Compact F-theory model building

- arXiv: 0906.0013 (JHEP) with Blumenhagen, Grimm, Jurke
- arXiv: 0908.1784 (NPB) with Blumenhagen, Grimm, Jurke
- arXiv: 0912.3524 (JHEP) with Grimm, Krause
- arXiv: 1006.0226 (PRD) with Grimm
- work in progress

Timo Weigand

ITP, University of Heidelberg

Why F-theory

Ultimate goal of modern physics:

Bring together cosmology and particle physics within **fundamental** theory

Prime question of modern string phenomenology:

Is this possible within the landscape of 4D string vacua

upon compactification $\mathcal{M}_{10} \rightarrow \mathbb{R}^{1,3} \times \mathcal{M}_6$?

Example:

Cosmological evolution $\leftrightarrow$ scalar fields (inflation, quintessence...)

Within string compactifications

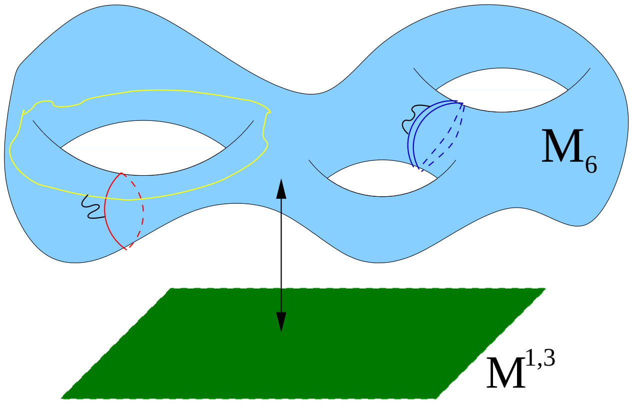
- scalar fields arise as side product: moduli of compactification
- studying the scalar dynamics requires non-trivial moduli potential $\leftrightarrow$ moduli stabilisation

So far moduli stabilisation is best controlled within **Type IIB compactifications** (but progess also in Type IIA and in heterotic)

Why F-theory

Culminated in various inflation models within Type IIB context

$\Longrightarrow$ quest for **Type IIB particle physics model building**



General framework:
Brane Worlds $\leftrightarrow$ localisation of gauge degrees of freedom on D-branes

Within Type IIB context:

- D7-branes: $(7+1)$ dim. subspace of 10 dim. spacetime
  $\Longrightarrow$ considerable backreaction on geometry
- system most reliably studied directly within **F-theory**

Why F-theory

Appreciated recently: `starting with [Beasley,Heckman,Vafa; Donagi,Wijnholt '08]`

F-theory shares favourable structure for GUT model building known from heterotic strings

$\leftrightarrow$ strong coupling effects that give rise to exceptional gauge symmetry

$\Rightarrow$ many **F-applications to GUT model building** proposed recently

local analysis:

Mechanisms to overcome challenges of conventional GUT models

But: realisation requires better understanding of F-theory technology

- at practical level: Develop methods to study compactification spaces
- at conceptual level: connection to perturbative Type IIB limit, description of gauge flux, brane motion, "loop" corrections ...

Why F-theory

Aim: Advance F-theory technology as goal by itself and for applications

$\leftrightarrow$ combining theme (with different emphasis) in research of string groups in Bonn, Heidelberg and Munich

$\leftrightarrow$ formal and phenomenological aspects go hand in hand

This talk:

- SU(5) GUT model building in compact settings
- physics of abelian gauge symmetries

Outline

I. Motivation

II. F-theory basics

III. ADE singularities

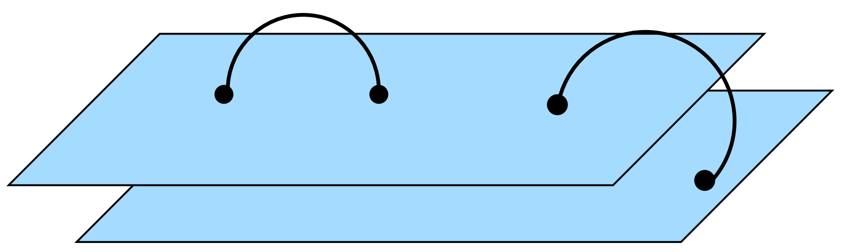
IV. SU(5) GUTs from F-theory

V. The geometry and physics of U(1) symmetries

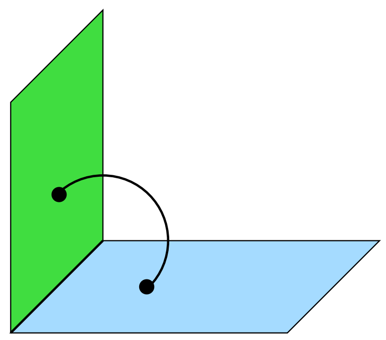
Intersecting Brane Models

Most of the structure of F-theory models is present already perturbatively

stacks of N coincident Dp-branes
$\rightarrow U(N)$ gauge symmetry



2 D7-branes intersecting at an angle:
$\rightarrow$ matter fields in bifundamental representation $(\overline{N}_a, N_b)$

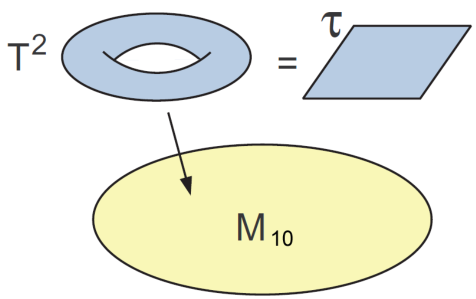


Yukawa couplings from triple overlap of wavefunction at intersection of matter loci

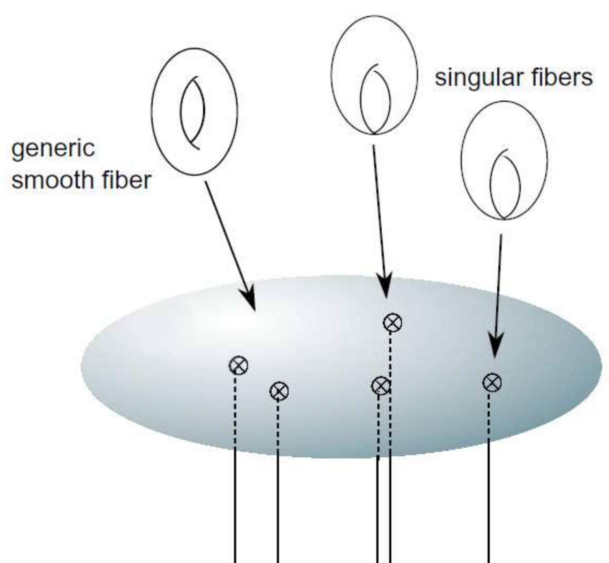
From branes to F-theory

F-theory geometrises Type IIB orientifolds with 7-branes: Vafa 1996

- 7-brane = source for varying axio-dilaton field $\tau = C_0 + \frac{i}{g_s}$
- locally near position of D-brane $\tau \simeq \frac{1}{2\pi i} \ln(z - z_0) \Rightarrow$ monodromy $\tau \to \tau + 1$
- Interpret τ as complex structure of auxiliary torus T^2
- τ varies $\leftrightarrow$ shape of T^2 varies $\Longrightarrow$ fibration of $T^2 \to \mathcal{M}_{10}$



pic adapted from: Denef, 0803.1194



at position of D-brane

$\tau = \frac{1}{2\pi i} \ln(z - z_0) + \ldots \to i\infty$

$\leftrightarrow$ T^2 fiber degenerates as 1-cycle $\to 0$

Elliptic fibrations

Case of phenomenological interest: Compactification $\mathcal{M}_{10} \to \mathbb{R}^{1,3} \times \mathcal{M}_6$

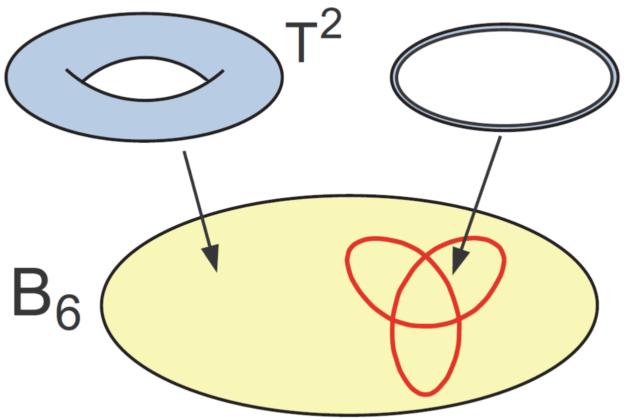
- physics encoded in geometry of $Y : T_2 \to B_6$
- $\mathcal{N} = 1$ SUSY requires Y to be Calabi-Yau
- jargon: F-theory on elliptic fourfold Y $=$ effective 4D theory obtained by compactification of Type IIB strings with D7-branes on "B_6"

IIB language:

7-branes wrap 4-cycle $\Gamma_a \subset$ "B_6"

F-theory language:

Γ_a = locus of fiber degeneration



pic adapted from: Denef, 0803.1194

ADE groups in F-theory

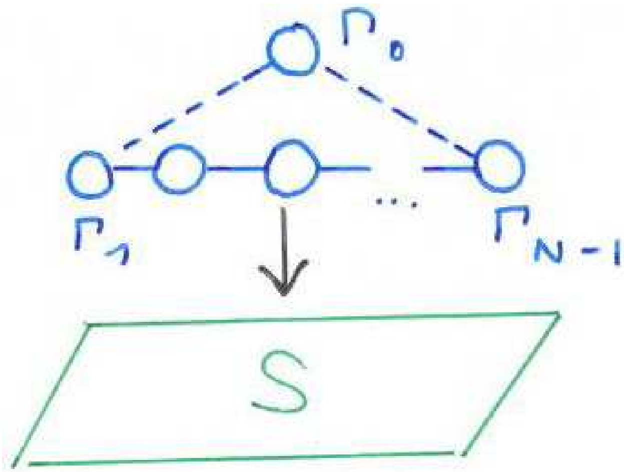
Non-abelian ADE gauge groups $\leftrightarrow$ singularity structure of elliptic fibration

[Bershadsky et al.][Morrison,Vafa I+II] '96, ...

$\rightsquigarrow$ Y_G: singular 4-fold $T^2 \to B_6$ with ADE group G along divisor $S \subset B_6$

$\rightsquigarrow$ singularities best studied by resolution $Y_G \to \overline{Y}_G$ within M-theory

- paste in tree of $\mathbb{P}^1$s fibered over S $\quad \Gamma_i^G \quad i = 1, \ldots, \mathrm{rk}(G)$
  singular Y_G $\leftrightarrow$ zero size limit of Γ_i^G
- resolution divisors D_i^G $\iff$ fibration $\Gamma_i^G \to S$
- Group theory of G
  $\Leftrightarrow$ extended Dynkin diagram



resolved $\overline{Y}_G$ $\iff$ Coulomb branch $G \to U(1)^{\mathrm{rk(G)}}$ in M-theory

ADE groups in F-theory

2 sources of gauge bosons along S from M-theory reduction

- off-diagonal elements in $ad(G)$:
  M2-branes along chains of $\mathbb{P}^1$ $\Gamma_i^G \cup \ldots \cup \Gamma_j^G$, $i \leq j$
  $\Longrightarrow$ massless only in singular limit

- Cartan $U(1)^{\mathrm{rk(G)}}$ generators:
  3-form C_3 expanded in $\omega_i^G = [D_i^G] \in H^2(\overline{Y}_G, \mathbb{Z})$

$$C_3 = \sum_{i=1}^{\mathrm{rk}(G)} A^i \wedge \omega_i^G + \ldots \qquad\qquad A^i \leftrightarrow \text{gauge field along } S$$

Gauge flux of Cartan U(1): $G_4 = \sum_i F_i \wedge \omega_i^G$, $\qquad F_i \in H^2(S)$

The quest for U(1)

✓ Non-abelian ADE gauge symmetry built in geometrically

✓ Cartan U(1)s easy to study in Coulomb phase in M-theory

What about extra U(1) gauge symmetries not tied to non-ab. groups?

General fact from expansion $C_3 = \sum_{i=1}^{\mathrm{rk}(G)} A^i \wedge \omega_i^G$:

total rank of gauge group [Morrison,Vafa I+II '96]

$$n_v = h^{1,1}(\overline{Y}_G) - h^{1,1}(B) - 1$$

$$n_{U(1)} = n_v - \mathrm{rk}(G)$$

$\leftrightarrow$ can be computed in **global models** with **resolution of singularity**

elliptic 3-folds: [Candelas,Font '96] [Candelas,Perevalov,Rajesh '97]

elliptic 4-folds: [Blumenhagen,Grimm,Jurke,TW 0908.1784][Grimm,Krause,TW 0912.3524]

Tate model for SU(5)

Application: Compactifications with MSSM gauge group and matter

Most efficient: SU(5) F-theory GUTs [Donagi,Wijnholt; Beasley,Heckman,Vafa]'08

- Constrain compl. structure of $Y_4 \leftrightarrow$ SU(5) singularity on GUT brane S

- Technically: choice of sections in **Tate model**

  T^2: coordinates $(x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z)$ B_6 coordinates: u_i

  $$P_W = x^3 - y^2 + x\,y\,z\,a_1 + x^2\,z^2\,a_2 + y\,z^3\,a_3 + x\,z^4\,a_4 + z^6\,a_6 = 0$$

  $a_n \equiv a_n(u_i) \leftrightarrow$ varying complex structure

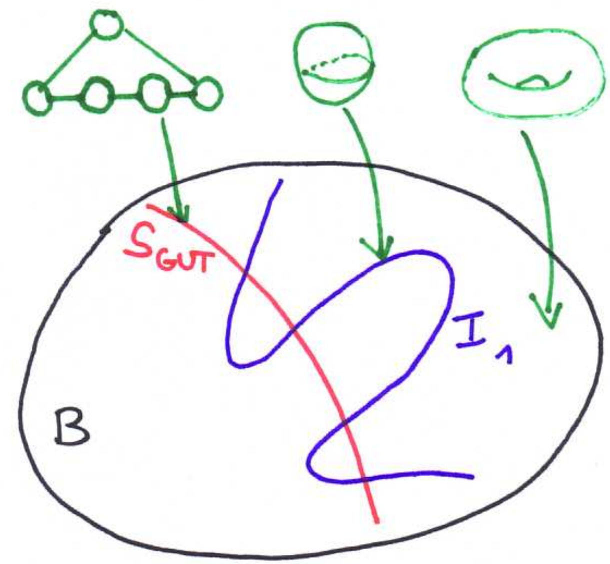
- Branes = discriminant $\Delta(u_i)$

- $[\mathbf{\Delta}] = \mathbf{5}[\mathbf{S}] + [\mathbf{D_1}]$ D_1: I_1 locus

  generic gauge group $SU(5) \times \emptyset$

- no extra U(1) factors due to maximal Higgsing of underlying $E_8 \to SU(5)$

  [Grimm, TW 1006.0226]



Tate model for SU(5)

Further singularity enhancement at **intersection of S and D_1**
$\leftrightarrow$ collision of vanishing $\mathbb{P}^1$s in fiber

a) matter: enhancement of singularity type on intersection $S \cap D_1$

[Katz,Vafa '96]

- $SU(5) \times U(1) \to SU(6)$
  $\mathbf{35} \to \mathbf{24} + 1 + \mathbf{5} + \overline{\mathbf{5}}$ $\Longrightarrow$ $\overline{\mathbf{5}}_m = (d^c_R, L)$ or $\mathbf{5}_H + \overline{\mathbf{5}}_H$
- $SU(5) \times U(1) \to SO(10)$
  $\mathbf{45} \to \mathbf{24} + 1 + \mathbf{10} + \overline{\mathbf{10}}$ $\Longrightarrow$ $\mathbf{10} = (Q_L, u^c_R, e^c_R)$

N^c_R: any SU(5) singlet with suitable couplings

b) Yukawas: Intersection of curves at points [BHV; DW] '08

- $\langle \mathbf{10}\, \overline{\mathbf{5}}\, \overline{\mathbf{5}} \rangle \subset \langle (\mathbf{66})^3 \rangle$ of SO(12) as in perturbative Type IIB
- $\langle \mathbf{10}\, \mathbf{10}\, \mathbf{5} \rangle \subset \langle (\mathbf{78})^3 \rangle$ of $\mathbf{E_6}$ **(only) truly F-theoretic input**

SU(5) GUT model building

Main model building ideas:

- SU(5) GUT breaking via $U(1)_Y$ flux [BHV],[DW] '08
  $\rightsquigarrow$ no need for GUT Higgs/ brane moduli
  $\rightsquigarrow$ requires global information of embedding GUT cycle
  $\rightsquigarrow$ preservation of unification ??? [Donagi, Wijnholt], [Blumenhagen]'08; [Conlon, Palti], [Saulina et al.]'09

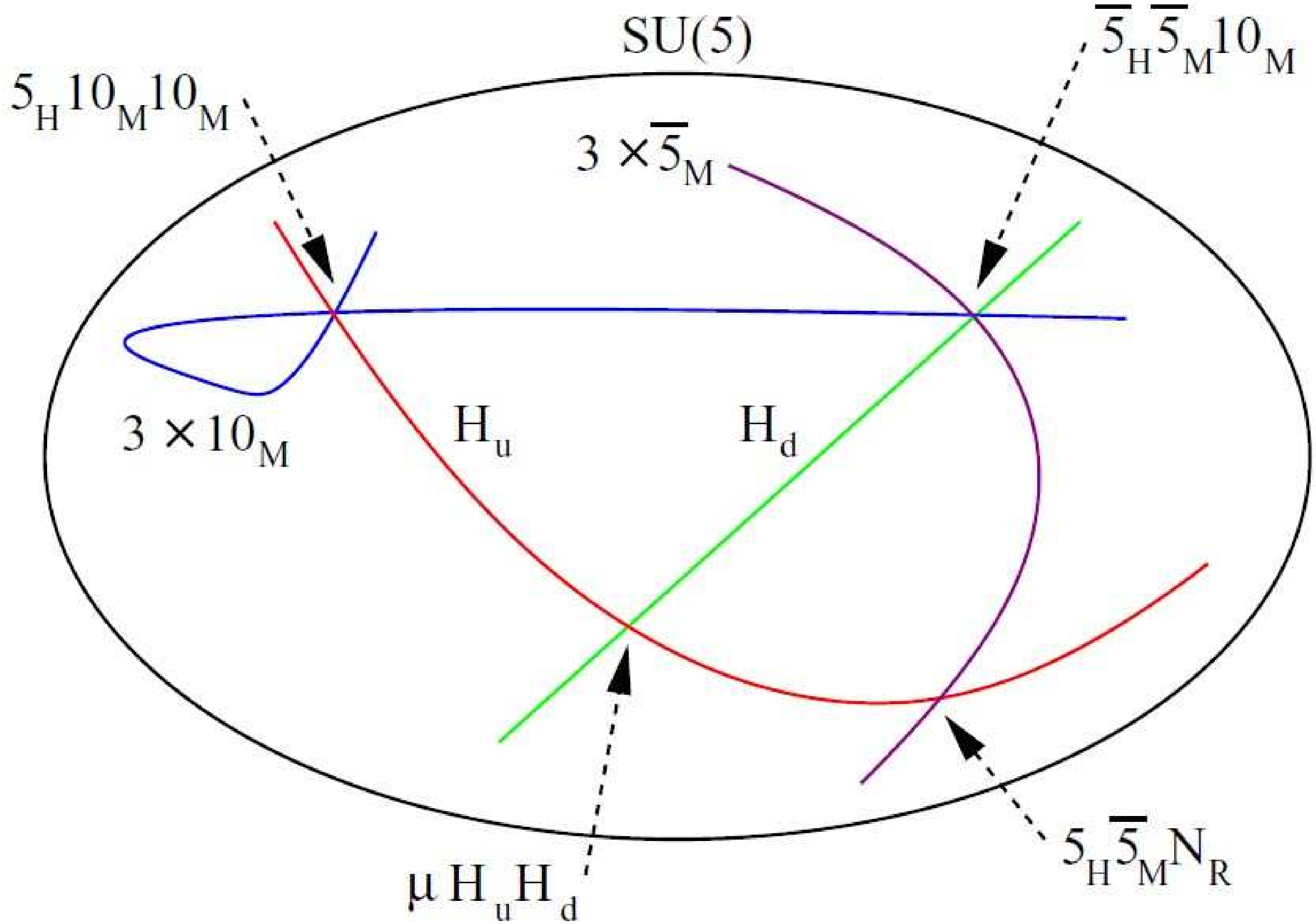
- no dimension 4 proton decay by split of $\mathbf{5}$-matter curves
  otherwise: $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{H}}$ implies $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{m}}$
  Note: **This is not sufficient** - see later

- no dimension 5 proton decay by missing partner mechanism
  $\Longleftrightarrow$ split H_u and H_d curve

- studies of flavour structure include [BHV], [Ibanez,Font] '08, [Palti,Dudas], [Conlon,Palti],[Cecotti,Cheng,Heckman,Vafa],[Marchesano,Martucci] '09

SU(5) GUTs

Necessary conditions on local geometry of GUT brane:



Beasely,Heckman,Vafa, 0806.0102

Why go global....

... if all these nice mechanisms involve local physics?

1) Existence proof:

Can we really get all the proposed geometric features (singularity enhancements...) in a well-defined compact geometry?

2) Coupling to cosmology

$\leftrightarrow$ SUSY breaking, moduli stabilisation

only possible within globally defined framework

3) All issues involving U(1) symmetries are global:

- $U(1)_Y$ breaking requires flux along cycles on S that are boundaries of chains in Y $\Rightarrow$ full information required
- $U(1)$ selection rules turn out crucial e.g. for dimension-4 proton decay Will see: This requires global information!

F-theory model building

2 ingredients for construction of global models:

- singular Calabi-Yau 4-fold $\Rightarrow$ gauge group, matter curves, Yukawas
- gauge flux $\Rightarrow$ 3 generations of chiral matter

Status: Geometries ✓ Gauge flux: still not fully understood

Approaches to geometry:

a) Explicit control of singularities possible within toric framework

$\Rightarrow$ Classes of compact singular fourfolds Y_G and their **resolutions** $\overline{Y}_G$

- [Blumenhagen,Grimm,Jurke,TW 0908.1784], [Grimm,Krause,TW 0912.3524] for SU(5)
- [Chen,Knapp,Kreuzer,Mayrhofer 1005.5735] for SO(10) models

$\Rightarrow$ only examples with reliable computation of Euler characteristic $\chi(\overline{Y}_G)$
$\Leftrightarrow$ prerequisite to study global consistency conditions

b) Non-torically realised singular fourfold constructed in

- [Marsano,Saulina,Schäfer-Nameki 0904.3932] for SU(5)

U(1) selection rules

- **Generic Tate model:** single 5-matter curve

$\Longrightarrow \quad \mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{H}} \leftrightarrow \mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{m}}$: dimension 4 proton decay

- Remedy: split $P_5 \rightarrow P_m + P_H$ + effective U(1) selection rule

[Beasley,Heckman,Vafa II '08] [Hayashi,Kawano,Tatar,Watari 0901.4941]

Group theory: $E_8 \rightarrow G \times H, \qquad G = SU(5)_{\mathrm{GUT}}$

- $H = SU(5)_{\perp} \rightarrow S[U(4) \times U(1)_X]$
- $U(1)_X$ charges: $\mathbf{10}_1 \qquad (\overline{\mathbf{5}}_m)_{-3} \qquad (\mathbf{5}_H)_{-2} + (\overline{\mathbf{5}}_H)_2 \qquad 1_{-5}$
- If $U(1)_X$ unbroken in fully fledged model, then $\mathbf{10}\,\overline{\mathbf{5}}_{\mathbf{m}}\,\overline{\mathbf{5}}_{\mathbf{m}}$ forbidden ✓
- Necessary condition for $U(1)_X$: split $P_5 \rightarrow P_m + P_H$
achieved by split spectral cover → realises $S[U(4) \times U(1)_X]$ "locally"

[Marsano,Saulina,Schäfer-Nameki 0906.4672] [Tatar,Tsuchiya,Watari 0905.2289]

Global caveats

Applied to construction of 3-generation models:

[Marsano,Saulina,Schäfer-Nameki 0906.4672] [Blumenhagen,Grimm,Jurke,TW 0908.1784]

[Marsano,Saulina,Schäfer-Nameki 0912.0272] [Grimm,Krause,TW 0912.3524]

[Chen,Knapp,Kreuzer,Mayrhofer 1005.5735]

Caveat:

- $U(1)_X$ might be higgsed by GUT singlets Φ

  [Tatar,Tsuchiya,Watari 0905.2289],[Grimm, TW 1006.0226]

- happens away from GUT brane $\Longleftrightarrow$ beyond regime of spectral cover

- This is a **global question of direct physical relevance**:
  If $U(1)_X$ higgsed, effective proton decay operators generated

$$W \supset \frac{1}{M}\,\mathbf{10}\,\overline{\mathbf{5}}_m\,\overline{\mathbf{5}}_m\,\boldsymbol{\Phi} \qquad \Longrightarrow \qquad \frac{\langle\boldsymbol{\Phi}\rangle}{M}\,\mathbf{10}\,\overline{\mathbf{5}}_m\,\overline{\mathbf{5}}_m$$

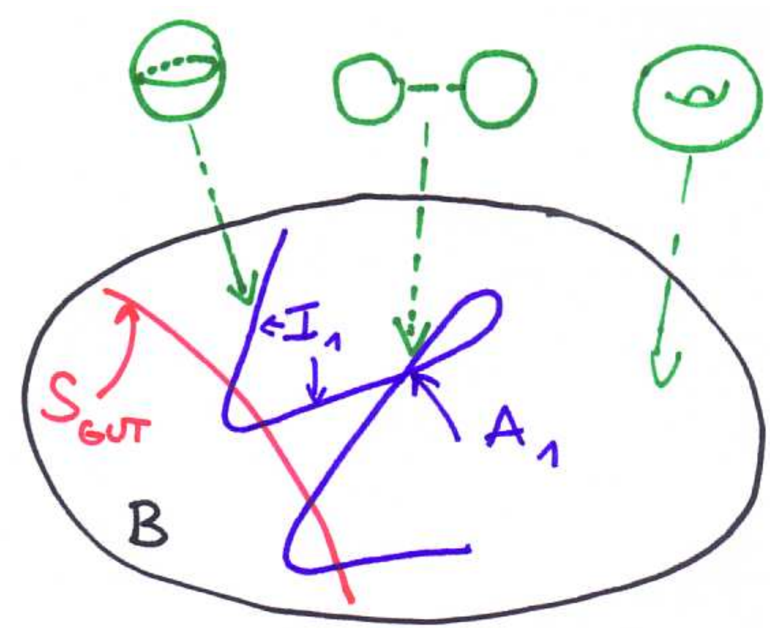
Independent analysis via monodromies: [Hayashi,Kawano,Tsuchiya,Watari 1004.3870]

U(1) restricted Tate model

Constructive method to ensure presence of abelian gauge symmetries:

[Grimm, TW 1006.0226]

- In generic models $U(1)$ absent due to maximal Higgsing compatible with non-abelian gauge symmetries $\leftrightarrow$ VEV for gauge singlets $\langle \mathbf{1_{-5}} \rangle$
- $U(1)$ symmetries recovered by unhiggsing
  $\rightarrow$ massless gauge singlets away from GUT brane
- requires singular matter curves away from GUT brane
- self-intersection of I_1 locus
  enhancement $I_1 \rightarrow A_1 \simeq SU(2)$
- further specification of complex structure:
  U(1) restricted Tate model



U(1) restricted Tate model

Safe way to check for $U(1)_X$: [Grimm,TW 1006.0226]

Resolve space $Y_G \to \overline{Y}_G$ and count $h^{1,1}(\overline{Y}_G)$

Result: $n_{U(1)} = h^{1,1}(\overline{Y}_G) - h^{1,1}(B) - 1 - rk(G) = 1$

Reason: Resolution divisor D_C for singular curve $C \leftrightarrow$ dual 2-form ω_C

$$C_3 = A_X \wedge \omega_X + \sum_i A_i \wedge \omega_i^G + \ldots \qquad \omega_X \leftrightarrow \omega_C$$

- presence of U(1) does not hinge on any factorisation of the discriminant
- Compatible with appearance of U(1) symmetries in IIB limit
  F-theory lift does not destroy U(1) - only affects split O7-plane/D7-brane intersection
- U(1) are non-generic both in IIB and in F-theory

U(1) restricted Tate model

Practical consequence of U(1) restriction: $\chi(\overline{Y})$ decreases drastically

GUT example of [Grimm,Krause,TW 0912.3524]

Generic SU(5) Tate model: $\chi = 5718 \leftrightarrow$ U(1)-restricted model: $\chi = 2556$

What about $U(1)_X$ flux?

- bundle is now of type $S[U(4) \times U(1)_X]$
- global description in terms of G_4 complicated

  tempting: $G = F_X \wedge \omega_X + \ldots$? details still under investigation

  For different approach see [Marsano,Saulina,Schäfer-Nameki 1006.0483]

- $U(1)_X$ flux $\Longrightarrow$ Fayet-Iliopoulos D-term

  $\leftrightarrow$ $U(1)_X$ acquires Stückelberg mass in presence of suitable gauge flux

  $\Longrightarrow$ **global selection rule** $\leftrightarrow$ instantons $\rightarrow$ work in progress

Conclusions and outlook

F-theory yields a geometric description of 7-brane physics

interesting prospects for particle physics:
$\leftrightarrow$ combination of the sunny sides of heterotic and Type II

- Recent progress in construction of compact F-theory GUT models
- Detailed phenomenology requires progress in F-theory technology
  Example: understanding of U(1) symmetries $\leftrightarrow$ proton decay
- Many exciting conceptual and phenomenological questions remain

Ultimate goal: Contact also with cosmology

- use calculable moduli stabilisation for inflation
- Current topic in TR33 A3: brane inflation in Type IIB/ F-theory
  work in progress with A. Hebecker, D. Lüst, Stephan Steinfurt, Sebastian Kraus