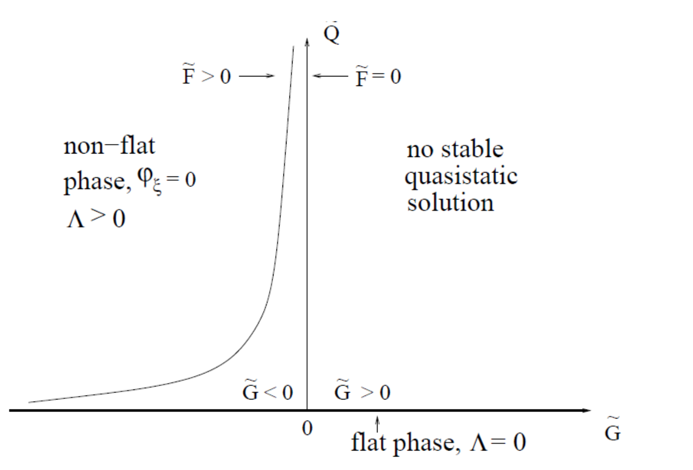
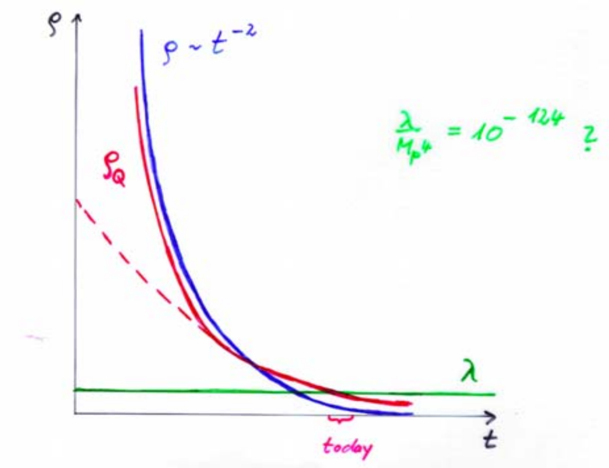
Asymptotically vanishing cosmological constant, Self-tuning and Dark Energy





Cosmological Constant
- Einstein -

- Constant λ compatible with all symmetries
- Constant λ compatible with all observations
- No time variation in contribution to energy density

- Why so small ? $\lambda/M^4 = 10^{-120}$

- Why important just today ?

Cosmological mass scales

- Energy density

$\rho \sim (2.4\times10^{-3}\ \mathrm{eV})^{-4}$

- Reduced Planck mass

$M=2.44\times10^{18}\mathrm{GeV}$

- Newton's constant

$G_N=(8\pi M^2)$

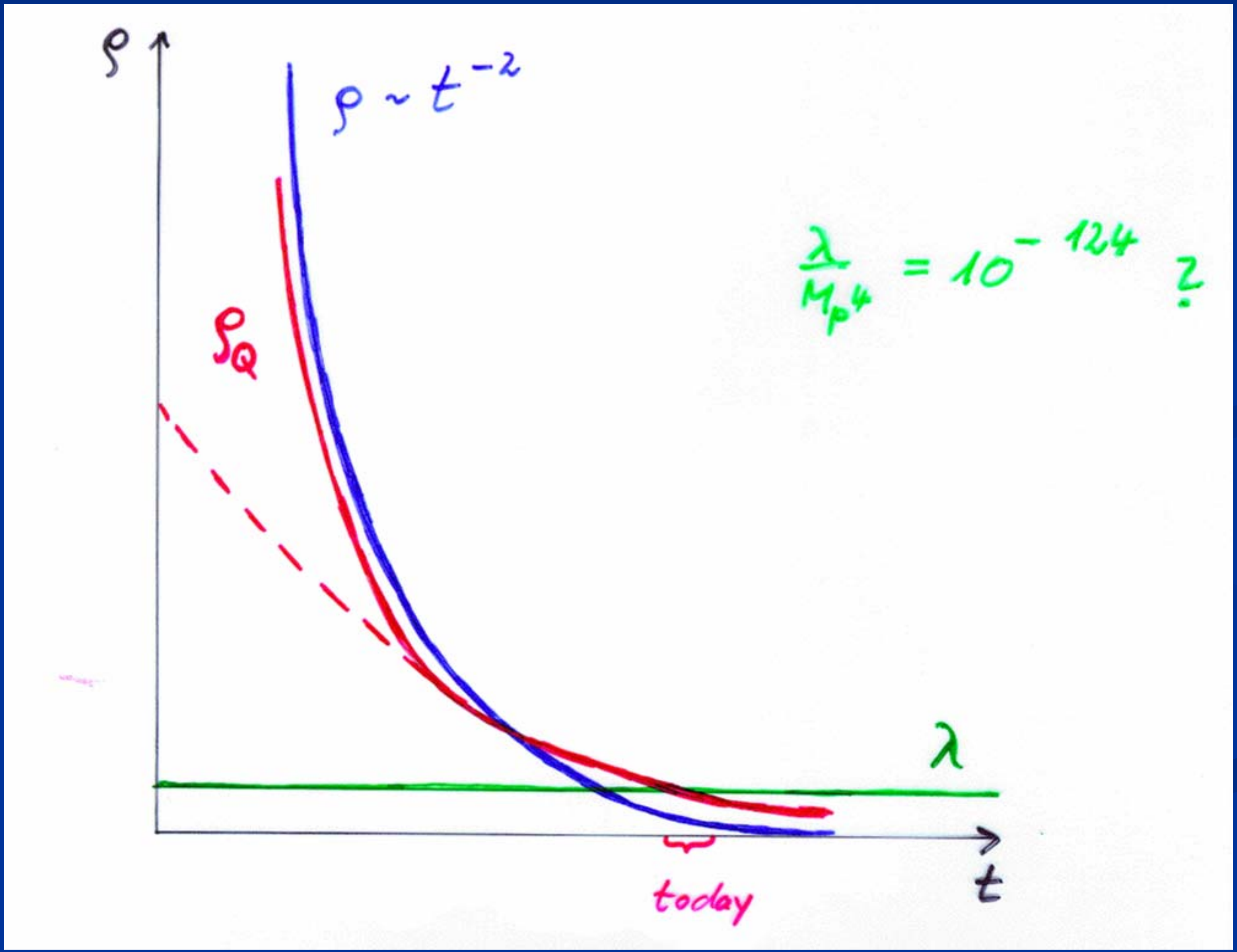
Only ratios of mass scales are observable !

homogeneous dark energy: $\rho_h/M^4 = 7 \cdot 10^{-121}$

matter: $\rho_m/M^4 = 3 \cdot 10^{-121}$

Cosm. Const | **Quintessence**

static | dynamical



Quintessence

Dynamical dark energy , generated by scalar **field** (cosmon)

Prediction :

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations
…. modifications

C.Wetterich,Nucl.Phys.B302(1988)668, 24.9.87
P.J.E.Peebles,B.Ratra,ApJ.Lett.325(1988)L17, 20.10.87

Cosmon

- *Scalar field changes its value even in the* ***present*** *cosmological epoch*
- *Potential und kinetic energy of cosmon contribute to the energy density of the Universe*

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

- *Time - variable dark energy :*

  $\rho_h(t)$ decreases with time !

$V(\varphi) = M^4 \exp(- \alpha\varphi/M)$

two key features for realistic cosmology

1) Exponential cosmon potential and scaling solution

$$V(\varphi) = M^4 \exp(- \alpha\varphi / M)$$

$$V(\varphi \to \infty) \to 0 \;\; !$$

2) Stop of cosmon evolution by cosmological trigger

e.g. growing neutrino quintessence

Evolution of cosmon field

Field equations

$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

Potential V(φ) determines details of the model

$V(\varphi) = M^4 \exp(-\alpha\varphi/M)$

for increasing φ the potential decreases towards zero !

exponential potential ⟹ constant fraction in dark energy

$$\Omega_h = 3(4)/\alpha^2$$

can explain order of magnitude of dark energy !

Asymptotic solution

explain $V(\varphi \rightarrow \infty) = 0$!

effective field equations should have generic solution of this type

setting : quantum effective action , all quantum fluctuations included: investigate generic form

Higher –dimensional dilatation symmetry solves cosmological constant problem

Higher dimensional dilatation symmetry

- for arbitrary values of effective couplings within a certain range : higher dimensional dilatation symmetry implies existence of a large class of solutions with vanishing four –dimensional cosmological constant

- all stable quasi-static solutions of higher dimensional field equations , which admit a finite four-dimensional gravitational constant and non-zero value for the dilaton , have V=0

- self-tuning mechanism

effective dilatation symmetry in full quantum theory

realized for fixed points

Cosmic runaway

- large class of cosmological solutions which never reach a static state : runaway solutions
- some characteristic scale χ changes with time
- effective dimensionless couplings flow with χ
(similar to renormalization group)
- couplings either diverge or reach fixed point
- for fixed point : exact dilatation symmetry of full quantum field equations and corresponding quantum effective action

approach to fixed point

- dilatation symmetry not yet realized
- dilatation anomaly
- effective potential **V(φ)**
- exponential potential reflects anomalous dimension for vicinity of fixed point

$$V(\varphi) = M^4 \exp(- \alpha\varphi/M)$$

cosmic runaway and the problem of time varying constants

- It is not difficult to obtain quintessence potentials from higher dimensional (or string ?) theories
- Exponential form rather generic

  (after Weyl scaling)
- Potential goes to zero for $\varphi \to \infty$
- But most models show too strong time dependence of constants !

higher dimensional dilatation symmetry

generic class of solutions with

vanishing effective four-dimensional cosmological constant

and

constant effective dimensionless couplings

graviton and dilaton

dilatation symmetric effective action

$$\Gamma = \int_{\hat{x}} \hat{g}^{1/2} \Big\{ -\frac{1}{2}\xi^2 \hat{R} + \frac{\zeta}{2}\partial^{\hat{\mu}}\xi\partial_{\hat{\mu}}\xi + F(\hat{R}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}) \Big\}$$

simple example $F = \tau \hat{R}^{\frac{d}{2}}$

in general : many dimensionless parameters characterize effective action

dilatation transformations

$$\Gamma = \int_{\hat{x}} \hat{g}^{1/2} \mathcal{L}.$$

$$\hat{g}_{\hat{\mu}\hat{\nu}} \to \alpha^2 \hat{g}_{\hat{\mu}\hat{\nu}} \ , \ \hat{g}^{1/2} \to \alpha^d \hat{g}^{1/2},$$
$$\xi \to \alpha^{-\frac{d-2}{2}} \xi \ , \ \mathcal{L} \to \alpha^{-d} \mathcal{L}.$$

$$\Gamma = \int_{\hat{x}} \hat{g}^{1/2} \mathcal{L}.$$

is invariant

flat phase

generic existence of solutions of higher dimensional field equations with

effective four –dimensional gravity and

vanishing cosmological constant

torus solution

example :

Minkowski space x D-dimensional torus

$\xi = \text{const}$

- solves higher dimensional field equations
- extremum of effective action

$$\Gamma = \int_{\hat{x}} \hat{g}^{1/2} \left\{ -\frac{1}{2}\xi^2 \hat{R} + \frac{\zeta}{2}\partial^{\hat{\mu}}\xi \partial_{\hat{\mu}}\xi + F(\hat{R}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}) \right\}$$

- finite four- dimensional gauge couplings
- dilatation symmetry spontaneously broken

generically many more solutions in flat phase !

look for extrema of effective action
for more general field configurations

warping

$$\hat{g}_{\hat{\mu}\hat{\nu}}(x,y) = \begin{pmatrix} \sigma(y) g^{(4)}_{\mu\nu}(x) & , & 0 \\ 0 & , & g^{(D)}_{\alpha\beta}(y) \end{pmatrix}$$

most general metric with maximal
four – dimensional symmetry

general form of quasi – static solutions
(non-zero or zero cosmological constant)

effective four – dimensional action

$$W(x) = \int_y (g^{(D)}(y))^{1/2} \sigma^2(y) \mathcal{L}(x,y)$$

$$\Gamma = \int_x (g^{(4)})^{1/2} W.$$

flat phase : extrema of W

in higher dimensions , those exist generically !

extrema of W

- provide large class of solutions with vanishing four – dimensional constant

- dilatation transformation

$$W \to \alpha^{-4} W.$$

- extremum of W must occur for W=0 !
- effective cosmological constant is given by W

$$\Gamma = \int_x (g^{(4)})^{1/2} W.$$

$$W(x) = \int_y (g^{(D)}(y))^{1/2} \sigma^2(y) \mathcal{L}(x,y)$$

extremum of W must occur for W = 0

for any given solution : rescaled metric and dilaton is again a solution

$$\hat{g}_{\hat{\mu}\hat{\nu}} \to \alpha^2 \hat{g}_{\hat{\mu}\hat{\nu}} \qquad \xi \to \alpha^{-\frac{d-2}{2}} \xi$$

for rescaled solution : $W \to \alpha^{-4} W.$

use $\alpha = 1 + \epsilon$

extremum condition :

$$\partial_\epsilon (1+\epsilon)^{-4} W_0 = 0 \quad \Longrightarrow \quad W_0 = 0$$

extremum of W is extremum of effective action

$$\delta\Gamma = \int_{\hat{x}} (\hat{g}_0^{1/2}\delta W + \delta\hat{g}^{1/2} W_0) = 0,$$

effective four – dimensional cosmological constant vanishes for extrema of W

expand effective 4 – d - action $\Gamma = \int_x (g^{(4)})^{1/2} W.$

in derivatives : $\Gamma^{(4)} = \int_x (g^{(4)})^{1/2} \left\{ V - \frac{\chi^2}{2} R^{(4)} + \dots \right\}$

4 - d - field equation

$$\chi^2 \left(R^{(4)}_{\mu\nu} - \frac{1}{2} R^{(4)} g^{(4)}_{\mu\nu} \right) = -V g^{(4)}_{\mu\nu}$$

$$\Gamma = \Gamma^{(4)} = -V \int_x (g^{(4)})^{1/2}$$

$$\Gamma_0 = -\int_x (g^{(4)})^{1/2} \chi^2 \Lambda$$

Quasi-static solutions

- for arbitrary parameters of dilatation symmetric effective action :
- large classes of solutions with extremum of W and $W_{ext} = 0$ are explicitly known (flat phase)

  example : Minkowski space x D-dimensional torus
- only for certain parameter regions : further solutions without extremum of W exist :

  (non-flat phase)

sufficient condition for vanishing cosmological constant

extremum of W exists

effective four – dimensional theory

characteristic length scales

l : scale of internal space

$$\int_y (g^{(D)})^{1/2} \sigma^2 = l^D .$$

$\bar{\xi}$: dilaton scale

$$\int_y (g^{(D)})^{1/2} \sigma^2 \xi^2 = l^D \bar{\xi}^2 .$$

effective Planck mass

$$\Gamma^{(4)} = \int_x (g^{(4)})^{1/2} \left\{ V - \frac{\chi^2}{2} R^{(4)} + \dots \right\}$$

$$\chi^2 = l^D \bar{\xi}^2 - 2\tilde{G} l^{-2}$$

$$\tilde{G} = l^2 \int_y (g^{(D)})^{1/2} \sigma G.$$

dimensionless ,
depends on internal geometry ,
from expansion of F in R

effective potential

$$V = \tilde{Q}\bar{\xi}^2 l^{D-2} + \tilde{F} l^{-4}$$

$$\tilde{F} = l^4 \int_y (g^{(D)})^{1/2} \sigma^2 F(R^{(int)}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}})$$

$$\tilde{Q} = \frac{1}{2}\bar{\xi}^{-2} l^{2-D} \int_y (g^{(D)})^{1/2} \sigma^2 (\zeta \partial^\alpha \xi \partial_\alpha \xi - \xi^2 R^{(int)})$$

canonical scalar fields

consider field configurations with rescaled internal length scale and dilaton value

$$\varphi_\xi = \bar{\xi} l^{\frac{D}{2}} \ , \ \varphi_l = l^{-1}$$

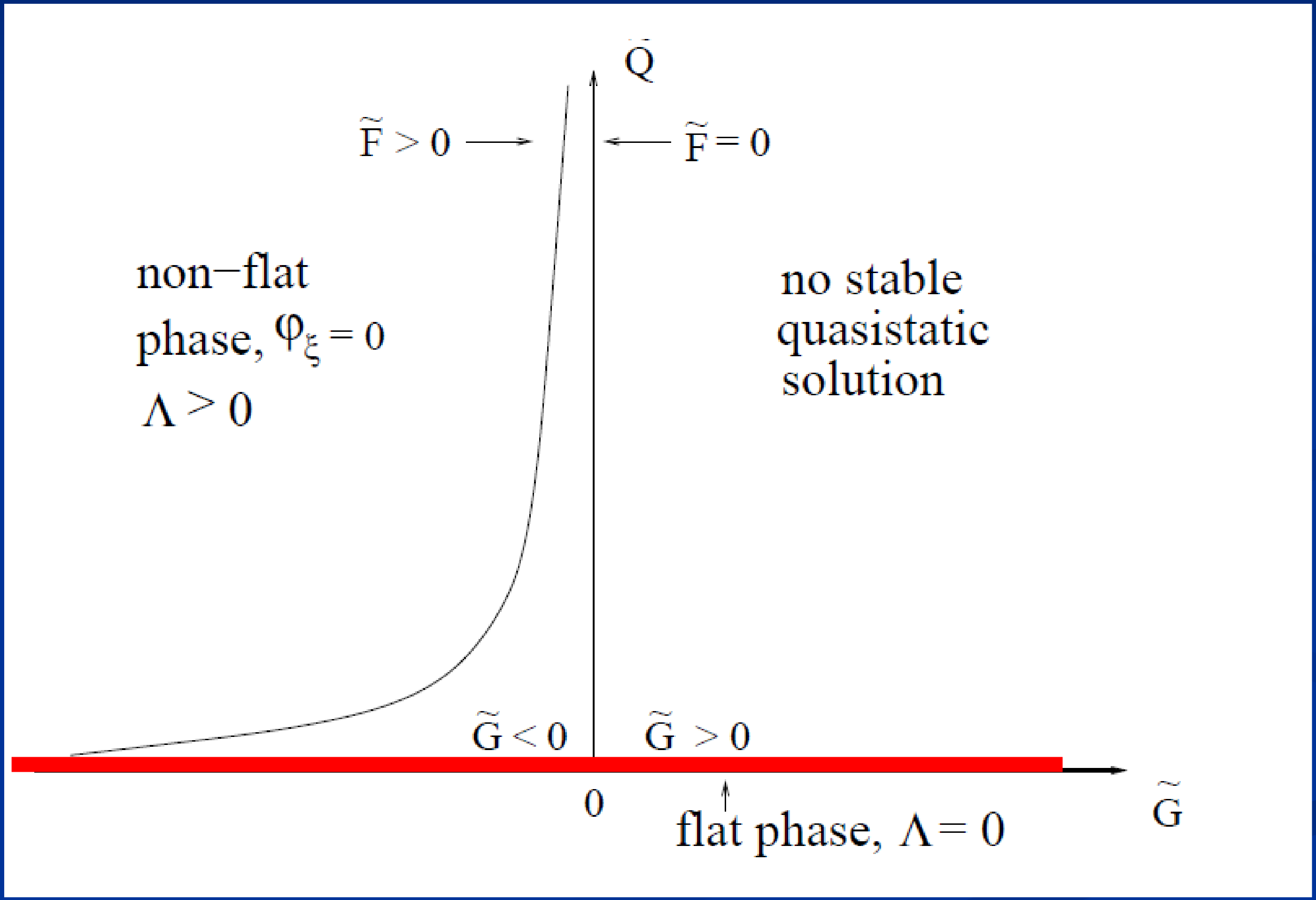
potential and effective Planck mass depend on scalar fields

$$V = \tilde{Q}\varphi_\xi^2\varphi_l^2 + \tilde{F}\varphi_l^4$$

$$\chi^2 = \varphi_\xi^2 - 2\tilde{G}\varphi_l^2$$

$$W = \tilde{Q}\varphi_\xi^2\varphi_l^2 + \tilde{F}\varphi_l^4 - 2\Lambda\varphi_\xi^2 + 4\tilde{G}\Lambda\varphi_l^2$$

phase diagram



stable solutions

phase structure of solutions

- solutions in flat phase exist for arbitrary values of effective parameters of higher dimensional effective action

- question : how “big” is flat phase

  (which internal geometries and warpings are possible beyond torus solutions)

- solutions in non-flat phase only exist for restricted parameter ranges

self tuning

for all solutions in flat phase :

self tuning of cosmological constant to zero !

self tuning

for simplicity : no contribution of F to V

$$V = \tilde{Q}\bar{\xi}^2 l^{D-2} + \tilde{F} l^{-4}$$

assume Q depends on parameter α , which characterizes internal geometry:

tuning required : $$\frac{\partial \tilde{Q}(\alpha)}{\partial \alpha}_{|\alpha_0} = 0$$ **and** $$\tilde{Q}(\alpha_0) = 0.$$

self tuning in higher dimensions

Q depends on higher dimensional fields $$\tilde{Q} = \tilde{R}[\alpha(y)]$$

extremum condition
amounts to field equations $$\frac{\delta \tilde{R}}{\delta \alpha(y)} = 0$$

typical solutions depend on integration constants γ

solutions obeying boundary condition exist :

$$\tilde{R}[\alpha_0(y;\gamma_i)] = 0$$

self tuning in higher dimensions

- involves infinitely many degrees of freedom !
- for arbitrary parameters in effective action :
  flat phase solutions are present
- extrema of W exist $\bar{W} = \int_y (g^{(D)}(y))^{1/2} \sigma^2 \mathcal{L}(y)$
- for flat 4-d-space : W is functional of internal geometry, independent of x $\hat{g}_{\hat{\mu}\hat{\nu}}(y) = \begin{pmatrix} \sigma(y)\eta_{\mu\nu} & , & 0 \\ 0 & , & g^{(D)}_{\alpha\beta}(y) \end{pmatrix}$
- solve field equations for internal metric and σ and ξ

Dark energy

if cosmic runaway solution has not yet reached fixed point :

dilatation symmetry of field equations

not yet exact

“ dilatation anomaly “

non-vanishing effective potential V in reduced four –dimensional theory

Time dependent Dark Energy : Quintessence

- What changes in time ?

- **Only dimensionless ratios of mass scales are observable !**

- V : potential energy of scalar field or cosmological constant
- V/M^4 is observable

- **Imagine the Planck mass M increases …**

Cosmon and fundamental mass scale

- Assume all mass parameters are proportional to scalar field χ (GUTs, superstrings,…)
- $M_p \sim \chi$, $m_{proton} \sim \chi$, $\Lambda_{QCD} \sim \chi$, $M_W \sim \chi$,…

- χ may evolve with time : **cosmon**
- m_n/M : (almost) constant - ***observation*** !

Only ratios of mass scales are observable

theory without explicit mass scale

- Lagrange density:

$$L = \sqrt{g}\left(-\frac{1}{2}\chi^2 R + \frac{1}{2}(\delta - 6)\partial^\mu \chi \partial_\mu \chi + V(\chi) + h\chi\overline{\psi}\psi\right)$$

recall : $\chi^2 = l^D \bar{\xi}^2 - 2\tilde{G} l^{-2}$

realistic theory

- χ has no gauge interactions
- χ is effective scalar field after "integrating out" all other scalar fields

four dimensional dilatation symmetry

- Lagrange density:

$$L = \sqrt{g}\left(-\frac{1}{2}\chi^2 R + \frac{1}{2}(\delta - 6)\partial^\mu \chi \partial_\mu \chi + V(\chi) + h\chi\overline{\psi}\psi\right)$$

- Dilatation symmetry for

$$V = \lambda\chi^4, \lambda = const., \delta = const., h = const.$$

- Conformal symmetry for $\delta=0$
- Asymptotic flat phase solution : $\lambda = 0$

Asymptotically vanishing effective "cosmological constant"

- Effective cosmological constant ~ V/M^4
- dilatation anomaly : $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A}\ \chi^4$ → $\mathbf{V/M^4 \sim (\chi/\mu)^{-A}}$
- $M = \chi$

It is sufficient that V increases less fast than χ^4 !

dilatation anomaly

example :

higher dimensional cosmological constant $\sim \mu^d$

$A = d$

for $\chi \rightarrow \infty$: contribution becomes irrelevant

Cosmology

Cosmology : χ increases with time !

(due to coupling of χ to curvature scalar)

for large χ the ratio V/M^4 decreases to zero

$\Rightarrow$

Effective cosmological constant vanishes asymptotically for large t !

Weyl scaling

Weyl scaling : $g_{\mu\nu} \rightarrow (M/\chi)^2 g_{\mu\nu}$,

$$\varphi/M = \ln(\chi^4/V(\chi))$$

$$L = \sqrt{g}\left(-\frac{1}{2}M^2R + \frac{1}{2}k^2(\phi)\partial^\mu\phi\partial_\mu\phi + V(\phi) + m(\phi)\overline{\psi}\psi\right)$$

Exponential potential : $V = M^4 \exp(-\varphi/M)$

No additional constant !

quantum fluctuations and dilatation anomaly

Dilatation anomaly

- Quantum fluctuations responsible both for fixed point and dilatation anomaly close to fxed point
- Running couplings: **hypothesis**

$$\partial\lambda/\partial \ln\chi = -A\lambda$$

- Renormalization scale μ : (momentum scale)
- $\lambda \sim (\chi/\mu)^{-A}$

Asymptotic behavior of effective potential

- $\lambda \sim (\chi/\mu)^{-A}$
- $V \sim (\chi/\mu)^{-A}\ \chi^4$

$$\mathbf{V \sim \chi^{4-A}}$$

crucial : behavior for large χ !

Without dilatation – anomaly :

V= const.

Massless Goldstone boson = dilaton

Dilatation – anomaly :

V (φ)

Scalar with tiny time dependent mass :

cosmon

Dilatation anomaly and quantum fluctuations

- Computation of running couplings (beta functions) needs unified theory !
- Dominant contribution from modes with momenta ~χ !
- No prejudice on “natural value “ of location of fixed point or anomalous dimension should be inferred from tiny contributions at QCD-momentum scale !

quantum fluctuations and naturalness

- Jordan- and Einstein frame completely equivalent on level of effective action and field equations (**after** computation of quantum fluctuations !)
- Treatment of quantum fluctuations depends on frame : Jacobian for variable transformation in functional integral
- What is natural in one frame may look unnatural in another frame

quantum fluctuations and frames

- Einstein frame : quantum fluctuations make zero cosmological constant look unnatural
- Jordan frame : quantum fluctuations can be the origin of dilatation anomaly;
- may be key ingredient for solution of cosmological constant problem !

fixed points and fluctuation contributions of individual components

If running couplings influenced by fixed points:
individual fluctuation contribution can be huge overestimate !

here : fixed point at vanishing quartic coupling and anomalous dimension ➡ $V \sim \chi^{4-A}$

it makes no sense to use naïve scaling argument to infer individual contribution $V \sim h\,\chi^{4}$

conclusions

- naturalness of cosmological constant and cosmon potential should be discussed in the light of dilatation symmetry and its anomalies
- Jordan frame
- higher dimensional setting
- four dimensional Einstein frame and naïve estimate of individual contributions can be very misleading !

conclusions

cosmic runaway towards fixed point may

solve the cosmological constant problem

and

account for dynamical Dark Energy

End