivial $J(\alpha)$

Green-Schwarz anomaly cancellation

- Under 'anomalous' $\mathrm{U}(1)$ symmetry the path integral measure exhibits non-trivial transformation Fujikawa (1979)
- One can absorb the change of the path integral measure in a change of Lagrangean

$$\Delta\mathscr{L}_{\text{anomaly}} = \frac{\alpha}{32\pi^2} F_{\text{anom}} \widetilde{F}_{\text{anom}}\, A_{\mathrm{U}(1)^3_{\text{anom}}} + \underbrace{\sum_G}_{\text{sum over all gauge factors}} \frac{\alpha}{32\pi^2} F^a \widetilde{F}^a\, A_{G-G-\mathrm{U}(1)_{\text{anom}}} - \frac{\alpha}{384\pi^2} \mathcal{R}\widetilde{\mathcal{R}}\, A_{\text{grav}-\text{grav}-\mathrm{U}(1)_{\text{anom}}}$$

sum over all gauge factors

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- Provided the Lagrangean also includes axion couplings

$$\mathscr{L} \supset -\frac{a}{8} F_{\mathrm{anom}} \widetilde{F}_{\mathrm{anom}} - \frac{a}{8} F^a \widetilde{F}^a + \frac{a}{4} \mathcal{R}\widetilde{\mathcal{R}}$$

$\Delta\mathscr{L}_{\text{anomaly}}$ can be compensated by a shift of the axion a if the anomaly coefficients are universal

Green & Schwarz (1984)

Discrete GS anomaly cancellation

☞ The analysis applies also for discrete symmetries

Discrete GS anomaly cancellation

- The analysis applies also for discrete symmetries
- Specifically for a $\mathbb{Z}_N$ transformation

$$\Phi^{(f)} \;\to\; \mathrm{e}^{-\mathrm{i}\,\frac{2\pi}{N}\,q^{(f)}}\,\Phi^{(f)}$$

the dilaton (containing the axion) has to transform as

$$S \;\to\; S + \frac{\mathrm{i}}{2}\Delta_{\mathrm{GS}}$$

where

$$\pi\,N\,\Delta_{\mathrm{GS}} \;\equiv\; \frac{1}{24}A_{\mathrm{grav-grav-}\mathbb{Z}_N} \;=\; A_{G-G-\mathbb{Z}_N} \mod \eta \quad \forall\; G$$

A unique $\mathbb{Z}_4^R$ symmetry

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- Yukawa couplings and Weinberg neutrino mass operator allowed

A unique $\mathbb{Z}_4^R$ symmetry

☞ Assumptions:

- anomaly universality $\Longleftrightarrow$ GS anomaly cancellation
- universal charges for quarks and leptons
- μ term forbidden at perturbative level
- Yukawa couplings and Weinberg neutrino mass operator allowed

Want to prove:

There is a unique $\mathbb{Z}_4^R$ symmetry in the MSSM with these features

Claim 1: it has to be an R symmetry

☞ Anomaly coefficients for non-R symmetry with SU(5) relations for matter charges

$$
\begin{aligned}
A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} &= \frac{9}{2} q_{\mathbf{10}} + \frac{3}{2} q_{\overline{\mathbf{5}}} \\
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- Anomaly universality

$$\begin{aligned} A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} &= 0 \\ \curvearrowright \quad \frac{1}{2}\left(q_H + q_{\overline{H}}\right) &= 0 \mod \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases} \end{aligned}$$

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bottom-line:

non-R $\mathbb{Z}_N$ symmetry cannot forbid μ term

Claim 2: Higgs discrete charges have to vanish

- Assumption: quarks and leptons have universal charge q

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Claim 3: The order has to be 4 (or 2)

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$$\begin{aligned} A_{\mathrm{SU}(3)^2-\mathbb{Z}_N^R} &= 6(q-1)+3 = 6q-3 \\ A_{\mathrm{SU}(2)^2-\mathbb{Z}_N^R} &= 6q+\frac{1}{2}\left(q_H+q_{\overline{H}}\right)-5 \end{aligned}$$

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$N = 2$ or $N = 4$

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$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N^R} - A$$

however: there is no meaningful $\mathbb{Z}_2^R$ symmetry

cf. e.g. Dine & Kehayias (2009)

$$\Leftrightarrow \; q_H + q_{\overline{H}} = 1 \mod \left\{ \; N \quad \text{for } N \text{ even} \right.$$

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$N = 4$ unique

Unique $\mathbb{Z}_4^R$ symmetry

☞ We know:

- it is a $\mathbb{Z}_4^R$ symmetry
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- Consistent with anomaly universality

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$\boxed{\text{e.g. } q_H = q_{\overline{H}} = 16}$

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\frac{1}{24}A_{\mathrm{grav}^2-\mathbb{Z}_N^R} &= \frac{1}{24}\left[-21 + 8 + 3 + 1 + 48(q-1) + 2(q_H + q_{\overline{H}} - 2) - 1\right]
\end{aligned}
$$

gravitino contribution

gaugino contributions

only defined $\mod 4$

axino contribution

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A ... 2

$\frac{1}{24}$ A ...

bottom-line:

- $\mathbb{Z}_4^R$ is anomaly free via GS mechanism
- GS axino contribution important for gravitational anomaly

$\mathbb{Z}_4^R$ literature

- Anomaly-free version of this $\mathbb{Z}_4^R$ with extra matter has been discussed previously Kurosawa, Maru & Yanagida (2001)

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  - no discussion of mixed hypercharge nor gravitational anomalies

Comment on schemes with SU(5) relations

- Using a similar strategy and demanding only SU(5) rather than SO(10) relations one can show that the order N of possible $\mathbb{Z}_N^R$ symmetries has to divide 24

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- There are only five viable charge assignments

N	$q_{\mathbf{10}}$	$q_{\overline{\mathbf{5}}}$	q_H	$q_{\overline{H}}$	ρ	$A_0^R(\mathrm{MSSM})$
4	1	1	0	0	1	1
6	5	3	4	0	0	1
8	1	5	0	4	1	3
12	5	9	4	0	3	1
24	5	9	16	12	9	7

Recall

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} q^{(f)} \stackrel{!}{=} \rho \mod \eta$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} \rho \mod \eta$$

Comment on schemes with SU(5) relations

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- N divides 24: hint at realization of $\mathbb{Z}_N^R$ as discrete rotational symmetry in orbifolds

(The geometry of orbifolds with $\mathcal{N} = 1$ SUSY is constrained that the order of discrete R symmetries also divides 24)

Implications of $\mathbb{Z}_4^R$

- Gauge invariant superpotential terms up to order 4

$$
\begin{aligned}
\mathscr{W} \;=\; & \mu\,\overline{H}H + \kappa_i\,L_iH \\
& + Y_e^{ij}\,L_i\overline{H}\overline{E}_j + Y_d^{ij}\,Q_i\overline{H}\overline{D}_j + Y_u^{ij}\,Q_iH\overline{U}_j \\
& + \lambda_{ijk}\,L_iL_j\overline{E}_k + \lambda'_{ijk}\,L_iQ_j\overline{D}_k + \lambda''_{ijk}\,\overline{U}_i\overline{D}_j\overline{D}_k \\
& + \kappa_{ij}^{(0)}\,HL_i\,HL_j + \kappa_{ijk\ell}^{(1)}\,Q_iQ_jQ_kL_\ell + \kappa_{ijk\ell}^{(2)}\,\overline{U}_i\overline{U}_j\overline{D}_k\overline{E}_\ell + \ldots
\end{aligned}
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forbidden at the perturbative level

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& + \kappa_{ij}^{(0)} H L_i H L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \dots
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$$

appear at non-perturbative level

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\end{aligned}
$$

also forbidden at
non-perturbative level by
non-anomalous $\mathbb{Z}_2$ subgroup
which is equivalent
to matter parity

Implications of $\mathbb{Z}_4^R$

- Gauge invariant superpotential terms up to order 4

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non-perturbative generation of μ solves the μ problem

Implications of $\mathbb{Z}_4^R$

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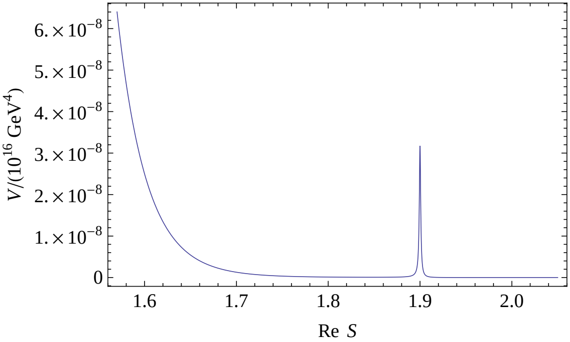
non-perturbatively generated terms harmless

Minimal realization of $\mathbb{Z}_4^R$

☞ MSSM + Kähler stabilized dilaton



- non-perturbative corrections to the Kähler potential lead to a bump in the potential of $\mathrm{Re}\,S$
- $\mathrm{Im}\,S$ has a flat potential $\curvearrowright$ GS axion remains light

Minimal realization of $\mathbb{Z}_4^R$

- MSSM + Kähler stabilized dilaton

- Non-perturbative superpotential

  $$\mathscr{W}_{\mathrm{np}} \supset M_{\mathrm{P}}^3\, \mathrm{e}^{-b\,S}$$

  is $\mathbb{Z}_4^R$ covariant (i.e. has R charge 2) as $S \to S + \frac{\mathrm{i}}{2}\Delta_{\mathrm{GS}}$
- Comments:
  - Of course $\mathscr{W}_{\mathrm{np}}$ is just the effective description of some hidden sector strong dynamics
  - $\mathbb{Z}_4^R$ anomaly universality leads to non-trivial constraints on the (β-function) coefficient b

Minimal realization of $\mathbb{Z}_4^R$

- MSSM + Kähler stabilized dilaton
- Non-perturbative superpotential

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is $\mathbb{Z}_4^R$ covariant (i.e. has R charge 2) as $S \to S + \frac{\mathrm{i}}{2}\Delta_{\mathrm{GS}}$

- Effective μ term and $QQQL$ coefficients

$$\mathscr{W}_{\mathrm{np}} \supset A\, M_{\mathrm{P}}\, \mathrm{e}^{-b\,S}\, \overline{H}H + M_{\mathrm{P}}^{-1}\, \mathrm{e}^{-b\,S}\, \overline{\kappa}_{ijk\ell}^{(1)}\, Q_iQ_jQ_kL_\ell + \dots$$

are also $\mathbb{Z}_4^R$ covariant

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- MSSM + Kähler stabilized dilaton
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$$\mathscr{W}_{\mathrm{np}} \supset A\,M_{\mathrm{P}}\,\mathrm{e}^{-b\,S}\,\overline{H}H + M_{\mathrm{P}}^{-1}\,\mathrm{e}^{-b\,S}\,\widetilde{\kappa}_{ijk\ell}^{(1)}\,Q_iQ_jQ_kL_\ell + \dots$$

are also $\mathbb{Z}_4^R$ covariant

- Non-trivial vacuum expectation value of $\mathscr{W}_{\mathrm{np}}$ is a measure for $\mathbb{Z}_4^R$ breaking and the gravitino mass

Minimal realization of $\mathbb{Z}_4^R$

- MSSM + Kähler stabilized dilaton
- Non-perturbative superpotential

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- $\langle\mathscr{W}\rangle$ breaks $\mathbb{Z}_4^R$ down to matter parity
- Why is $\mu \sim \langle\mathscr{W}\rangle$?

Explicit

string theory

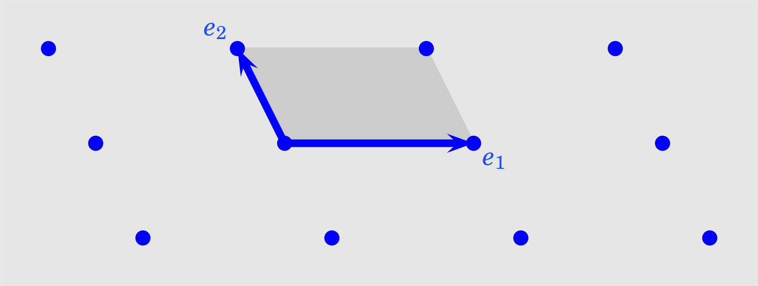
realization

- origin of $\mathbb{Z}_4^R$
- higher-dimensional operators (effective μ term etc.)

The $\mathbb{Z}_2$ orbifold plane

2D space with SO(2) rotational symmetry

The $\mathbb{Z}_2$ orbifold plane



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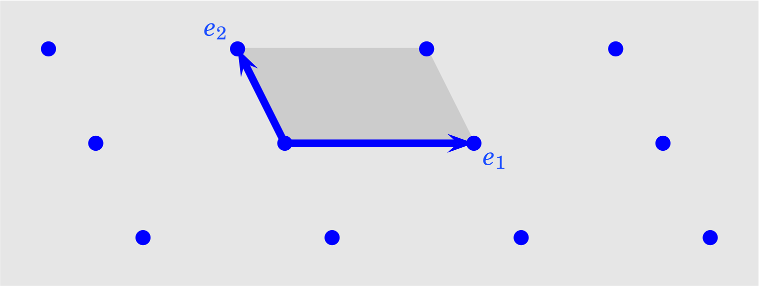

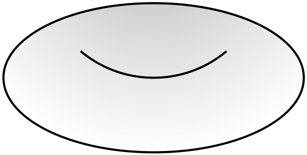

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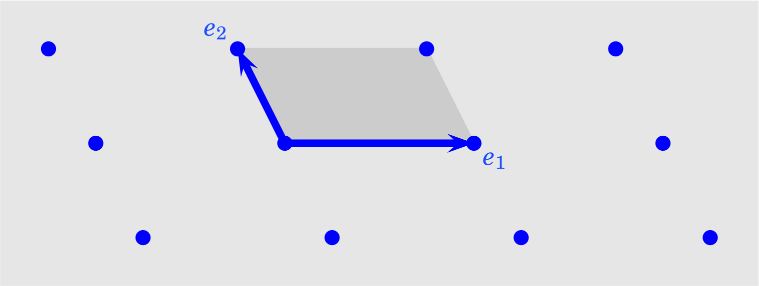

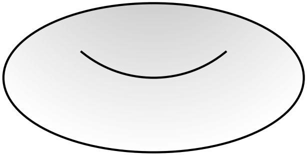

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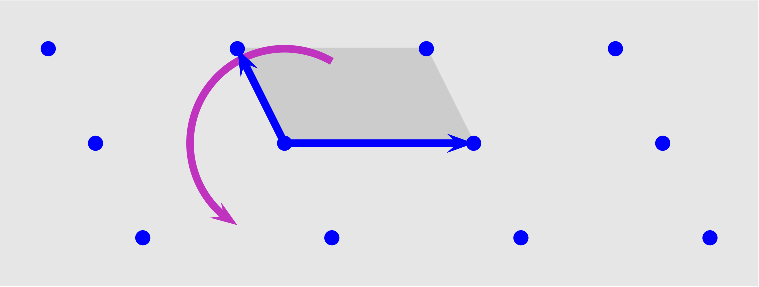

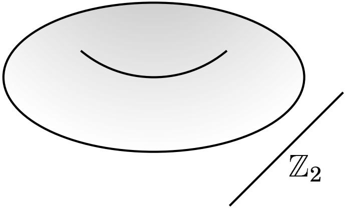

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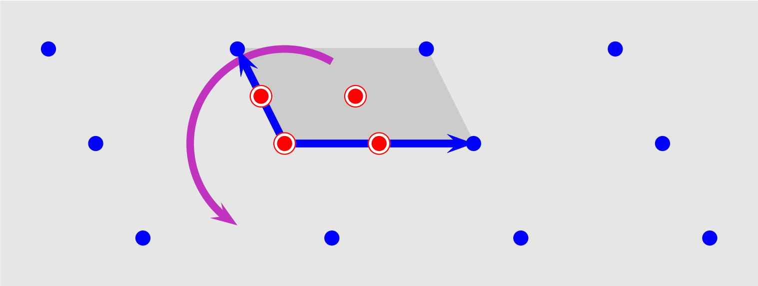

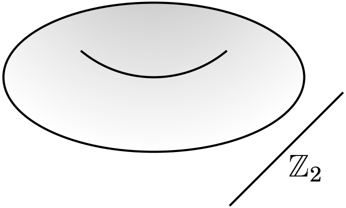

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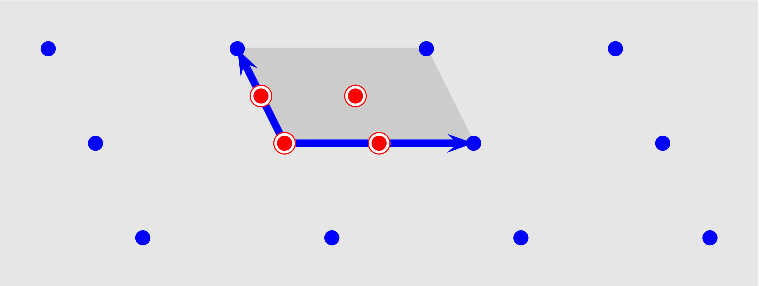
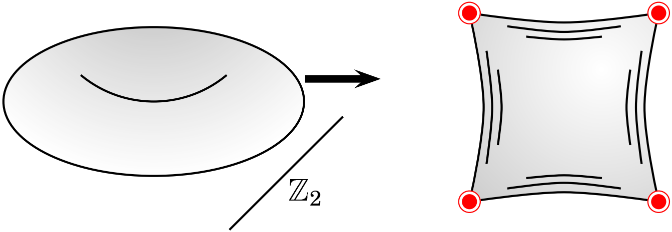
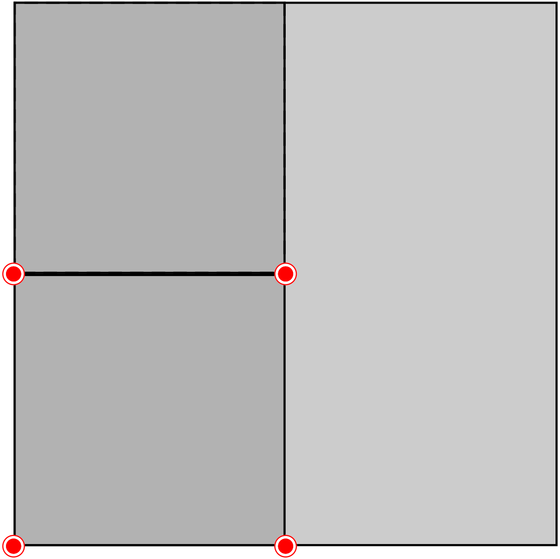




$\mathbb{Z}_2$ orbifold pillow



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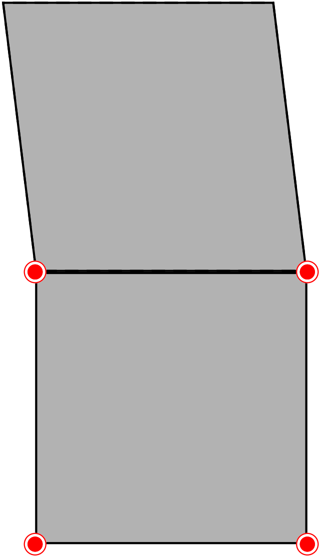

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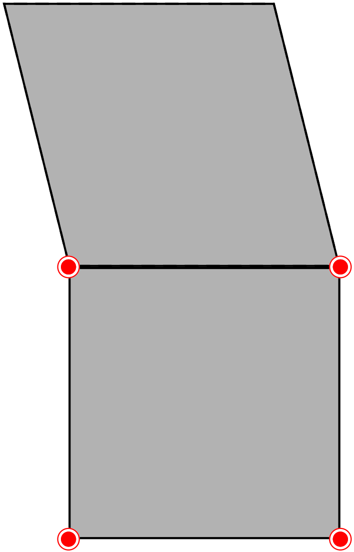

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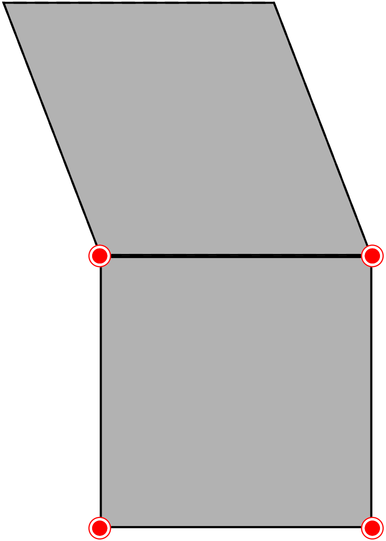

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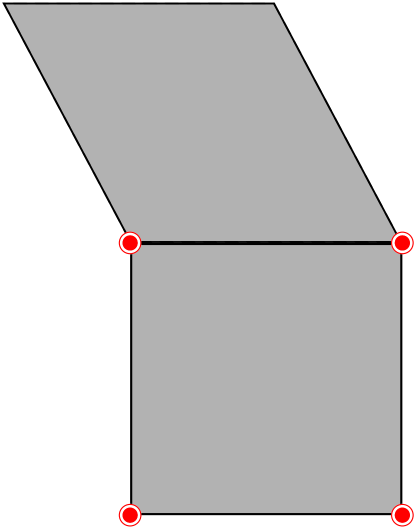

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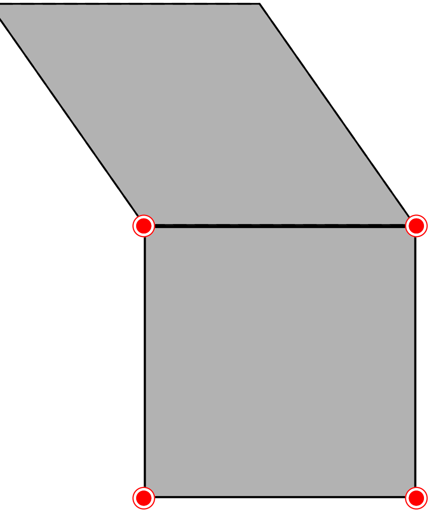

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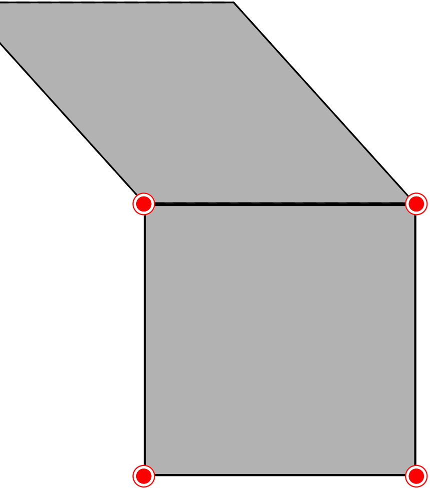

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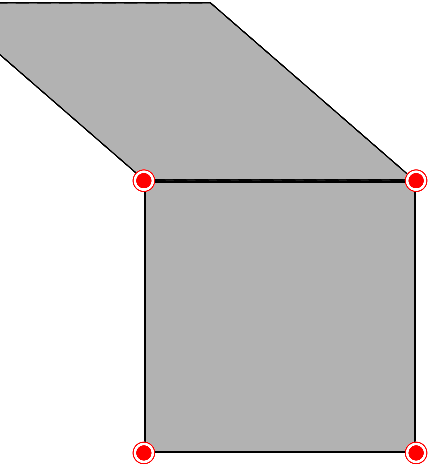

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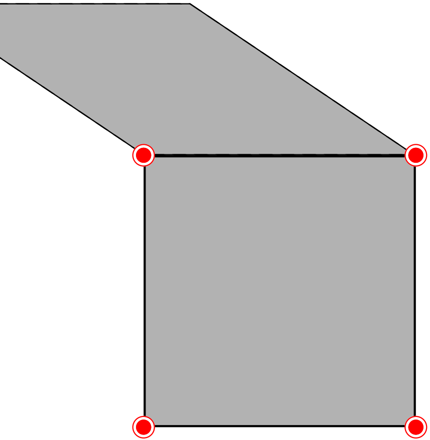

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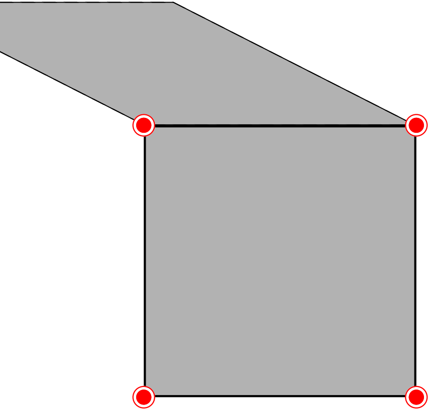

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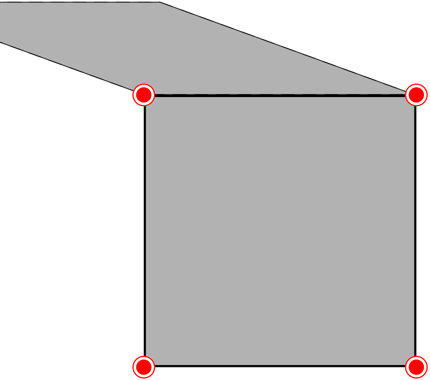

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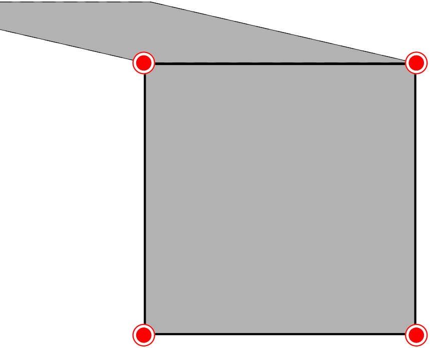

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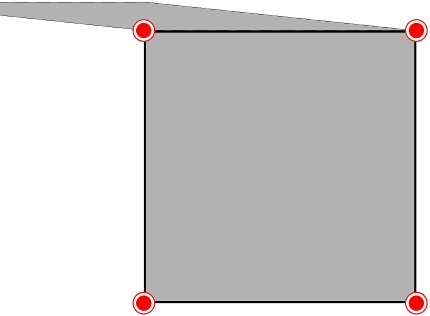

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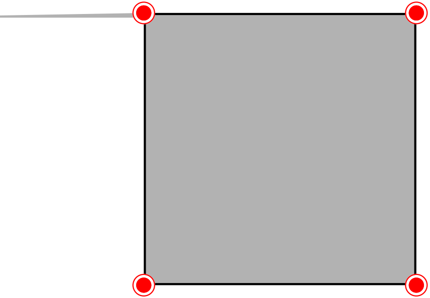

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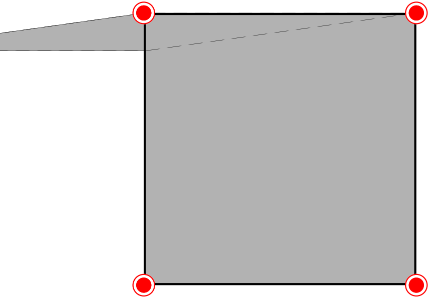

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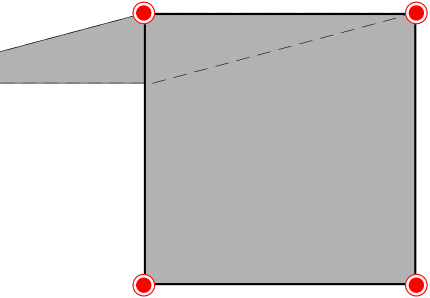

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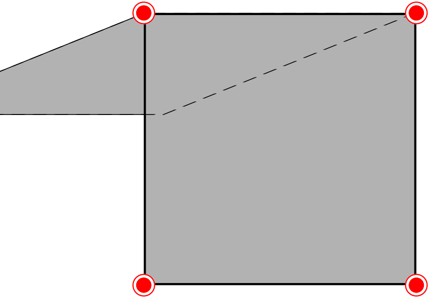

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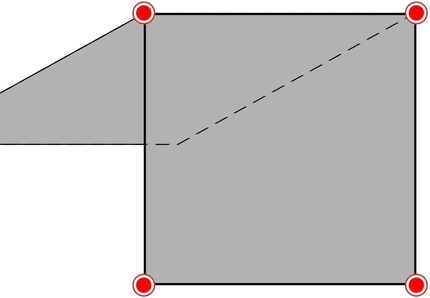

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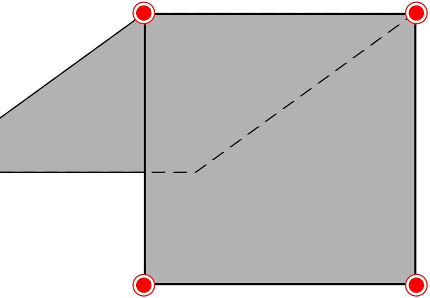

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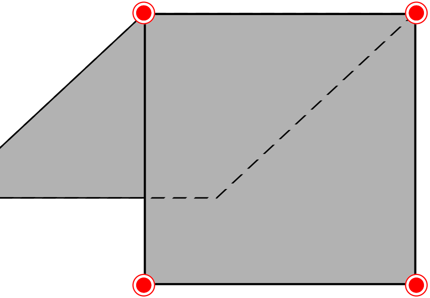

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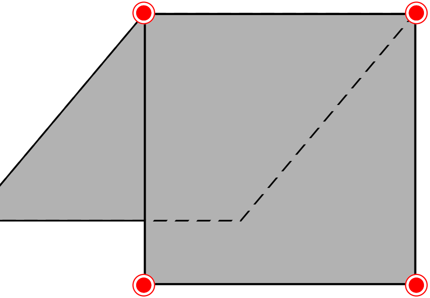

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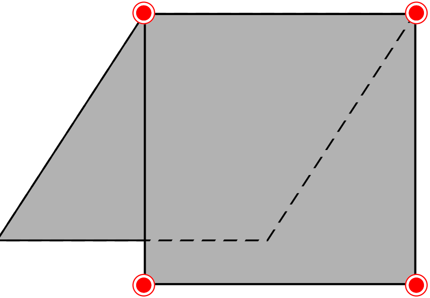

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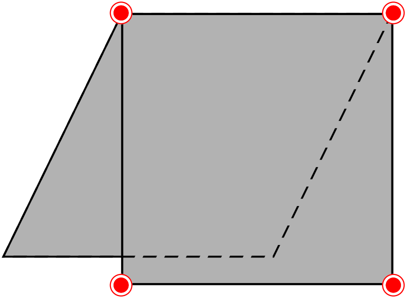

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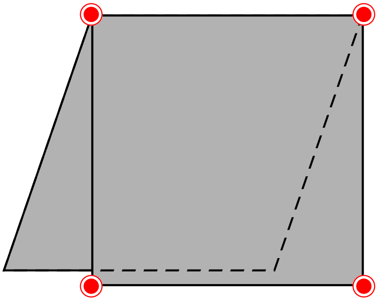

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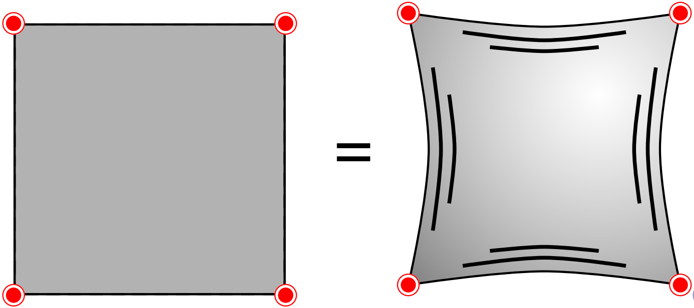


back

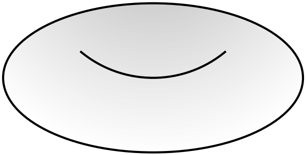
The $\mathbb{Z}_2$ orbifold plane

- Orbifolds with $\mathbb{Z}_2$ plane have three important properties:

The $\mathbb{Z}_2$ orbifold plane

☞ Orbifolds with $\mathbb{Z}_2$ plane have three important properties:

1. $\mathbb{Z}_4^R$ symmetry arises as a remnant of the Lorentz group in compact dimensions



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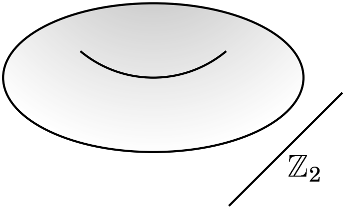

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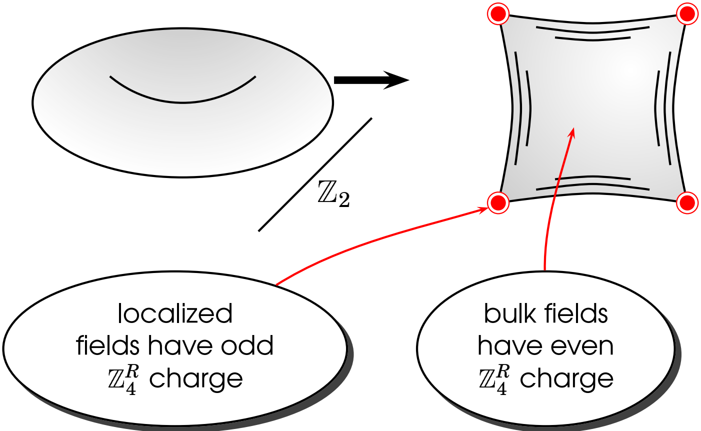

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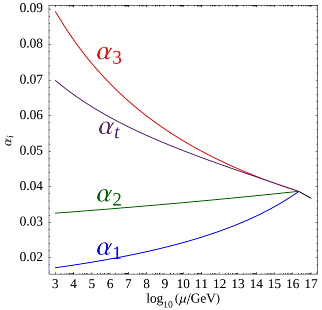
❷ Orbifold GUT limit with $\mathrm{SU}(6)$ bulk symmetry gives us gauge-top unification

P. Hosteins, R. Kappl, M.R., K. Schmidt-Hoberg (2009)



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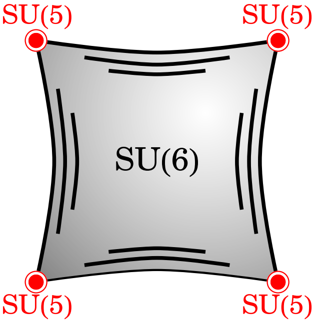
➥ Rest of this talk: discuss globally consistent string model with these features

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold example

M. Blaszczyk, S. Groot Nibbelink, M.R., F. Ruehle, M. Trapletti, P. Vaudrevange (2009)



1. step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry

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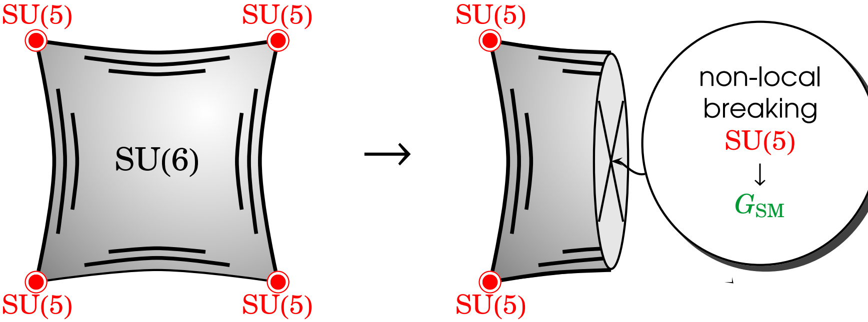
M. Blaszczyk, S. Groot Nibbelink, M.R., F. Ruehle, M. Trapletti, P. Vaudrevange (2009)



1. step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry
2. step: mod out a freely acting $\mathbb{Z}_2$ symmetry which:
   - breaks $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$
   - reduces the number of generations to 3

analogous mechanism in CY MSSMs Bouchard & Donagi (2005)
Braun, He, Ovrut, Pantev (2005)

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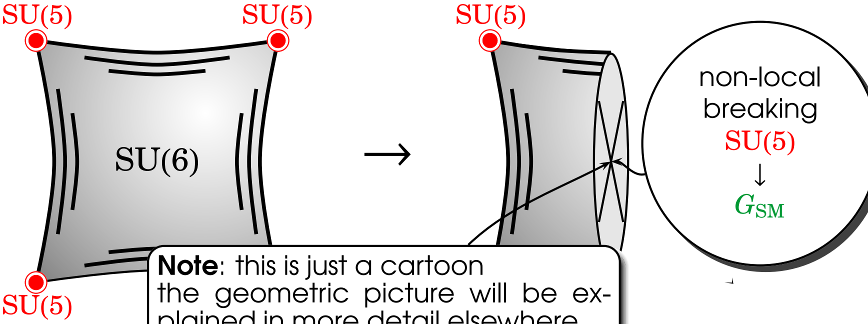

Note: this is just a cartoon
the geometric picture will be explained in more detail elsewhere

M. Fischer, M.R., P. Vaudrevange (to appear)

❶ step: 6 ge… …mmetry

❷ step: mod out a freely acting $\mathbb{Z}_2$ symmetry which:

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Main features

1. GUT symmetry breaking non-local
   $\curvearrowright$ no 'logarithmic running above the GUT scale'

   Hebecker, Trapletti (2004)

   $\curvearrowright$ precision gauge unification
   with distinctive pattern of soft masses

   Raby, M.R., Schmidt-Hoberg (2009)

Main features

1. GUT symmetry breaking non-local
2. No localized flux in hypercharge direction
$\curvearrowright$ complete blow-up without breaking SM gauge symmetry in principle possible

Main features

1. GUT symmetry breaking non-local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_Y \times \mathsf{U}(1)_{B-L} \times [\mathsf{SU}(3) \times \mathsf{SU}(2)^2 \times \mathsf{U}(1)^7]$

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4. massless spectrum

   spectrum $=$ $3 \times$ generation + vector-like

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5. Various appealing features:
   - vacua where exotics decouple at the linear level in SM singlets
   - non-trivial Yukawa couplings
   - gauge-top unification
   - SU(5) relation $y_\tau \simeq y_b$ (but also for light generations)

P. Hosteins, R. Kappl, M.R., K. Schmidt-Hoberg (2009)

'Anomalous' $\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ model

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☺ There are similar $\mathbb{Z}_2 \times \mathbb{Z}_2$ models without this problem but all the good features

- ✓ F- and D-flatness explicitly verified
- ✓ exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$
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- ➥ Successful string embedding of $\mathbb{Z}_4^R$ possible!

'Anomalous' $\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ model

☞ We succeeded in finding vacua with the 'anomalous' $\mathbb{Z}_4^R$

☹ In the Blaszczyk et al. model all vacua have an extra $\mathbb{Z}_2^{\text{nasty}}$, which leads to rank 2 Y_e and Y_d Yukawa couplings

☺ There are similar $\mathbb{Z}_2 \times \mathbb{Z}_2$ models without this problem but all the good features

- ✓ F- and D-flatness explicitly verified
- ✓ exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$
- ✓ non-trivial full-rank Yukawa couplings
- ✓ gauge-top unification
- ✓ SU(5) relation $y_\tau \simeq y_b$ (but also for light generations)

➥ Successful string embedding of $\mathbb{Z}_4^R$ possible!

☞ Note: the 'anomalous' $\mathbb{Z}_4^R$ does **not** come from the 'anomalous' $\mathrm{U}(1)$!

...rather the discrete symmetries of the orbifold point can appear 'anomalous' by themselves

μ from $\mathscr{W}$ in models with $\mathbb{Z}_2$ plane

F. Brümmer, R. Kappl, M.R., K. Schmidt-Hoberg (2009)

☞ Higher-dimensional gauge invariance $\curvearrowright$ Kähler potential

Antoniadis, Gava, Narain & Taylor (1994); Choi et al. (2003)

$$K = -\ln\left[\left(T_3 + \overline{T_3}\right)\left(Z + \overline{Z}\right) - \left(H_u + \overline{H_d}\right)\left(H_d + \overline{H_u}\right)\right]$$

Kähler modulus

complex structure modulus

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Higgs fields
= extra components
of gauge fields

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&\quad+\frac{1}{\left(T_3+\overline{T_3}\right)\left(Z+\overline{Z}\right)}\left[|H_u|^2+|H_d|^2+(H_uH_d+\text{c.c.})\right]
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&= -\ln\left[\left(T_3+\overline{T_3}\right)\,\left(Z+\overline{Z}\right)\right]+\left[|\widehat{H}_u|^2+|\widehat{H}_d|^2+(\widehat{H}_u\widehat{H}_d+\text{c.c.})\right]
\end{aligned}
$$

normalized
Higgs fields

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  $\mathscr{W} \;=\; \Omega \;=\;$ independent of the monomial $\widehat{H}_u\widehat{H}_d$

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☞ K & $\mathscr{W}$ in leading order in $\widehat{H}_u\widehat{H}_d$ equivalent to

$$\begin{aligned} K' &= -\ln\left[\left(T_3+\overline{T_3}\right)\,\left(Z+\overline{Z}\right)\right] + \left[|\widehat{H}_u|^2 + |\widehat{H}_d|^2\right] \\ \mathscr{W}' &= \exp(\widehat{H}_u\,\widehat{H}_d)\,\Omega = \Omega\,\widehat{H}_u\,\widehat{H}_d + \dots \end{aligned}$$

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bottom-line:

μ term proportional to $\langle\Omega\rangle$

Non-perturbative violation of $\mathbb{Z}_4^R$ (cont'd)

- Since $H_u\,H_d$ is proportional to $\langle \mathscr{W} \rangle$ we will get a holomorphic contribution to the μ term of the right order

Kim & Nilles (1983); Casas & Muñoz (1992)

$$\mu \sim \frac{\langle \mathscr{W} \rangle}{M_\mathrm{P}^2} \simeq m_{3/2}$$

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☞ Whatever gives us $\langle \mathscr{W} \rangle$ will be a measure for $\mathbb{Z}_4^R$ breaking

... for instance, one may replace/describe hidden sector superpotential by gaugino condensate

Nilles (1982)

$$\langle \mathscr{W} \rangle \;\simeq\; \langle \lambda\lambda \rangle \;\simeq\; \Lambda^3$$

- this is consistent with a non-perturbative breaking of $\mathbb{Z}_4^R$
- this assumes that the dilaton is fixed somehow (e.g. Kähler stabilization)

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- Dimension 5 proton decay operators will have highly suppressed coefficients

$$\mathscr{W}_{QQQL}^{\mathrm{np}} \sim \frac{\langle \mathscr{W} \rangle}{M_\mathrm{P}^4} Q\,Q\,Q\,L \sim \frac{m_{3/2}}{M_\mathrm{P}} \frac{1}{M_\mathrm{P}} Q\,Q\,Q\,L \sim 10^{-15} \frac{1}{M_\mathrm{P}} Q\,Q\,Q\,L$$

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- No R parity violation because $\mathbb{Z}_4^R$ has a non-anomalous subgroup which is equivalent to matter parity

Summary

&

outlook

Summary – bottom-up

☞ A simple 'anomalous' $\mathbb{Z}_4^R$ symmetry can

- provide a solution to the μ problem
- suppress proton decay operators

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Summary – bottom-up

☞ A simple 'anomalous' $\mathbb{Z}_4^R$ symmetry can

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$$\mathbb{Z}_4^R \curvearrowright \left\{\begin{array}{l}\text{dim. 4 proton decay operators completely forbidden}\\ \text{dim. 5 proton decay operators highly suppressed}\\ \mu \text{ appears non-perturbatively}\end{array}\right.$$

Summary – top-down

- Embedding into string theory allows us to understand where the $\mathbb{Z}_4^R$ symmetry comes from: it may arise as a discrete remnant of Lorentz symmetry in extra dimensions

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Summary – top-down

- Embedding into string theory allows us to understand where the $\mathbb{Z}_4^R$ symmetry comes from: it may arise as a discrete remnant of Lorentz symmetry in extra dimensions
- Such symmetries are on the same footing as the fundamental symmetries C, P and T
- Guided by the (unique) $\mathbb{Z}_4^R$ symmetry we have constructed a globally consistent string model with:
  - exact MSSM spectrum
  - non-trivial Yukawa couplings
  - exact matter parity
  - $\mu \sim m_{3/2}$
  - dimension five proton decay operators sufficiently suppressed

Vielen Dank!