Axions: Models
Student Seminar on Non-Accelerator Particle Physics

Rahul Mehra

Rheinische Friedrich-Wilhelms-Universität Bonn

June 10 2016
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In the limit of massless quarks \((m_u, m_d, m_s \approx 0)\), QCD has a global flavor chiral symmetry \(SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A\).

The axial \(U(1)_A\) neither represents a symmetry nor is spontaneously broken.

Solution: \(U(1)_A\) exhibits a quantum anomaly!
\[ \partial^\mu J^5_\mu = \frac{g_s^2}{32\pi^2} G \cdot \tilde{G} \]

- This non-zero divergence for \( J^5_\mu \) and the complex non-perturbative nature of the QCD vacuum lead us to the \( \theta \) term of QCD and the **Strong CP problem**.

- An extra term to the QCD Lagrangian is added:

\[ \mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^{\mu\nu}_b \tilde{G}^{\mu\nu}_b \]

- Complete QCD Lagrangian: \( \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \theta \frac{g_s^2}{32\pi^2} G^{\mu\nu}_b \tilde{G}^{\mu\nu}_b \)
Strong CP problem

- Why does $G \cdot \tilde{G}$ matters whereas $F \cdot \tilde{F}$ does not?

- The non-perturbative effects in QCD give rise to **instantons** $\Rightarrow$ the true vacuum is a linear superposition of $n$ different configurations:

$$|\theta\rangle = \sum_n e^{-in\theta} |n\rangle$$

- If one also includes the weak interactions, the quark mass matrix:

$$\mathcal{L}_{\text{mass}} = \bar{q}_i M_{ij} q_j + h.c.$$
Strong CP problem

- Chiral rotations on the quarks changes $\theta$ by $\text{arg } \det M$, and hence
  \[ \bar{\theta} = \theta + \text{argdet } M \]
- The CP violating theta term induces a neutron dipole moment.
- The strong experimental bound $|d_n| < 3 \times 10^{-26} \text{ ecm}$ implies that $\bar{\theta} < 10^{-9}$

Strong CP problem: Why is this $\bar{\theta}$ angle coming from the strong and weak interaction, so small?
There can be three possible solutions to the strong CP problem:
- Unconventional dynamics
- Spontaneously broken CP
- An additional chiral symmetry

Examples of unconventional dynamics:
- Raising the number of spatial dimensions à la Kaluza-Klein leads to the vanishing of the $\theta$ problem but the $U(1)_A$ still remains! [Khlebnikov and Shaposhnikov, 1988]
- Use the periodicity of vacuum energy $E(\theta) \sim \cos\theta$ to deduce that $\theta$ vanishes but no motivation for minimisation of vacuum energy. [Schierholz, 1994]
Spontaneous CP violation (SCPV)

- Spontaneous CP breaking occurs when CP is a symmetry of the original Lagrangian but after SSB, no CP symmetry remains.

- To get $\bar{\theta} < 10^{-9}$, one needs to ensure that $\bar{\theta}$ also vanishes at the 1-loop level.

- One can have multiple Higgs doublets with complex VEVs achieving this but FCNCs rear its ugly head!
Only a symmetry based solution to the Strong CP problem is seen as natural one!

The idea is to take the parameter $\bar{\theta}$ and promote it to a dynamical variable such that $\bar{\theta} \neq 0$ emerges.

Add to the SM - a global $U(1)_{PQ}$ symmetry - known as the Peccei-Quinn symmetry.
The quarks and the Higgs multiplets transform non-trivially under this $U(1)_{PQ}$

Weinberg and Wilczek pointed that since a $U(1)_{PQ}$ is a global continuous symmetry spontaneously broken by the vacuum, there has to be a Goldstone boson! $\implies$ **Axion**

The axion is a pseudo Nambu-Goldstone boson of the 'anomalous' $U(1)_{PQ}$ symmetry.

[1]
$U(1)_{PQ}$ and Axions

- Under a $U(1)_{PQ}$ transformation, the axion field translates to
  \[ a(x) \xrightarrow{U(1)_{PQ}} a(x) + \alpha f_a \]

  where $f_a$ is the order parameter associated with the breaking of $U(1)_{PQ}$.

- To make SM $U(1)_{PQ}$ invariant, the Lagrangian needs to be augmented by axion interactions:
  \[
  \mathcal{L}_{\text{total}} = \mathcal{L}_{\text{SM}} + \bar{\theta} \frac{g_s^2}{32\pi^2} G_{a}^{\mu\nu} \tilde{G}_{a\mu\nu} - \frac{1}{2} \partial_{\mu} a \partial^{\mu} a + \mathcal{L}_{\text{int}}(\partial^{\mu} a / f_a, \psi)
  \]
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where $\xi$ is a model dependent parameter.

We have added an axion mass term!
**$U(1)_{PQ}$ and Axions**

- The added term is essential for ensuring that $U(1)_{PQ}$ has a chiral anomaly.

\[ \partial_\mu J^\mu_{PQ} = \xi \frac{g_S^2}{32\pi^2} G^\mu_\nu \tilde{G}_{\mu\nu} \]

- Non-pert. QCD effects generate a potential for the axion field which is periodic in the effective vacuum angle

\[ V_{\text{eff}} \sim \Lambda^4_{QCD} \left( 1 - \cos \left( \bar{\theta} + \xi \frac{\langle a \rangle}{f_a} \right) \right) \]

- Hence, the potential is minimized to give the PQ solution:

\[ \langle a \rangle = -\frac{f_a}{\xi} \bar{\theta} \]

- With the PQ solution, the Lagrangian no longer has a CP-violating $\bar{\theta}$ term.
In the original PQ model, the $U(1)_{PQ}$ breaking scale was taken to be the electroweak scale $f_a = v_F$, with $v_F \approx 250$ GeV.

If $f_a \gg v_F$ then the axion is very light, very weakly coupled and very long lived \(\Rightarrow\) Invisible axion models

To make the SM $U(1)_{PQ}$ invariant, one must introduce two Higgs fields to absorb independent chiral transformations of the up- and down-quarks (and leptons).

\[
\mathcal{L}_{Yukawa} = \Gamma_{ij}^u \bar{Q}_L \Phi_1 u_{Rj} + \Gamma_{ij}^d \bar{Q}_L \Phi_2 d_{Rj} + \Gamma_{ij}^l \bar{L}_L \Phi_2 l_{Rj} + h.c.
\]

\[
\begin{array}{cccccccc}
\Phi_1 & \Phi_2 & Q & \bar{u} & \bar{d} & L & \bar{e} \\
Y & 1 & -1 & \frac{1}{3} & -\frac{4}{3} & \frac{2}{3} & -1 & 2 \\
PQ & 1 & 1 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}
\end{array}
\]

**Table:** Assignment of Hypercharge $Y$ and Peccei-Quinn charge $PQ$
\[ \mathcal{L}_{\text{Yukawa}} = \Gamma_{ij}^u \bar{Q}_i \Phi_1 u_R^j + \Gamma_{ij}^d \bar{Q}_i \Phi_2 d_R^j + \Gamma_{ij}^l \bar{L}_i \Phi_2 l_R^j + \text{h.c.} \]

- Defining \( x = \frac{v_2}{v_1} \) and \( v_F = \sqrt{v_1^2 + v_2^2} \), the axion is the common phase field in \( \Phi_1 \) and \( \Phi_2 \):

\[
\Phi_1 = \frac{v_1}{\sqrt{2}} e^{iax/v_F} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \Phi_2 = \frac{v_2}{\sqrt{2}} e^{ia/xv_F} \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

- It is clear that \( \mathcal{L}_{\text{Yukawa}} \) is invariant under the \( U(1)_{PQ} \) transformations:

\[
a \to a + \alpha v_F; \quad u_R^j \to e^{-i\alpha x} u_R^j; \\
d_R^j \to e^{-i\alpha/x} d_R^j; \quad l_R^j \to e^{-i\alpha/x} l_R^j
\]
The symmetry current for $U(1)_{PQ}$ can then be calculated as:

$$J_{PQ}^\mu = -v_F \partial^\mu a + x \sum_i \bar{u}_{iR} \gamma^\mu u_{iR} + \frac{1}{x} \sum_i \bar{d}_{iR} \gamma^\mu d_{iR}$$

$$m_a^2 = \left\langle \frac{\partial^2 V_{eff}}{\partial a^2} \right\rangle = -\frac{\xi}{f_a} \frac{g_S^2}{32\pi^2} \partial \left\langle G_{\mu\nu}^b \tilde{G}_{b\mu\nu}^b \right\rangle \bigg|_{a=-\tilde{f}_a/\xi}$$

To compute the axion mass, one can either use current algebra techniques or use an effective Lagrangian approach.

The mass of the axion for the standard PQ model is:

$$m_a = m_\pi \frac{f_\pi}{v_F} \frac{\sqrt{m_u m_d}}{m_u + m_d} N_g \left( x + \frac{1}{x} \right) \simeq 75 \left( x + \frac{1}{x} \right) \text{keV}$$
The PQ model where \( f_a = v_F \) has been ruled out by experimental results.

For example: \( BR(K^+ \rightarrow \pi^+ + a)_{\text{theo}} = 3 \times 10^{-5} (x + 1/x)^2 \) which is well above the bounds \( BR(K^+ \rightarrow \pi^+ + X^0)_{\text{exp}} < 3.8 \times 10^{-8} \).

Invisible axion models introduce scalar fields that carry PQ charge but are \( SU(2) \times U(1) \) singlets, and hence \( f_a \gg v_F \) remains possible.

Two types of models:
- **KSVZ** (Kim, Shifman, Vainshtein and Zakharov) model
- **DFSZ** (Dine, Fischer, Srednicki and Zhitnisky) model
In this model, the following were introduced

- a scalar field \( \sigma \) with \( f_a = \langle \sigma \rangle \gg v_F \)
- a super-heavy quark \( X \) with \( M_X \sim f_a \)

as the only fields carrying PQ charge.

- Ordinary quarks and leptons are PQ singlets.

- The \( SU(2) \times U(1) \) singlet field \( \sigma \) interacts with the new heavy quark \( X \) which carry PQ charge via the Yukawa interaction:

\[
L^{KSVZ}_{\text{Yukawa}} = -h \bar{X}_L \sigma X_R - h^* \bar{X}_R \sigma^\dagger X_L
\]

- By construction, KVSZ axion doesn’t interact with leptons and interacts only with light quarks due to strong and electromagnetic anomalies:

\[
L_{\text{anomaly}} = \frac{a}{f_a} \left( \frac{g_S^2}{32\pi^2} G_b^{\mu\nu} \tilde{G}_{b\mu\nu} + 3q_X^2 \frac{\alpha}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu} \right)
\]
The Higgs potential is taken to be:

\[ V(\phi, \sigma) = -\mu_{\phi}^2 \phi^\dagger \phi - \mu_{\sigma}^2 \sigma^* \sigma + \lambda_{\phi} (\phi^\dagger \phi)^2 + \lambda_{\sigma} (\sigma^* \sigma)^2 + \lambda_{\phi\sigma} \phi^\dagger \phi \sigma^* \sigma \]

where

\[ f_a = \langle \sigma \rangle \gg v_F \]

For the Yukawa and the form of the potential to be \( U(1)_{PQ} \) invariant, the following transformation should hold:

\[ Q \rightarrow e^{i\gamma_5 \alpha} Q \quad \sigma \rightarrow e^{-2i\alpha} \sigma \]

The axion mass is given by:

\[ m_a = \left( m_{\pi} \frac{f_\pi \sqrt{m_u m_d}}{v_F (m_u + m_d)} \right) \frac{v_F}{f_a} = 6.3 \text{eV} \left( \frac{10^6}{f_a \text{GeV}} \right) \]
DFSZ model

- Adds to the PQ model a complex scalar field $\zeta$ which carries PQ charge but is a singlet under $SU(2) \times U(1)$

- As before, the model has two Higgs fields, $\Phi_1$ and $\Phi_2$ since both the quarks and leptons carry $U(1)_{PQ}$.

- The quarks and leptons feel the effects of the axions only through the interactions that the field $\zeta$ has with $\Phi_1$ and $\Phi_2$ in the Higgs potential. (quartic coupling $\mathcal{L}_{\zeta H} \supset \Phi_1^T C(\zeta^\dagger)^2 \Phi_2$)

\[
\Phi_1 = \frac{v_1}{\sqrt{2}} e^{i(x_1/f_a) a} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \Phi_2 = \frac{v_2}{\sqrt{2}} e^{i(x_2/f_a) a} \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

\[
\zeta = \frac{v_F}{\sqrt{2}} \exp \left( \frac{ia}{v_F} \right)
\]

where $v_F^2 = v_1^2 + v_2^2$, and $X_1 = \frac{2v_2^2}{v_F^2}; \quad X_2 = \frac{2v_1^2}{v_F^2}$
Idea is to represent the dynamical content of a theory in the low energy limit and incorporate heavy particles in a different manner.

Write the general Lagrangian consistent with symmetries and include only a few - relevant for low energies:

\[
\mathcal{L} = \bar{\psi} i \slashed{\partial} \psi + \frac{1}{2} \partial_\mu \pi \cdot \partial^\mu \pi + \frac{1}{2} \partial_\mu \sigma \cdot \partial^\mu \sigma - g (\bar{\psi} \sigma - i \bar{\pi} \cdot \vec{\sigma} \gamma_5)
+ \frac{1}{2} \mu^2 \left( \sigma^2 + \pi^2 \right) - \frac{\lambda}{4} \left( \sigma^2 + \pi^2 \right)^2
\]

If one works at low energy, all the matrix elements can be contained in \( \mathcal{L}_{\text{eff}} \):

\[
\mathcal{L}_{\text{eff}} = \frac{F^2}{4} \text{Tr} \left( \partial_\mu U \partial^\mu U^\dagger \right)
\]

where \( U = \exp \left( i \frac{\vec{\tau} \cdot \vec{\pi}}{F} \right) \) and \( F = v = \sqrt{\frac{\mu^2}{\lambda}} \).
For the interaction of axion with light quarks → construct an appropriate effective chiral Lagrangian

Effects of heavy quarks → contribution to the anomaly of $J_{PQ}^\mu$!

For two light quarks, one can introduce a $2 \times 2$ matrix of NG fields:

$$\Sigma = \exp \left( \frac{i \bar{\tau} \cdot \bar{\pi} + \eta}{f_\pi} \right)$$
Calculations in DFSZ

- The $U(2)_V \times U(2)_A$ invariant effective Lagrangian for the meson sector of the light-quarks is (no Yukawa yet!):

$$\mathcal{L}_{\text{chiral}} = -\frac{f_\pi^2}{4} \text{Tr} \left( \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right)$$

- We now add $U(2)_V \times U(2)_A$ breaking terms - mimicking the $U(1)_{PQ}$ invariant Yukawa interactions of the u- and d-quarks:

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} (f_\pi m_\pi^0)^2 \text{Tr} \left[ \Sigma A M + (\Sigma A M)^\dagger \right]$$

where

$$A = \begin{pmatrix} e^{-i a X_1 / v_F} & 0 \\ 0 & e^{-i a X_2 / v_F} \end{pmatrix} \quad M = \begin{pmatrix} m_u & 0 \\ m_u + m_d & 0 \\ 0 & m_d \end{pmatrix}$$

and $U(1)_{PQ}$ of $\mathcal{L}_{\text{mass}}$ requires:

$$\Sigma \longrightarrow \Sigma \begin{pmatrix} e^{i \alpha X_1} & 0 \\ 0 & e^{i \alpha X_2} \end{pmatrix}$$
The quadratic terms in $L_{mass}$ involving neutral fields still has a problem similar to $U(1)_A$ i.e. incorrect mass ratio of $\eta$ and $\pi$

$$L^{(2)}_{mass} = -\frac{(m_\pi^0)^2}{2} \left[ \frac{m_u}{m_u + m_d} \left( \pi^0 + \eta - \frac{X_1 f_\pi}{v_F} a \right)^2 + \frac{m_d}{m_u + m_d} \left( \eta - \pi^0 - \frac{X_2 f_\pi}{v_F} a \right)^2 \right]$$

Resolving this $U(1)_A$ problem in effective Lagrangian theory is done by adding another mass term that takes care of the anomaly in both $U(1)_A$ and $U(1)_{PQ}$

$$L_{anomaly} = -\frac{(m_\eta^0)^2}{2} \left[ \eta + \frac{f_\pi}{v_F} \frac{(N_g - 1)}{2} (X_1 + X_2) a \right]^2$$
Axion mass in DFSZ

- Diagonalization of $L_{\text{mass}}$ and $L_{\text{anomaly}}$ gives the axion mass and other parameters ($\xi_{a\pi}$, $\xi_{a\eta}$)

$$m_a = \left(m_\pi \frac{f_\pi}{v_F} \sqrt{m_u m_d} \right) \frac{v_F}{f_a} = 6.3\text{eV} \left( \frac{10^6}{f_a} \text{GeV} \right)$$

- Axion models are in addition characterised by the coupling of the axion to two photons: $K_{a\gamma\gamma}$

- Axion interactions with fermions also generate model dependent couplings

$$L_{af} = -g_{af} \bar{\psi}_f i\gamma^5 \psi_f a$$

$$g_{af} = \frac{C_f m_f}{f_a}; \quad \alpha_{af} = \frac{g_{af}^2}{4\pi}$$
Axions can decay to two photons; this allows stars like our sun to produce axions by transforming a photon into an axion \( \rightarrow \) searches for solar axion flux.

\[
\text{axion} \rightarrow \text{photon}
\]

Axion emission then also provides a pathway for energy loss in stars; bounds from lifetimes of stars, red giants and SN 1987a tell us that

\[
f_a \geq 10^9 \text{GeV} \implies m_a \leq 10^{-2} \text{eV}
\]
The lower bound on axion mass comes from cosmological arguments where the candidacy of axion for cold dark matter is used.

Recent WMAP data on cold dark matter tell us that:

\[ f_a \leq 10^{12}\, GeV \iff m_a \geq 10^{-6}\, eV \]

Hence, we have a small window on the mass of the axion:

\[ 10^{-2}\, eV \geq m_a \geq 10^{-6}\, eV \]
Microwave Cavity detection of axion at ADMX, University of Washington

$U(1)_{A}$ and Strong CP problem are intertwined.

- Peccei-Quinn symmetry - a natural solution to the Strong CP problem.
- Visible axion models (electroweak scale) have been ruled out.
- Invisible axion (KSVZ and DFSZ) models provide hope!
- Astrophysical and cosmological arguments provide bounds on axion mass and couplings.
Cleaning up the SM!

[Frank Wiczek]

A cartoon by N.N. Cabibbo.

*Proceedings of the XIth International Conference in High Energy Physics*
Extension to SM 1

THE ANOMALY AWAKENS

CP invariance has vanished. In its absence, the sinister Strong CP has risen from the ashes of QCD and will not rest until the CP invariance has been destroyed.

With the support of the SM, Pececi-Quinn with help from Weinberg and Wilzek use the anomaly to find the axion.

Desperate to find the axion and gain its help in restoring naturalness and symmetry to the SM, they start a daring search.
References

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A simple solution to the strong cp problem with a harmless axion.  

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