Learning from WIMPs

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Introduction: WIMPs as Dark Matter

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Galactic rotation curves imply $\Omega_{DM}h^2 \geq 0.05$.

$\Omega$: Mass density in units of critical density; $\Omega = 1$ means flat Universe.
$h$: Scaled Hubble constant. Observation: $h = 0.72 \pm 0.07$ (?)
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- Models of structure formation, X ray temperature of clusters of galaxies, . . .

- Cosmic Microwave Background anisotropies (WMAP) imply $\Omega_{DM} h^2 = 0.105^{+0.007}_{-0.013}$ Spergel et al., astro-ph/0603449
Weakly Interacting Massive Particles (WIMPs)

- Exist in well-motivated extensions of the SM: SUSY, (Little Higgs with $T$–Parity), ((Universal Extra Dimension))
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- In standard cosmology, roughly weak cross section automatically gives roughly right relic density for thermal WIMPs! (On logarithmic scale)
- Roughly weak interactions may allow both direct and indirect detection of WIMPs
Let $\chi$ be a generic DM particle, $n_\chi$ its number density (unit: GeV$^3$). Assume $\chi = \bar{\chi}$, i.e. $\chi\chi \leftrightarrow$ SM particles is possible, but single production of $\chi$ is forbidden by some symmetry.
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Evolution of $n_\chi$ determined by Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3H n_\chi = -\langle \sigma_{\text{ann}} v \rangle \left( n^2_\chi - n^2_{\chi, \text{eq}} \right)$$

$H = \dot{R}/R$ : Hubble parameter
$\langle \ldots \rangle$ : Thermal averaging
$\sigma_{\text{ann}} = \sigma(\chi\chi \rightarrow \text{SM particles})$
$v$ : relative velocity between $\chi$’s in their cms
$n_{\chi, \text{eq}}$ : $\chi$ density in full equilibrium
Thermal WIMP

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$$n_\chi \langle \sigma_{\text{ann}} v \rangle > H$$
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For $T < m_\chi$: $n_\chi \simeq n_{\chi, \text{eq}} \propto T^{3/2} e^{-m_\chi/T}$, \(H \propto T^2\).
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Inequality cannot be true for arbitrarily small $T$; point where inequality becomes (approximate) equality defines decoupling (freeze-out) temperature $T_F$. 
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For $T < T_F$: WIMP production negligible, only annihilation relevant in Boltzmann equation.
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Gives

$$\Omega_\chi h^2 \propto \frac{1}{\langle v \sigma_{\text{ann}} \rangle} \sim 0.1 \quad \text{for} \quad \sigma_{\text{ann}} \sim \text{pb}$$
Thermal WIMPs: Assumptions

- $\chi$ is effectively stable, $\tau_\chi \gg \tau_U$: partly testable at colliders
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Can we test these assumptions, if $\Omega_\chi$ and “all” particle physics properties of $\chi$ are known?
Low temperature scenario

Assume $T_0 \lesssim T_F$, $n_\chi(T_0) = 0$ ($T_0$: Initial temperature)
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Introduce dimensionless variables
\[ Y_\chi \equiv \frac{n_\chi}{s}, \quad x \equiv \frac{m_\chi}{T} \quad (s: \text{entropy density}). \]
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Use non–relativistic expansion of cross section:

$$ \sigma_{\text{ann}} = a + bv^2 + \mathcal{O}(v^4) \implies \langle \sigma_{\text{ann}} v \rangle = a + 6b/x $$
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![Graph showing $\Omega_\chi h^2$ vs. $a$ with $x_0 = 22$, $\Omega_\chi h^2_{\text{exact}}$, and $\Omega_\chi h^2_{\text{old}}$. The WMAP value is indicated.]
Using explicit form of $H$, $Y_{\chi, \text{eq}}$, Boltzmann eq. becomes

$$
\frac{dY_{\chi}}{dx} = -f \left( a + \frac{6b}{x} \right) x^{-2} \left( Y_{\chi}^2 - cx^3 e^{-2x} \right).
$$

\[ f = 1.32 \, m_\chi M_{\text{Pl}} \sqrt{g_*}, \quad c = 0.0210 \, g_{\chi}^2 / g_* \]
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$f = 1.32 \, m_{\chi} M_{Pl} \sqrt{g_*}$, $c = 0.0210 \, g_{\chi}^2 / g_*^2$

For $T_0 \ll T_F$: Annihilation term $\propto Y_{\chi}^2$ negligible: defines 0-th order solution $Y_0(x)$, with

$$Y_0(x \to \infty) = f \, c \left[ \frac{a}{2} x R e^{-2xR} + \left( \frac{a}{4} + 3b \right) e^{-2xR} \right].$$

Note: $\Omega_{\chi} h^2 \propto \sigma_{\text{ann}}$ in this case!
Low temperature scenario (cont.’d)

Using explicit form of $H$, $Y_{\chi,\text{eq}}$, Boltzmann eq. becomes

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For intermediate temperatures, $T_0 \lesssim T_F$: Define 1st–order solution

$$Y_1 = Y_0 + \delta.$$

$\delta < 0$ describes pure annihilation:

$$\frac{d\delta}{dx} = -f \left( a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}.$$

$\delta(x)$ can be calculated analytically: $\delta \propto \sigma_{\text{ann}}^3$.
Get good results for $\Omega \chi h^2$ for all $T_0 \leq T_F$ through “resummation”:

$$Y_1 = Y_0 \left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$

$Y_{1,r} \propto 1/\sigma_{\text{ann}}$ for $|\delta| \gg Y_0$  

MD, Imminiyaz, Kakizaki, hep-ph/0603165
Numerical comparison: $b = 0$

\begin{align*}
a &= 10^{-8} \text{ GeV}^{-2} \\
a &= 10^{-9} \text{ GeV}^{-2}
\end{align*}
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Can extend validity of new solution to all \( T \), including \( T \gg T_0 \), by using \( \Omega_\chi(T_{\text{max}}) \) if \( T_0 > T_{\text{max}} \approx T_F \).
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Can extend validity of new solution to all $T$, including $T \gg T_0$, by using $\Omega_\chi(T_{\text{max}})$ if $T_0 > T_{\text{max}} \simeq T_F$

Note: $\Omega_\chi(T_0) \leq \Omega_\chi(T_0 \gg T_F)$
If $n_\chi(T_0) = 0$, demanding $\Omega_\chi h^2 \sim 0.1$ imposes lower bound on $T_0$: 
Application: lower bound on $T_0$ for thermal WIMP

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\begin{align*}
\Omega_\chi h^2 &
\begin{array}{c}
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$$\Omega_\chi h^2$$

$\Omega_\chi h^2$ blows up at $T_0$ near $10^{-9}$ GeV.

$$\Rightarrow T_0 \geq \frac{m_\chi}{23}$$

Holds independent of $\sigma_{\text{ann}}$!
Application: lower bound on $T_0$ for thermal WIMP


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$$\implies T_0 \geq \frac{m_\chi}{23}$$

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If $T_0 \sim m_\chi/22$: Get right $\Omega_\chi h^2$ for wide range of cross sections!
Constraining $H(T)$

Assumptions
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- Parameterize modified expansion history:

\[
A(z) = \frac{H_{\text{st}}(z)}{H(z)}, \quad z = T/m_{\chi}
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Constraining $H(T)$

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**Parameterize modified expansion history:**

$$A(z) = \frac{H_{st}(z)}{H(z)} , \ z = \frac{T}{m_\chi}$$

**Around decoupling:** $z \ll 1 \implies$ use Taylor expansion

$$A(z) = A(z_{F, st}) + (z - z_{F, st}) A'(z_{F, st}) + (z - z_{F, st})^2 A''(z_{F, st}) / 2$$
Constraining $H(T)$

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- Successful BBN $\implies k \equiv A(z \to 0) = 1.0 \pm 0.2$
Constraining $H(T)$ (cont.d)

Assume $T_0 \gg T_F \implies \Omega_\chi h^2 \propto \frac{1}{\int_{0}^{z_F} A(z)(a+6bz) \, dz}$
Constraining $H(T)$ (cont.d)

Assume $T_0 \gg T_F \implies \Omega_\chi h^2 \propto \frac{1}{\int_0^{z_F} A(z)(a+6bz)\,dz}$
The case $A''(z_{F,st}) = 0$
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Relative constraint on $A(z_{F,st})$ weaker than that on $\Omega_\chi h^2$. 
Direct WIMP detection

- WIMPs are everywhere!
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  \[ \chi + N \rightarrow \chi + N \]
  Measured quantity: recoil energy of \( N \)
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- Detection needs ultrapure materials in deep–underground location; way to distinguish recoils from \( \beta, \gamma \) events; neutron screening; . . .
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  Measured quantity: recoil energy of \( N \)
- Detection needs ultrapure materials in deep-underground location; way to distinguish recoils from \( \beta, \gamma \) events; neutron screening; . . .
- Is being pursued vigorously around the world!
Direct WIMP detection: theory

Counting rate given by

\[
\frac{dR}{dQ} = AF^2(Q) \int_{v_{\text{min}}}^{v_{\text{esc}}} v f_1(v) \, dv
\]

\(Q\): recoil energy
\(A = \rho \sigma_0 / (2m_{\chi} m_r) = \text{const.: encodes particle physics}\)
\(F(Q)\): nuclear form factor
\(v\): WIMP velocity in lab frame
\(v_{\text{min}}^2 = m_N Q / (2m_r^2)\)
\(v_{\text{esc}}\): Escape velocity from galaxy
\(f_1(v)\): normalized one–dimensional WIMP velocity distribution
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In principle, can invert this relation to measure \( f_1(v) \)!
Direct reconstruction of $f_1$

$$f_1(v) = \mathcal{N} \left\{ -2Q \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \frac{dR}{dQ} \right] \right\}_{Q=2m_T v^2/m_N}$$
Direct reconstruction of $f_1$

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$\mathcal{N}$: Normalization ($\int_0^\infty f_1(v) dv = 1$).
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Need to know form factor $\Longrightarrow$ stick to spin–independent scattering.
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Need to know form factor $\Rightarrow$ stick to spin–independent scattering.
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Need to know slope of recoil spectrum!
$dR/dQ$ is approximately exponential: better work with logarithmic slope
Determining the logarithmic slope of $dR/dQ$

- Good local observable: Average energy transfer $\langle Q \rangle_i$ in $i$–th bin
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- Good local observable: Average energy transfer $\langle Q \rangle_i$ in $i$–th bin

- Stat. error on slope $\propto (\text{bin width})^{-1.5}$ $\implies$ need large bins
Determining the logarithmic slope of $dR/dQ$

- Good local observable: Average energy transfer $\langle Q \rangle_i$ in $i$–th bin
- Stat. error on slope $\propto (\text{bin width})^{-1.5} \implies$ need large bins
- To maximize information: use overlapping bins ("windows")
Recoil spectrum: prediction and simulated measurement

\[ f_1(v) \text{ [s/km]} \]

\( \chi^2 / \text{dof} = 0.73 \)

500 events, 5 bins, up to 3 bins per window

input distribution

v [km/s]
Recoil spectrum: prediction and simulated measurement

χ²/dof = 0.98

5,000 events, 10 bins, up to 4 bins per window

input distribution
Statistical exclusion of constant $f_1$

Average over 1,000 experiments

- Probability vs. $N_{ev}$
- Graph shows the mean and median probability values for different $N_{ev}$ values.
- The probability decreases as $N_{ev}$ increases.

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Statistical exclusion of constant $f_1$

Need several hundred events to begin direct reconstruction!
Determining moments of $f_1$

$$\langle v^n \rangle \equiv \int_0^\infty v^n f_1(v) dv$$
Determining moments of $f_1$

\[
\langle v^n \rangle \equiv \int_0^\infty v^n f_1(v) dv \\
\propto \int_0^\infty Q^{(n-1)/2} \frac{1}{F^2(Q)} \frac{dR}{dQ} dQ
\]
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$$\propto \int_0^\infty Q^{(n-1)/2} \frac{1}{F^2(Q)} \frac{dR}{dQ} dQ$$

$$\rightarrow \sum_{\text{events}} a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}$$
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Can incorporate finite energy (hence velocity) threshold
Determining moments of $f_1$

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Moments are strongly correlated!
Determining moments of $f_1$

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$$ \rightarrow \sum_{\text{events}} a \frac{Q_{a}^{(n-1)/2}}{F^2(Q_{a})}$$

Can incorporate finite energy (hence velocity) threshold

Moments are strongly correlated!

High moments, and their errors, are underestimated in “typical” experiment: get large contribution from large $Q$
Determination of first 10 moments

\[ \frac{\langle v^n \rangle}{\langle v^n \rangle_{\text{exact}}} \]

100 events
Constraining a “late infall” component

\[ \Delta \chi^2 = 1 \]
\[ \Delta \chi^2 = 4 \]

25 events, fit moments \( n = -1, 1, 2 \)
Constraining a “late infall” component

100 events, fit moments $n = -1, 1, 2, 3$

$\Delta \chi^2 = 1$

$\Delta \chi^2 = 4$

$v_{esc}$ [km/s]
Determining the WIMP mass

Can determine $m_\chi$ from requirement that different targets yield same moments of $f_1$.
Learning about the Early Universe:
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If all DM is thermal WIMPs: \( T_0 \geq m_\chi / 23 \sim 10^4 T_{BBN} \)
Learning about the Early Universe:

- If all DM is thermal WIMPs: $T_0 \geq m_\chi/23 \sim 10^4 T_{\text{BBN}}$
- Error on Hubble parameter during WIMP freeze–out somewhat bigger than that on $\Omega_\chi h^2$
Summary

- Learning about the Early Universe:
  - If all DM is thermal WIMPs: \( T_0 \geq \frac{m_\chi}{23} \sim 10^4 T_{\text{BBN}} \)
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- **Learning about WIMPs:** Can determine $m_\chi$ from moments of $f_1$ measured with two different targets.