WIMP Velocity Distribution and Mass from Direct Detection Experiments

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Based on MD, C.–L. Shan, astro-ph/0703651, JCAP **0706**, 011 (2007), and arXiv:0803.4477 [hep-ph] (JCAP, to appear).

Introduction: WIMPs as Dark Matter

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- Models of structure formation, X ray temperature of clusters of galaxies, ...
- Cosmic Microwave Background anisotropies (WMAP) imply $\Omega_{\rm DM} h^2 = 0.105^{+0.007}_{-0.013}$ **Spergel et al., astro–ph/0603449**

Weakly Interacting Massive Particles (WIMPs)

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- Roughly weak interactions may allow both indirect and *direct* detection of WIMPs

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- Is being pursued vigorously around the world!

Direct WIMP detection: theory

Counting rate given by

$$\frac{dR}{dQ} = AF^2(Q) \int_{v_{\rm min}}^{v_{\rm max}} \frac{f_1(v)}{v} dv$$

Q: recoil energy

$A = \rho\sigma_0/(2m_\chi m_r) =$ const.: encodes particle physics

$F(Q)$: nuclear form factor

v: WIMP velocity in lab frame

$v_{\rm min}^2 = m_N Q/(2m_r^2)$

$v_{\rm max}$: Maximal velocity if WIMPs bound to galaxy

$f_1(v)$: normalized one–dimensional WIMP velocity distribution

Note: $Q^2 \propto v^2(1-\cos\theta^*) \;\Rightarrow \frac{d\sigma}{dQ} \propto \frac{1}{v^2}\frac{d\sigma}{d\cos\theta^*}$.

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In principle, can invert this relation to measure $f_1(v)$!

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- Might teach us something about merger history of our own galaxy (e.g. if tidal stream is detected)
- Necessary to check whether this WIMP forms all (local) DM

Direct reconstruction of f_1

$$f_1(v) = \mathcal{N} \left\{ -2Q \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \frac{dR}{dQ} \right] \right\}_{Q=2m_r^2 v^2/m_N}$$

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Need to know *slope* of recoil spectrum!
dR/dQ is approximately exponential: better work with logarithmic slope

Determining the logarithmic slope of dR/dQ

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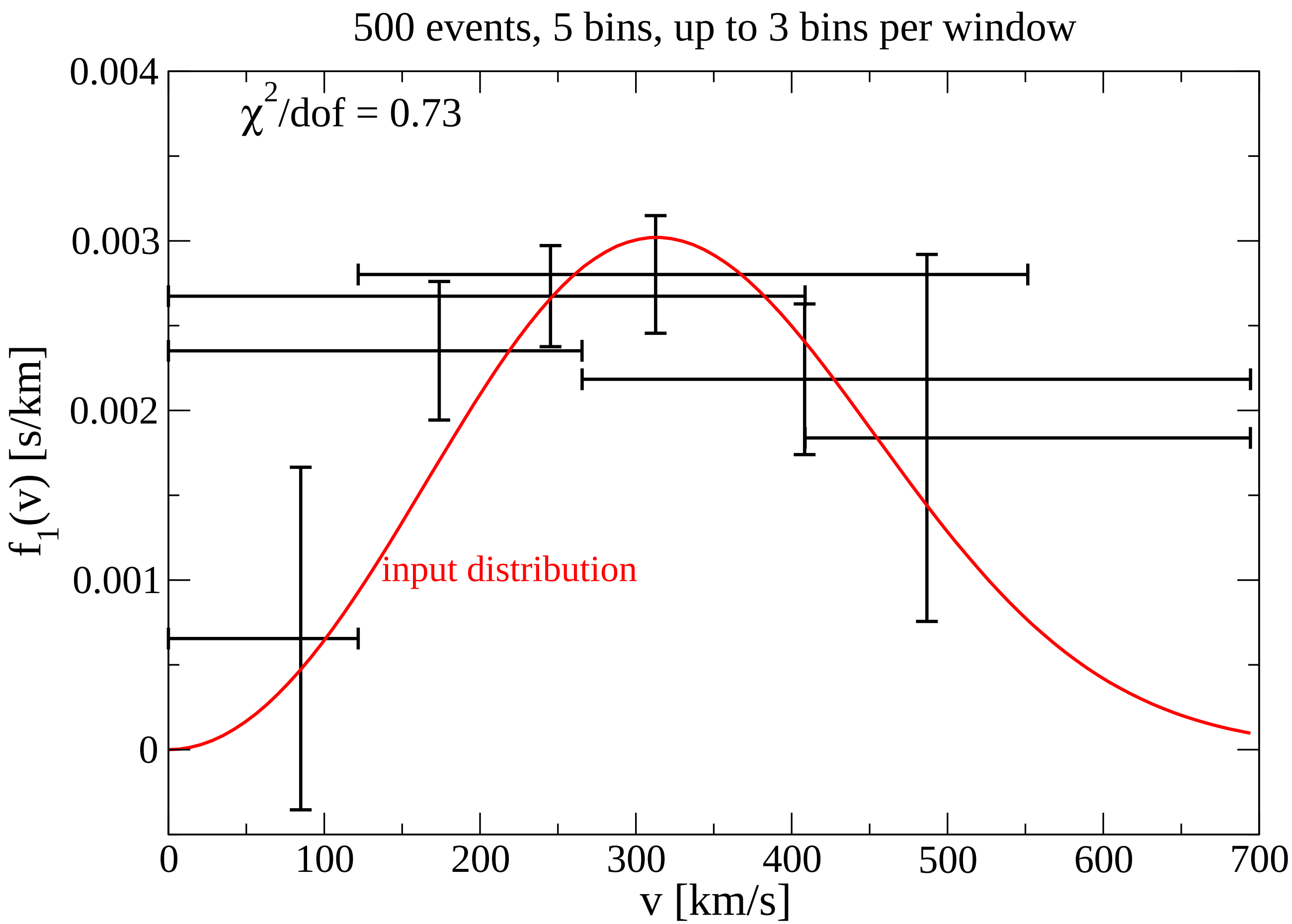
- To maximize information: use overlapping bins (“windows”)

Recoil spectrum: prediction and simulated measurement



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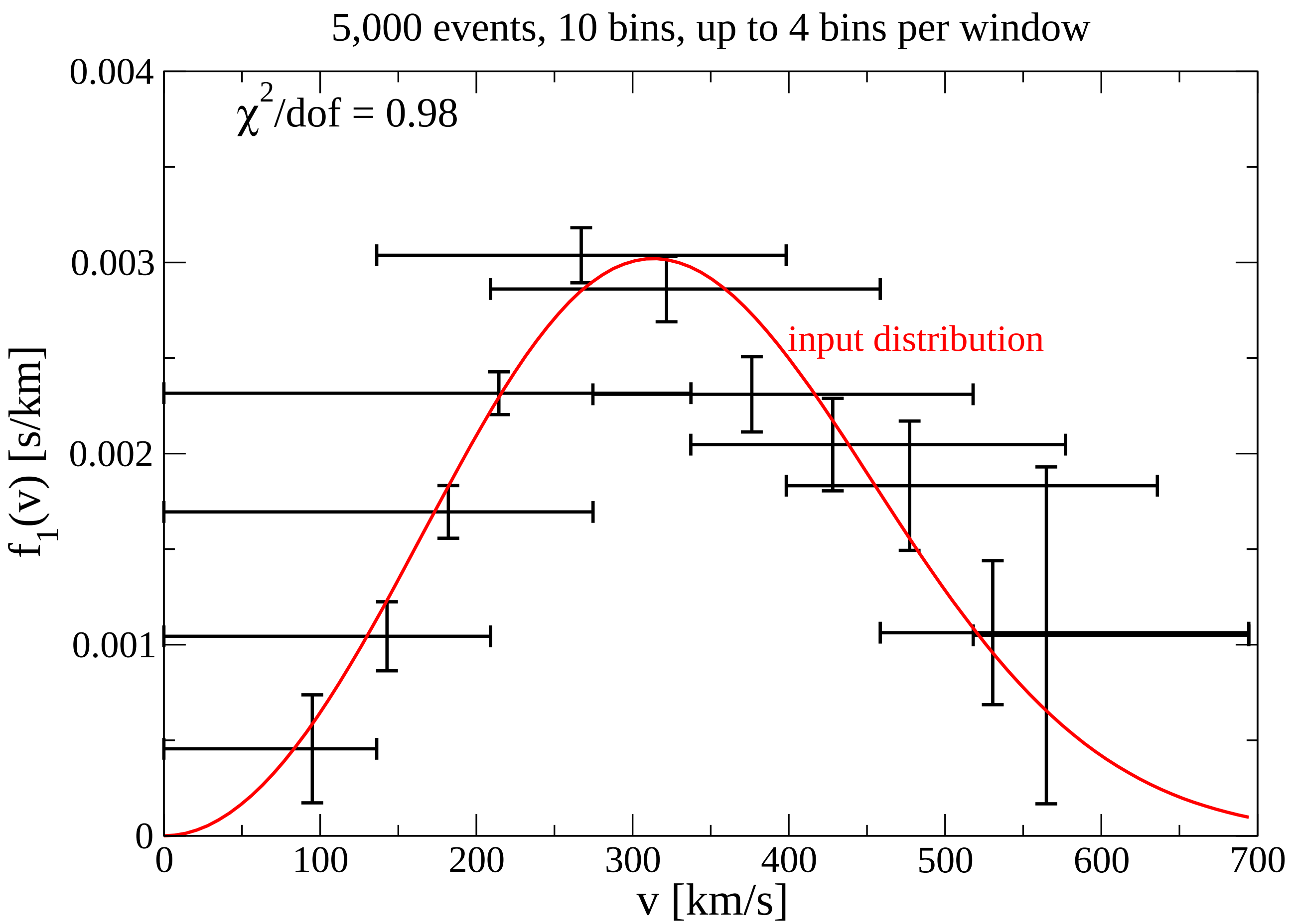
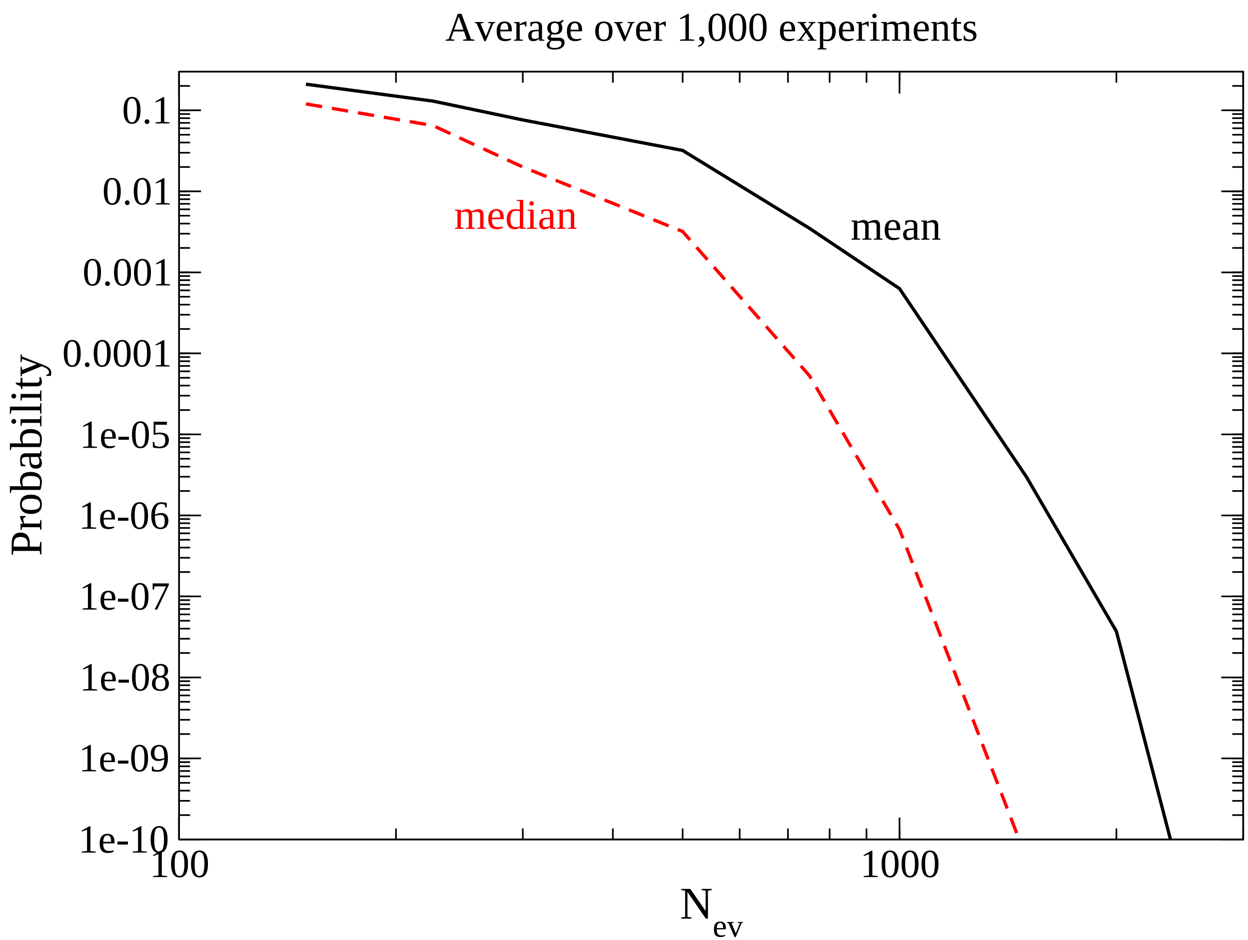


Statistical exclusion of constant f_1



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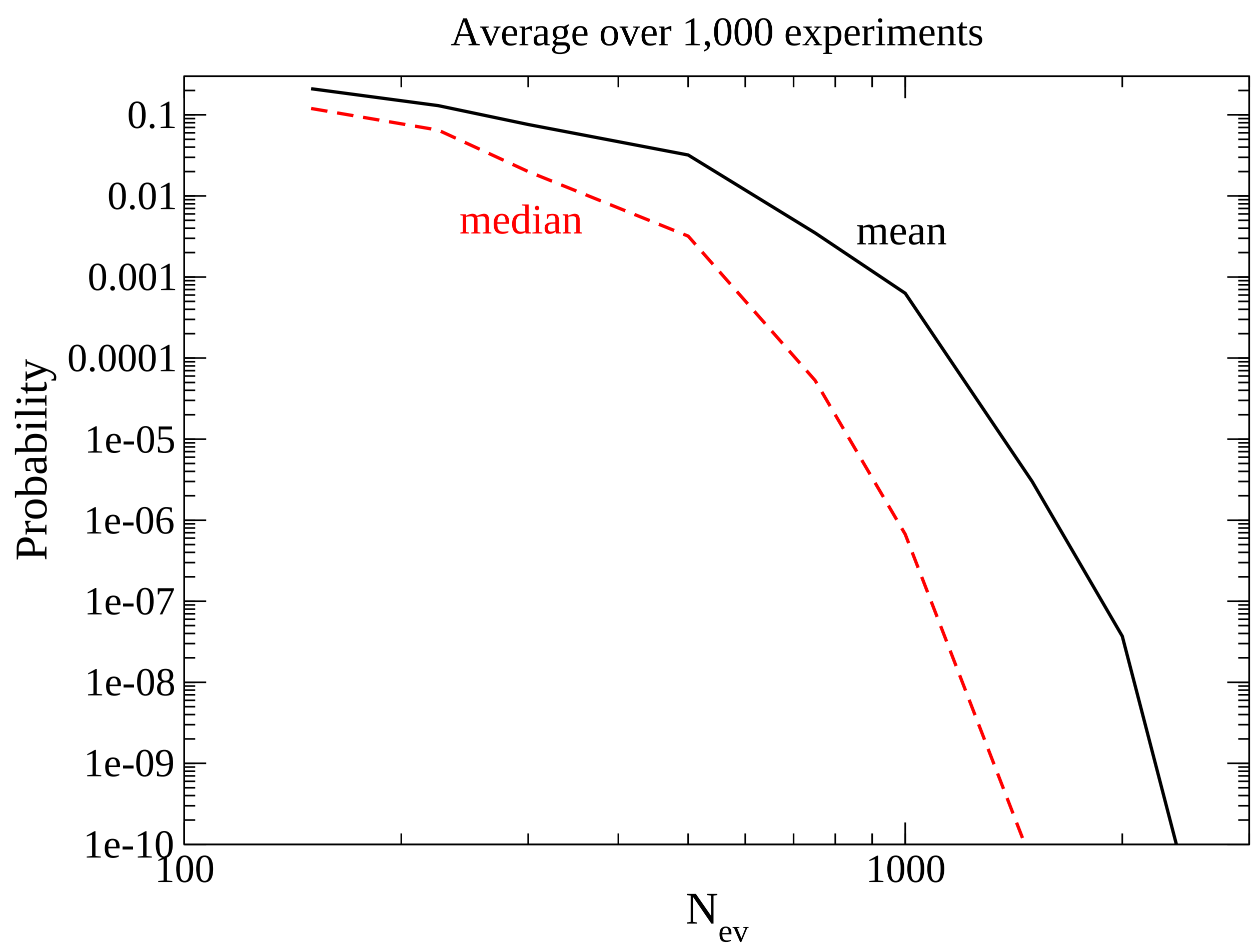

Need several hundred events to begin direct reconstruction!

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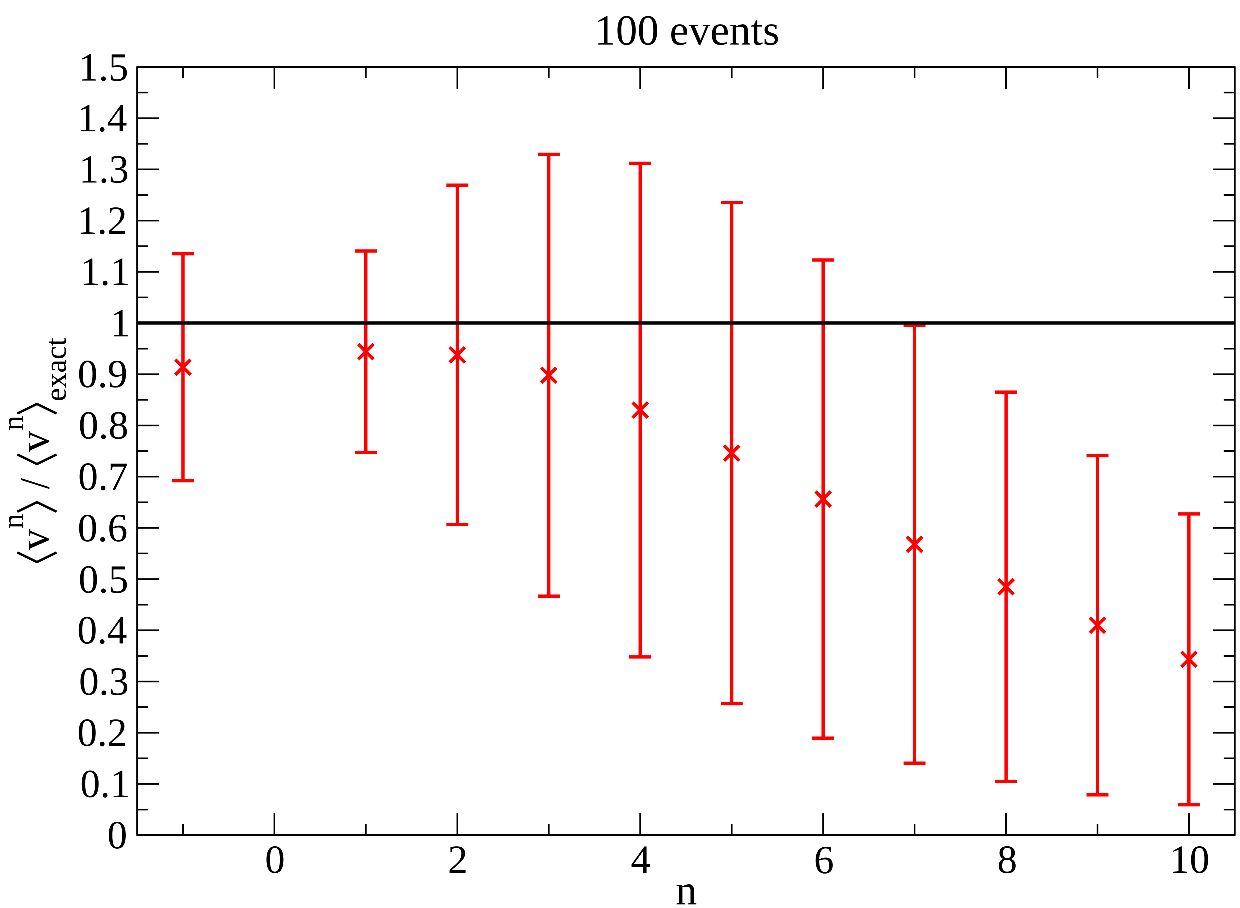
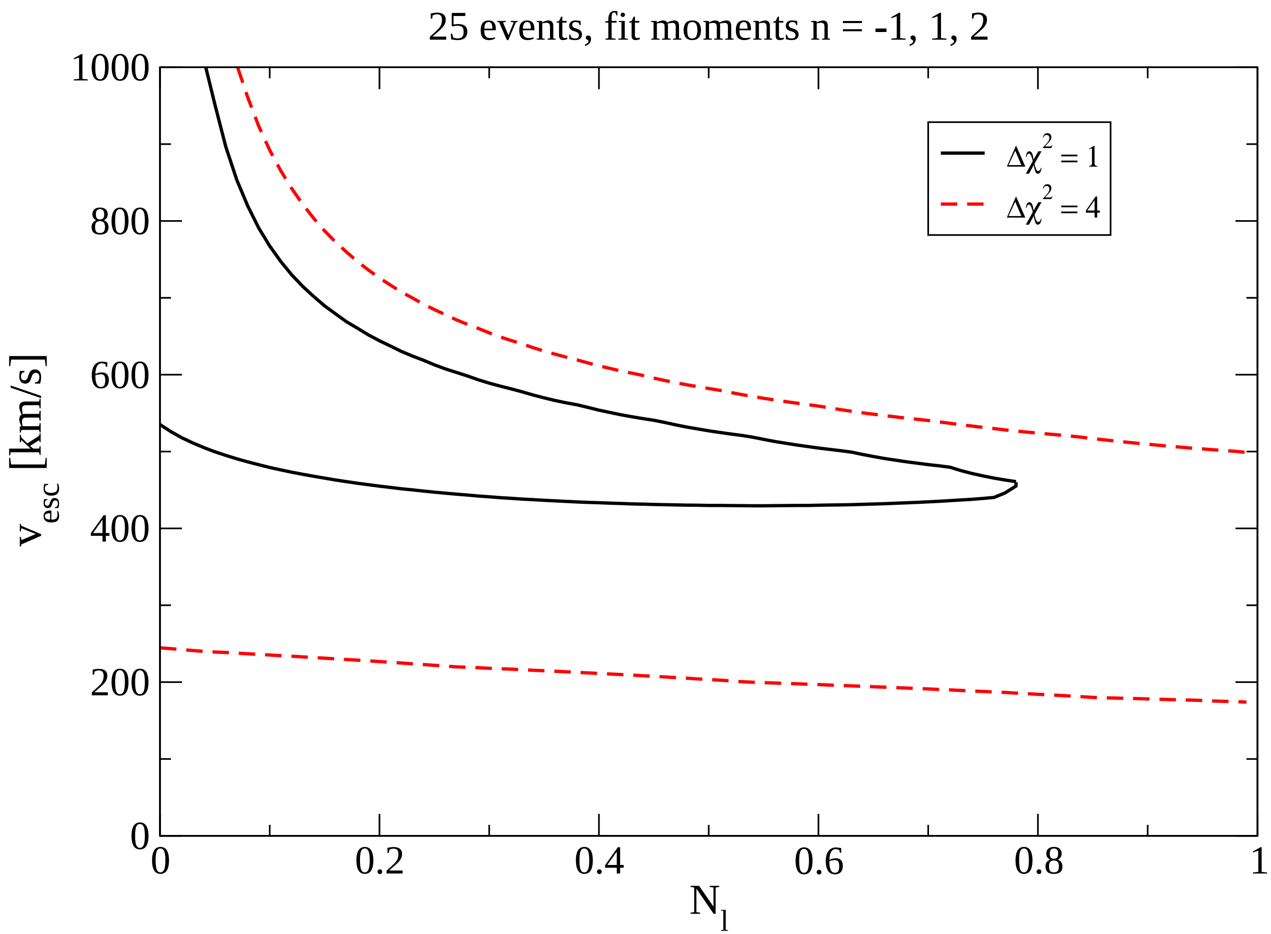
High moments, and their errors, are underestimated in "typical" experiment: get large contribution from large Q

Determination of first 10 moments



Constraining a “late infall” component



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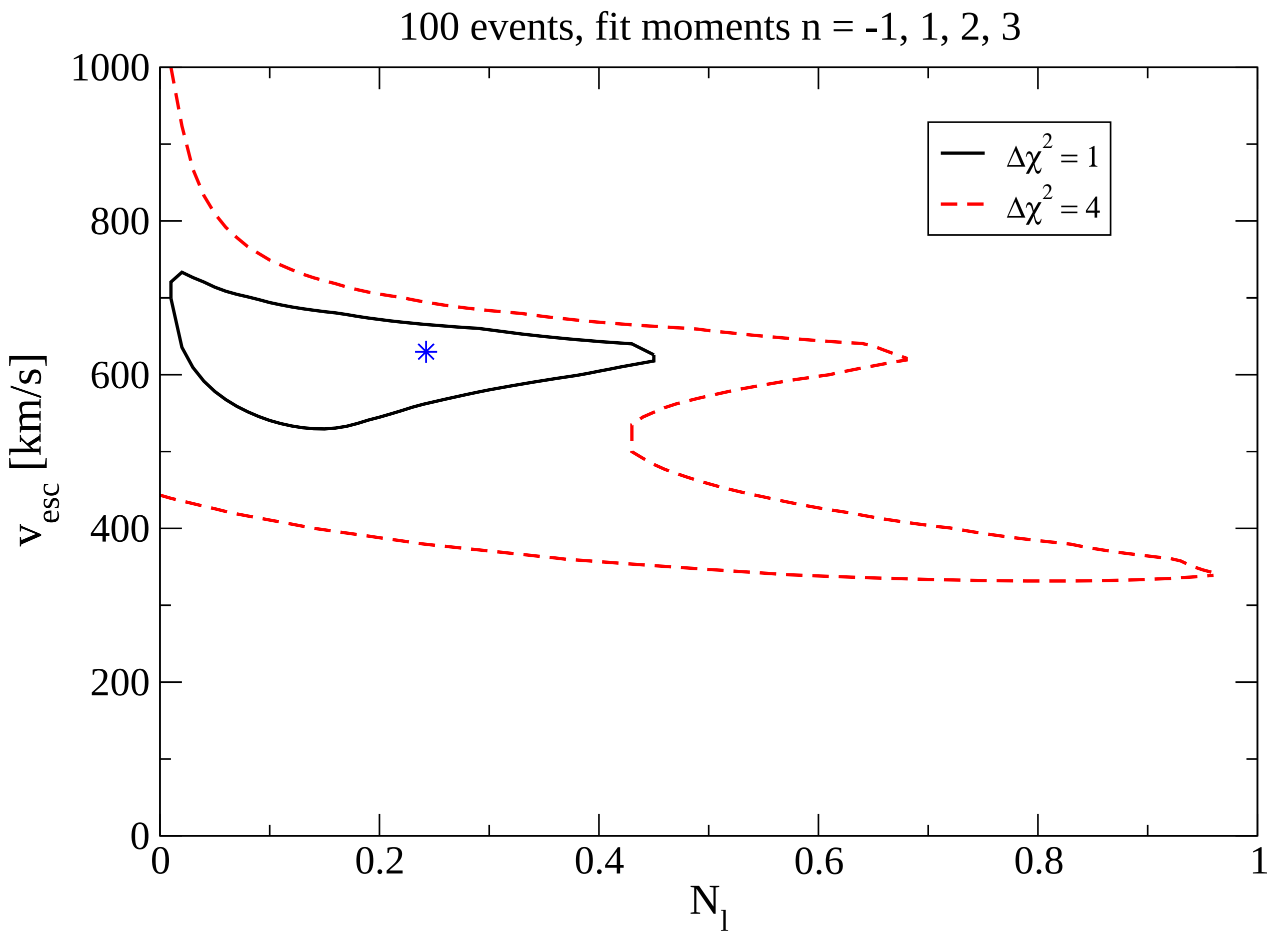

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- Can also get m_χ from comparison of event rates, assuming equal cross section on neutrons and protons.

Formalism

$$\langle v^n \rangle = \alpha^n (n+1) \frac{I_n}{I_0}$$

$$\alpha = \sqrt{\frac{m_N}{2m_{\mathrm{red},N}^2}} \quad , \quad I_n = \int_0^\infty \frac{Q^{(n-1)/2}}{F^2(Q)} \frac{dR}{dQ} \, dQ$$

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$$\Rightarrow m_\chi = \frac{\sqrt{m_X m_Y} - m_X \mathcal{R}_n}{\mathcal{R}_n - \sqrt{m_X / m_Y}}, \quad \mathcal{R}_n \equiv \frac{\alpha_Y}{\alpha_X} = \frac{I_{n,X} I_{0,Y}}{I_{n,Y} I_{0,X}}$$

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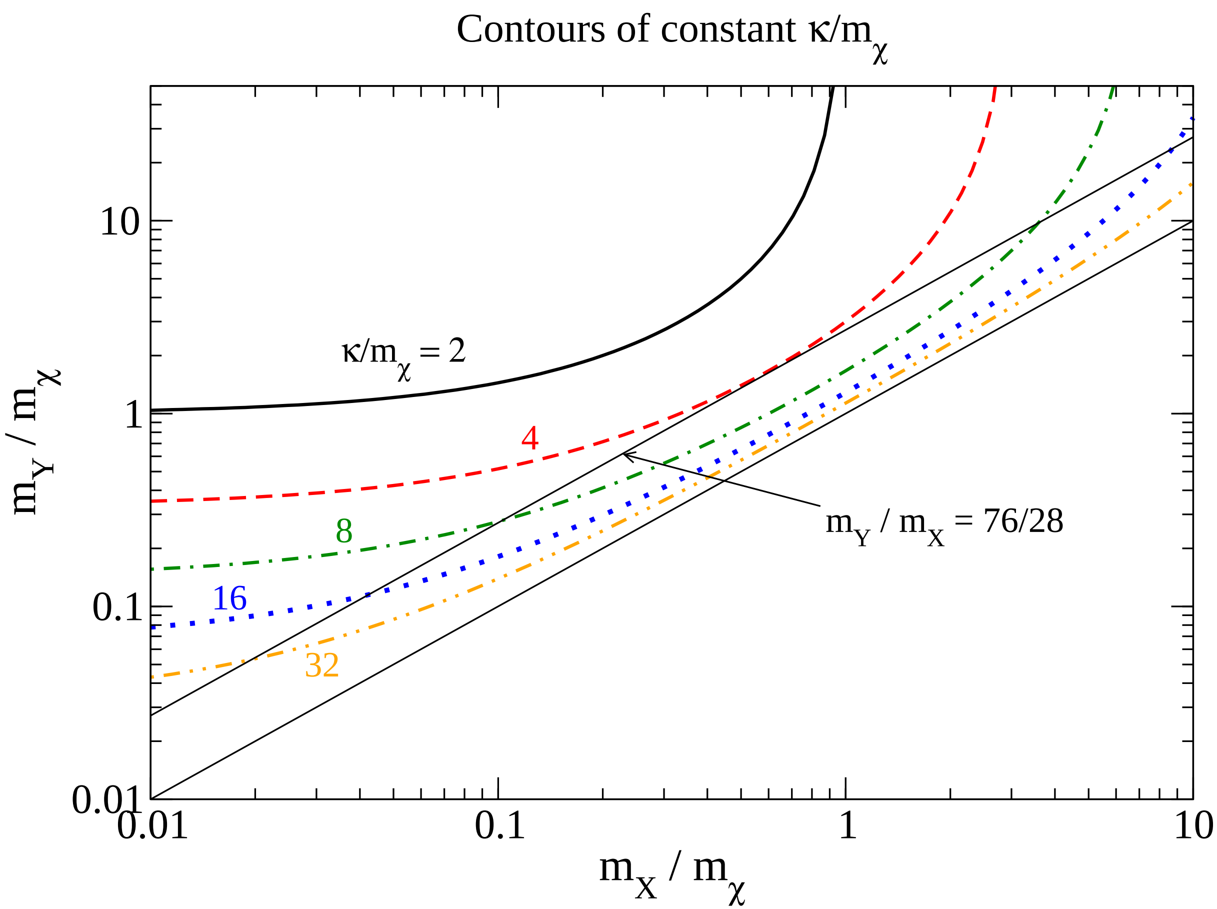
$$\Rightarrow \sigma(m_\chi)\big|_{\langle v^n \rangle} \propto \frac{\mathcal{R}_n \sqrt{m_X/m_Y}\, |m_X - m_Y|}{\left(\mathcal{R}_n - \sqrt{m_X/m_Y}\right)^2}$$

$$\propto \frac{(m_\chi + m_X)(m_\chi + m_Y)}{|m_X - m_Y|} \equiv \kappa$$

Selecting target materials



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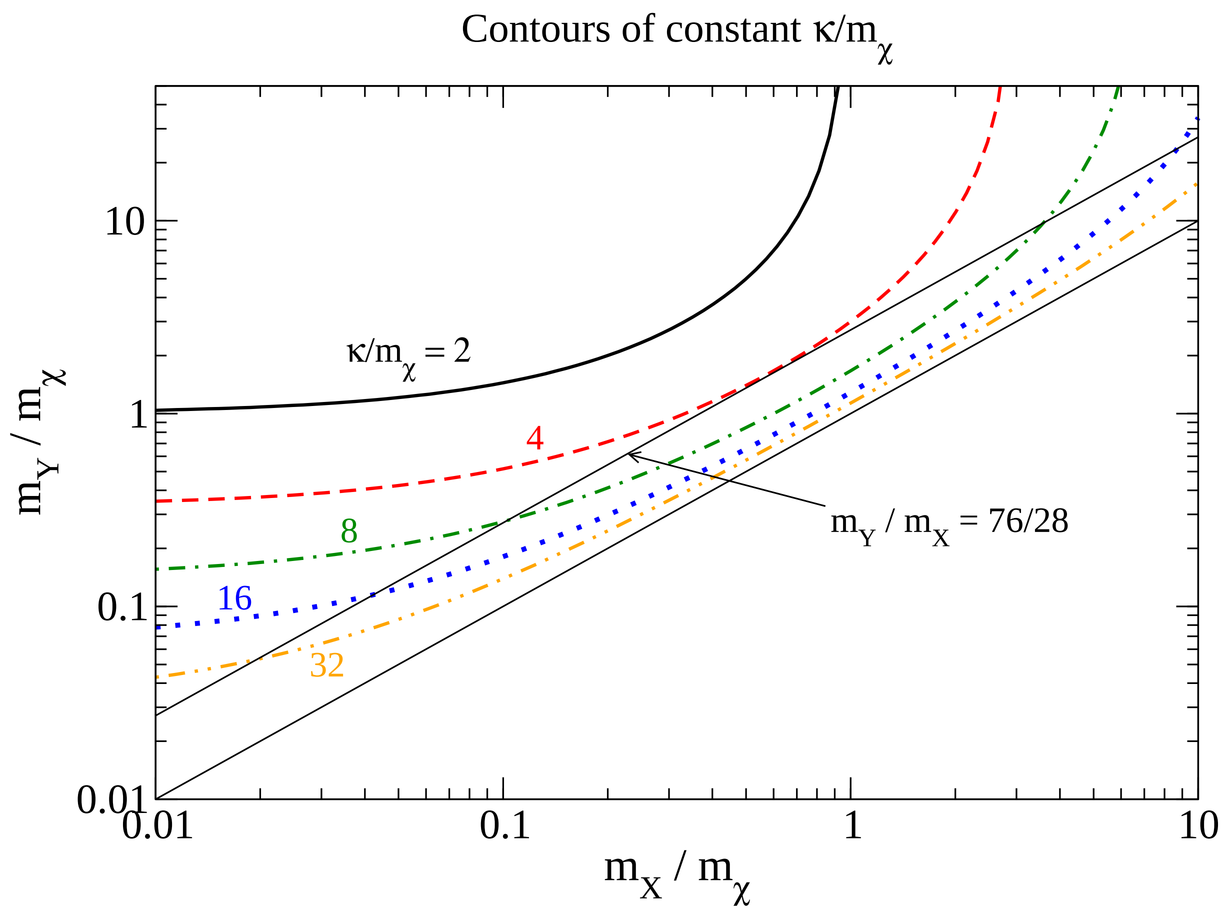

Target nuclei should have quite different masses, preferably bracketing WIMP mass

Systematic errors

- Equality of moments of f_1 holds only if integrals run over identical ranges of v, e.g. $v_{\rm min} = 0$, $v_{\rm max} = \infty$.

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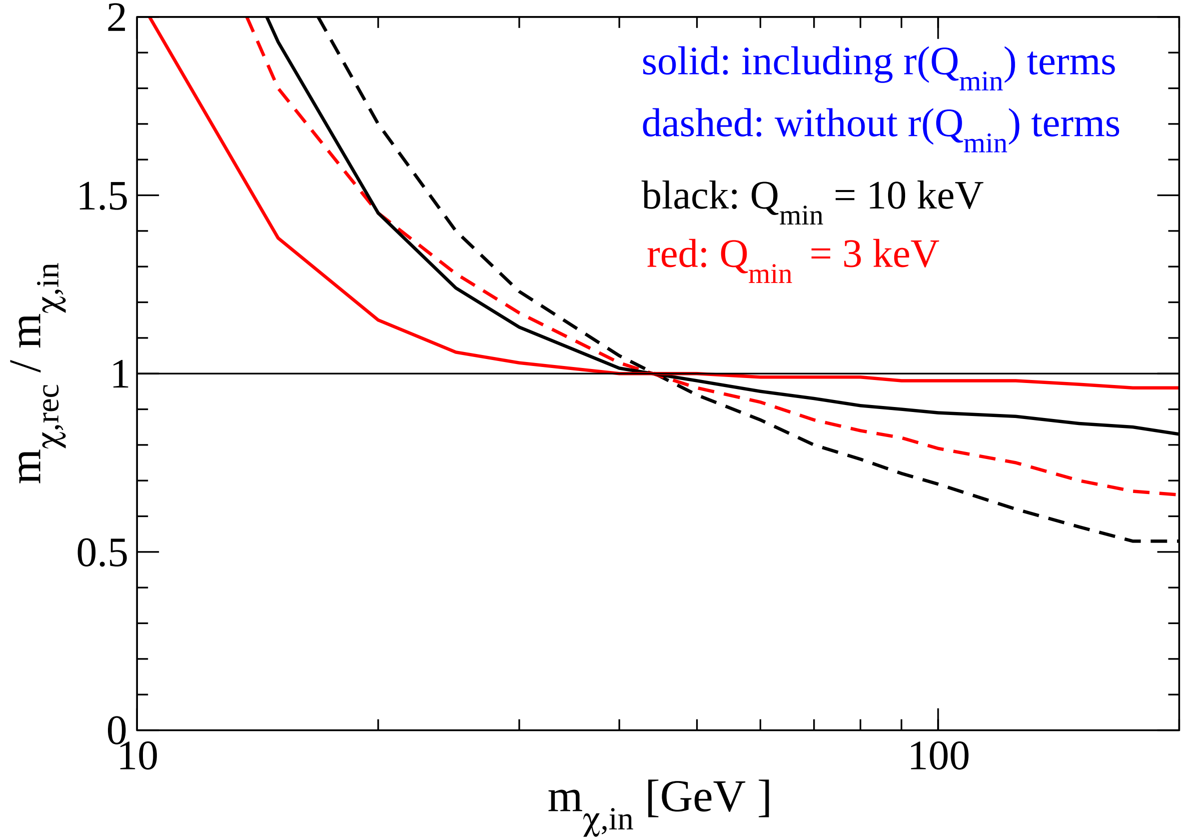
- Ensuring $v_{{\rm min},X} = v_{{\rm min},Y}$ and $v_{{\rm max},X} = v_{{\rm max},Y}$ only possible if m_χ is known
- For $v_{\rm min}$: Systematic effect not very large if $m_\chi \gtrsim 20$ GeV, $Q_{\rm min} \lesssim 3$ keV, $Q_{{\rm min},X} = Q_{{\rm min},Y}$ terms included in I_n.

Effect of $Q_{\min} \neq 0$



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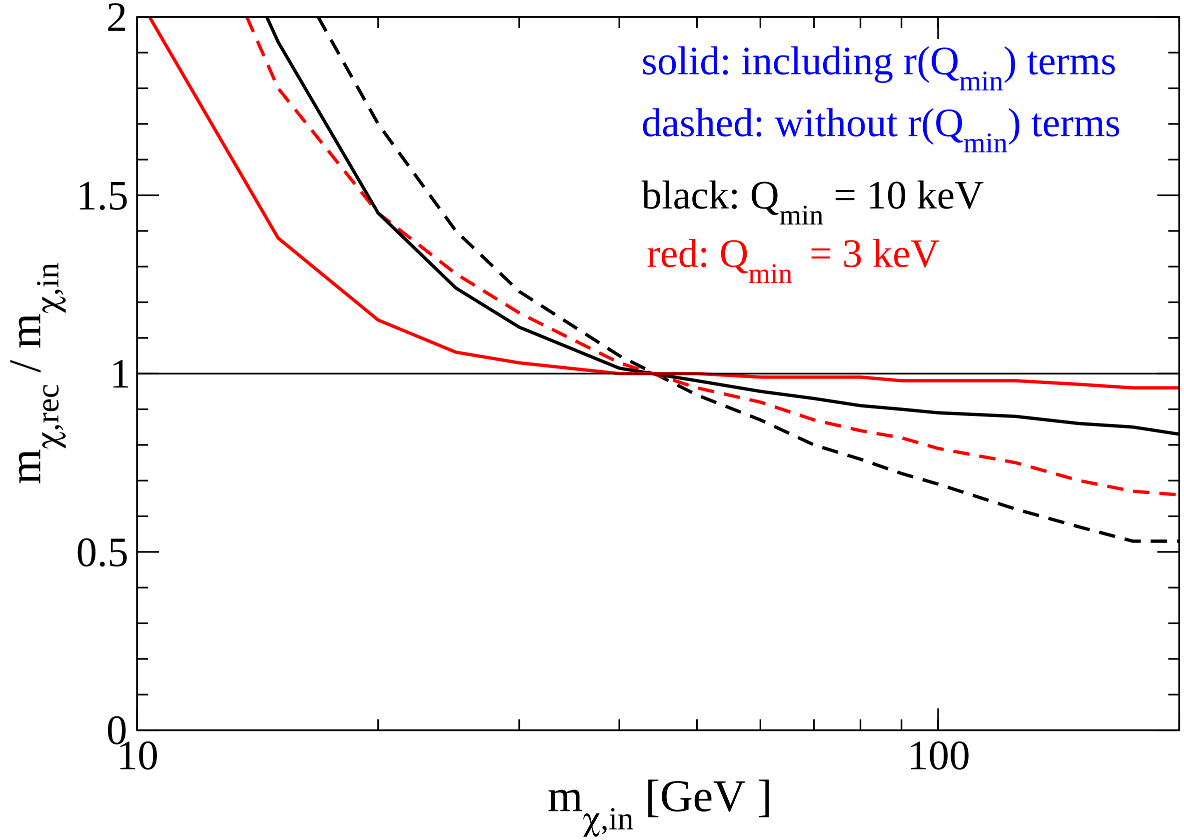

Use $Q_{\min} = 0$ from now on.

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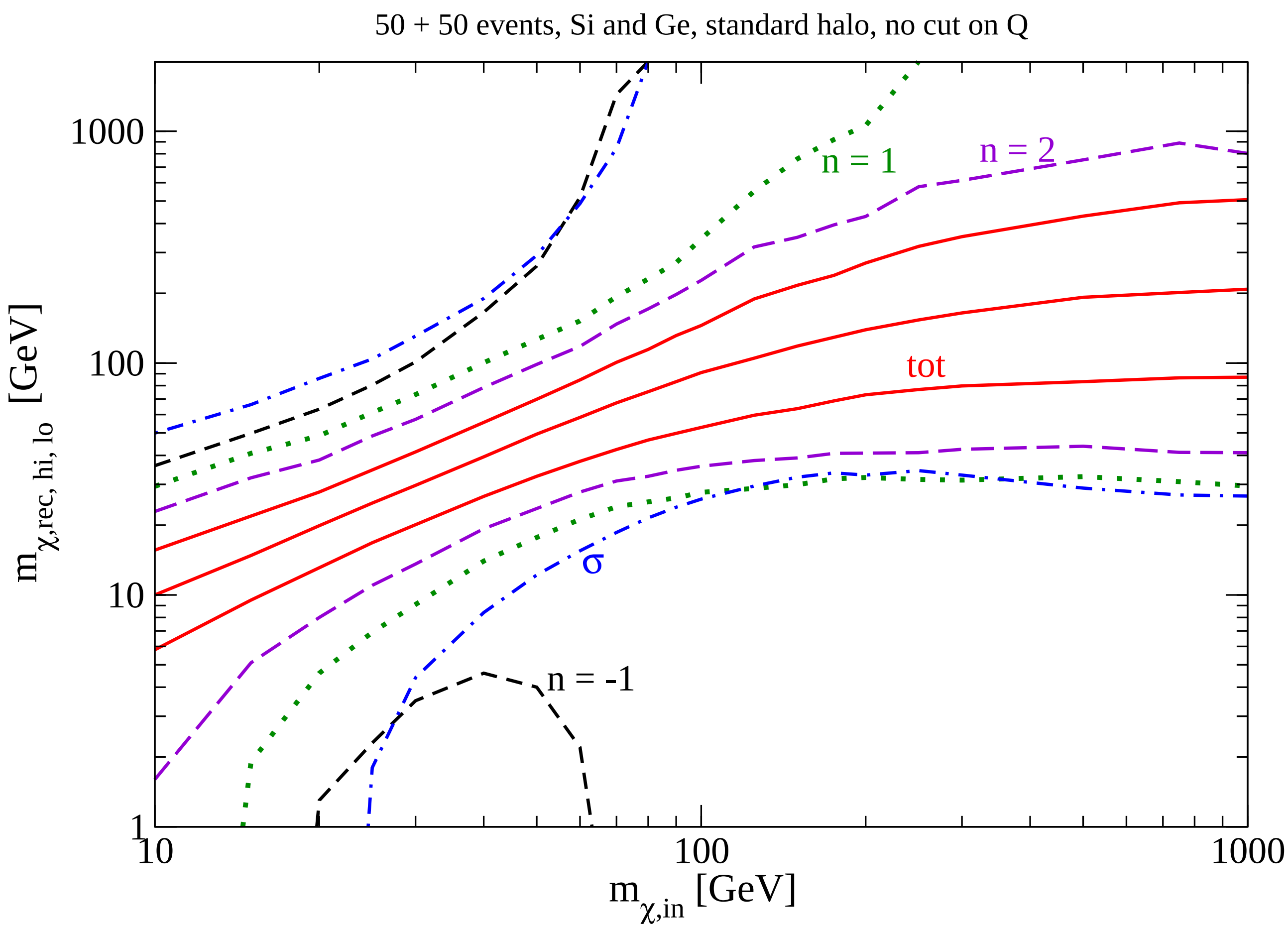
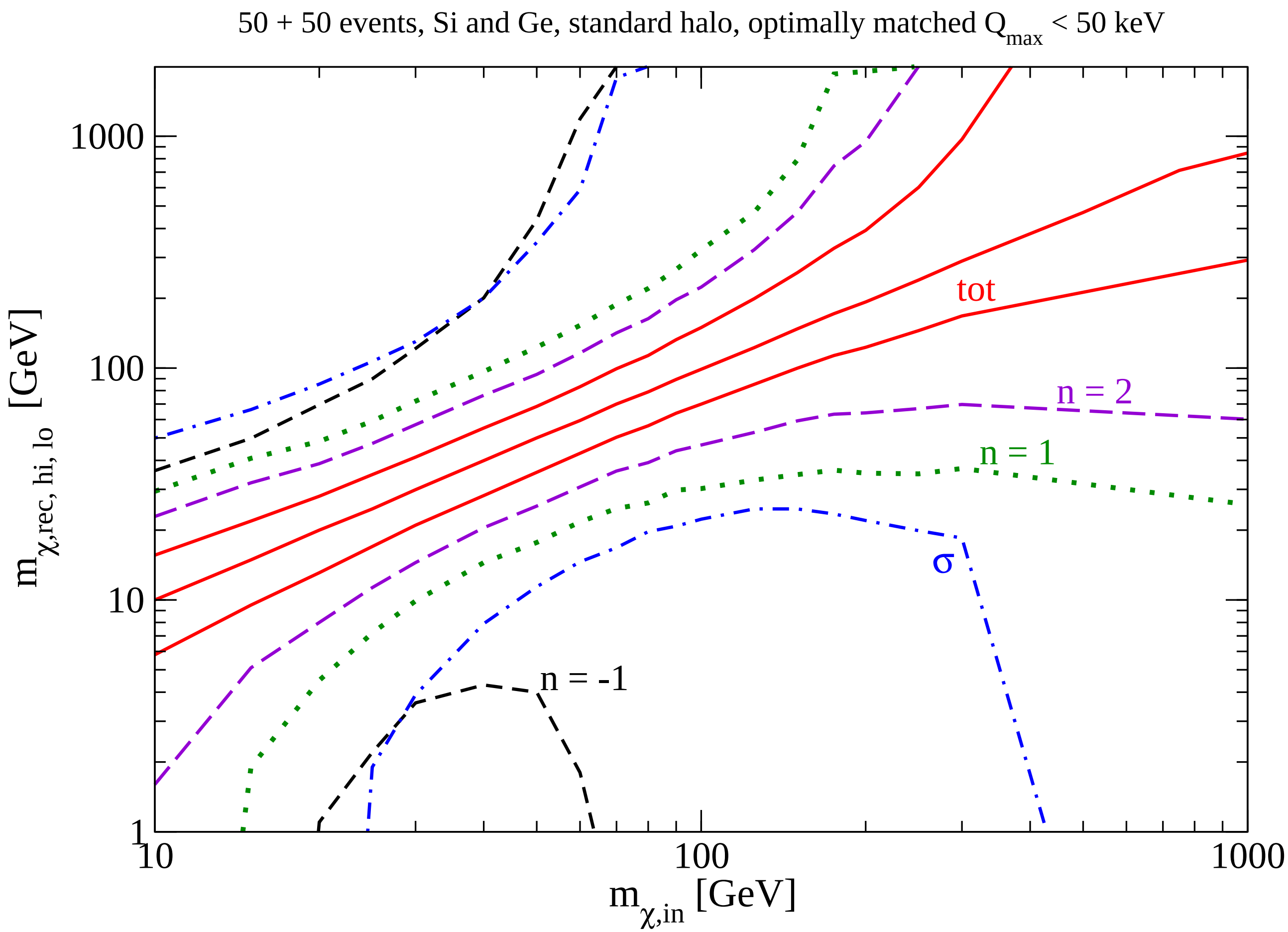
- Imposing finite $Q_{\rm max}$ can alleviate this problem,
- but introduces systematic error unless $Q_{\rm max}$ values of two targets are matched; matching depends on m_χ.

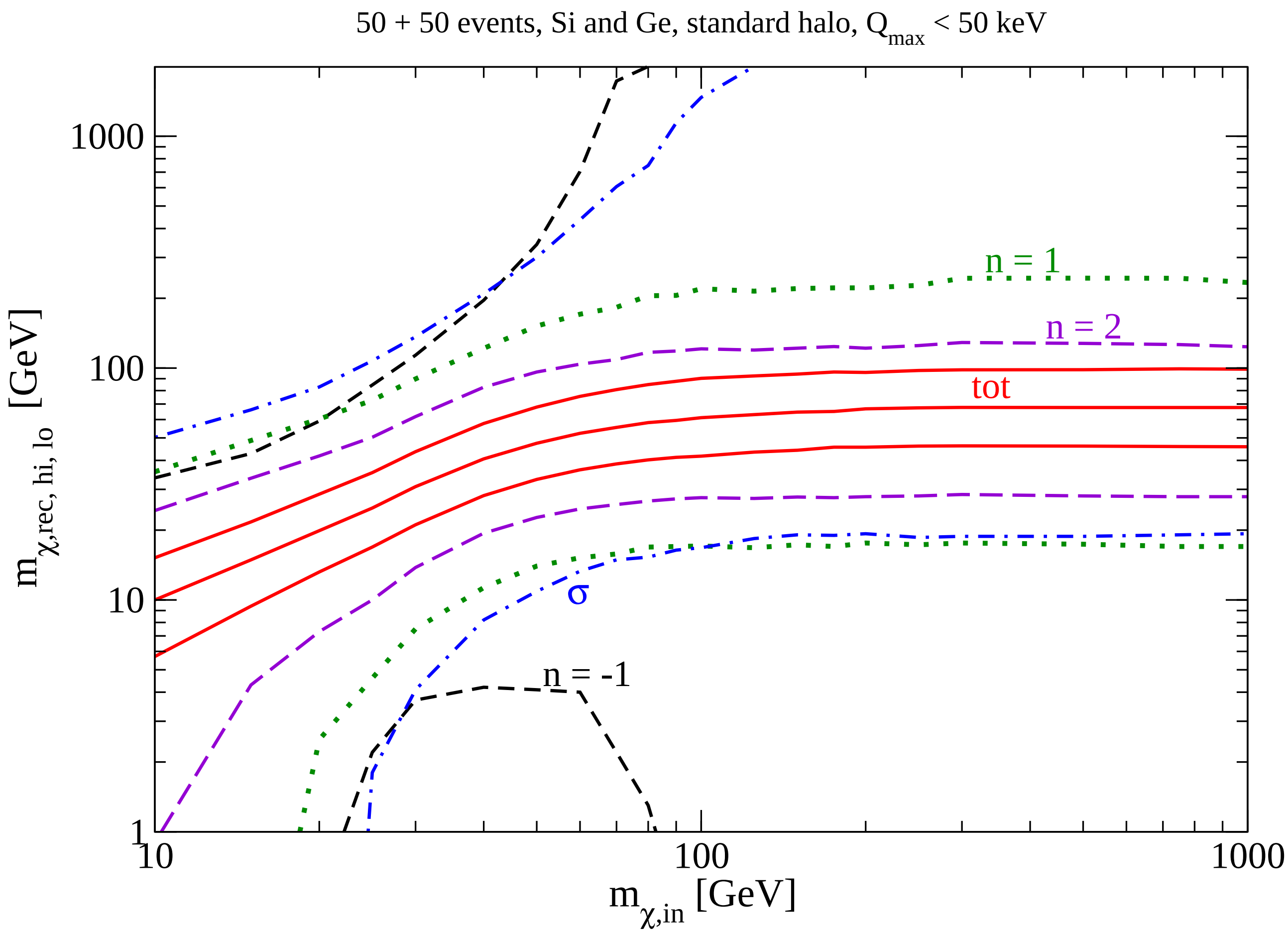
Median reconstructed WIMP mass: no cut on Q



Median reconstructed WIMP mass: optimal $Q_{\max}$ matching



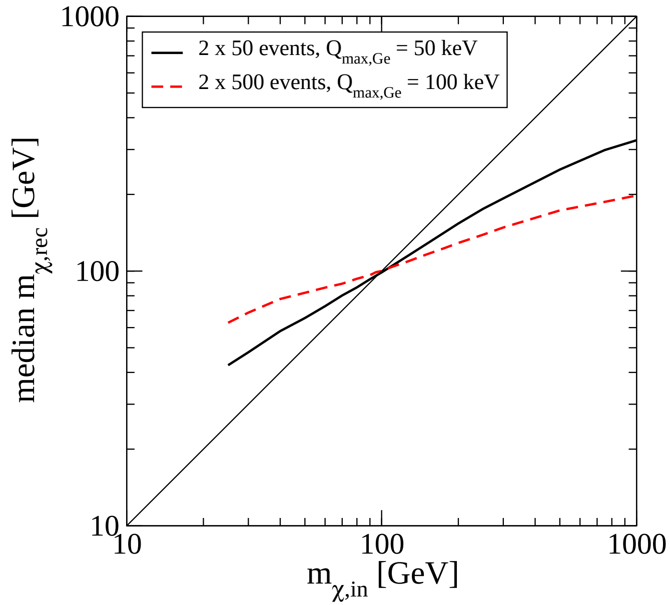
Median reconstructed WIMP mass: equal $Q_{\max}$



Matching procedures

Iterative: $m_{\chi,0}$ used for matching $\rightarrow m_{\chi,\mathrm{rec},1}$, used as new input $\rightarrow \ldots$: converges "on average"

Ge and Si, true $m_\chi = 100$ GeV



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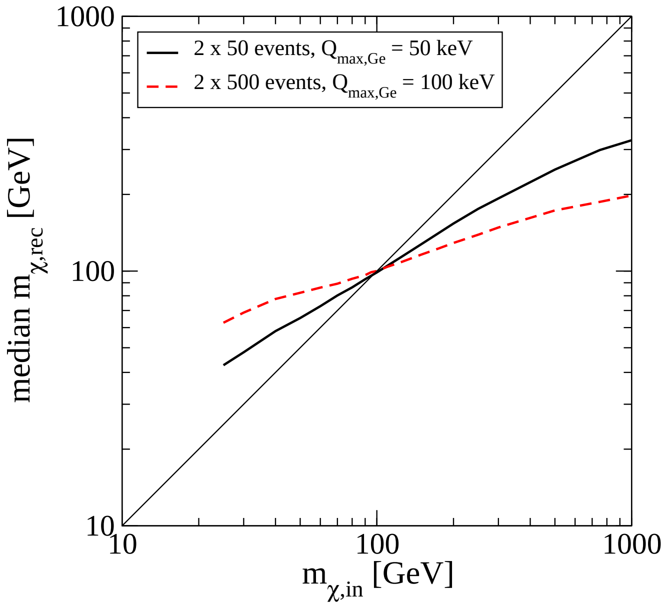

Unfortunately, in given experiment often leads to endless loop!

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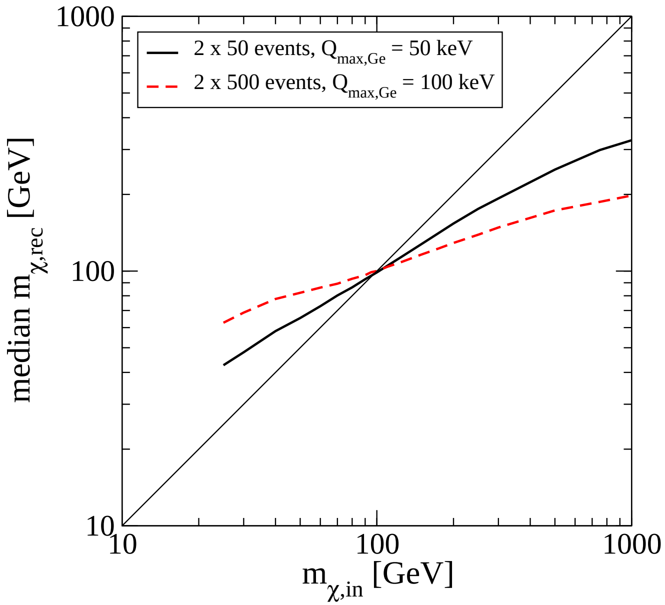

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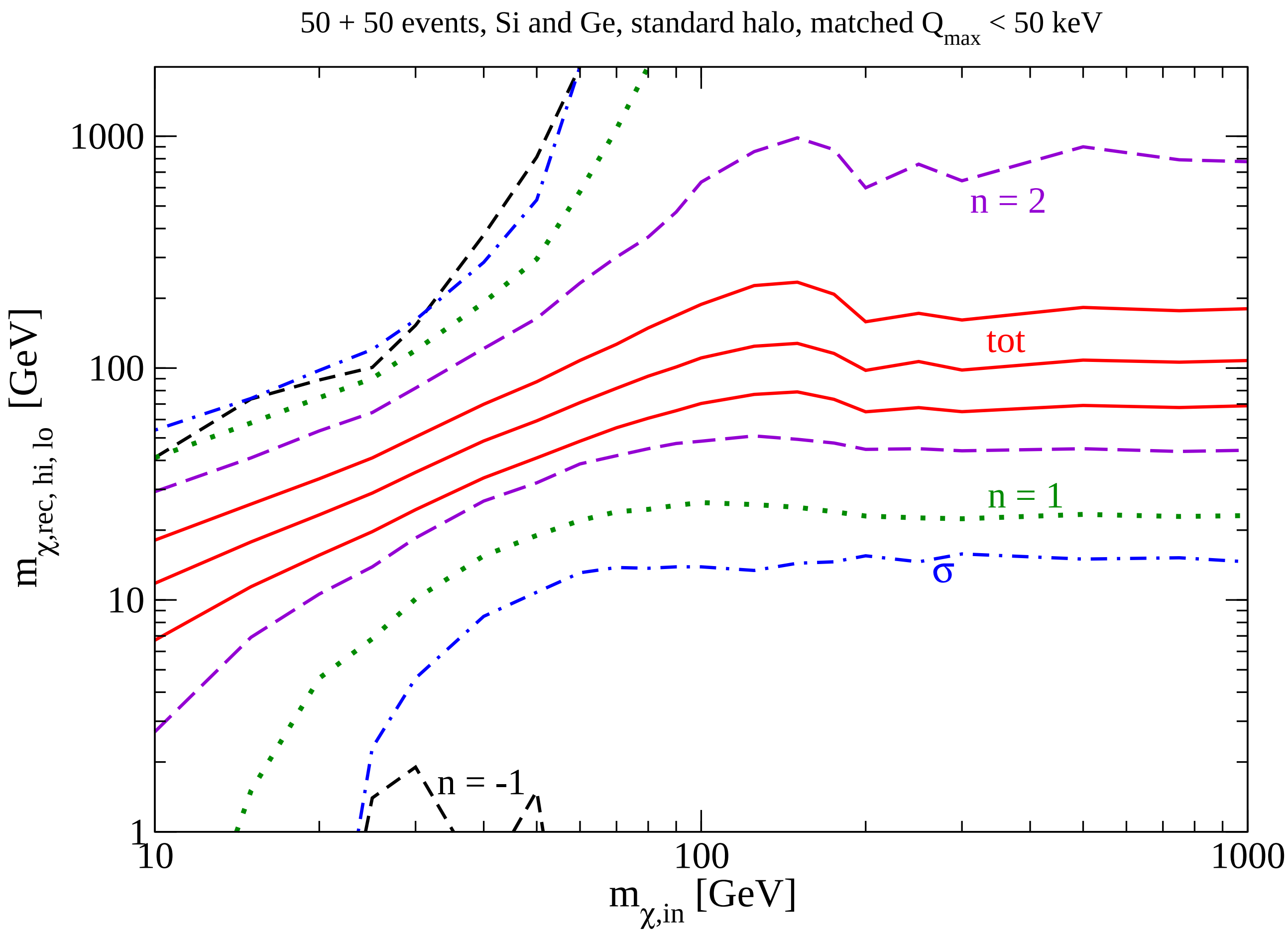
Instead developed matching procedure based on total χ^2 fit

Median reconstructed WIMP mass: χ^2 matching



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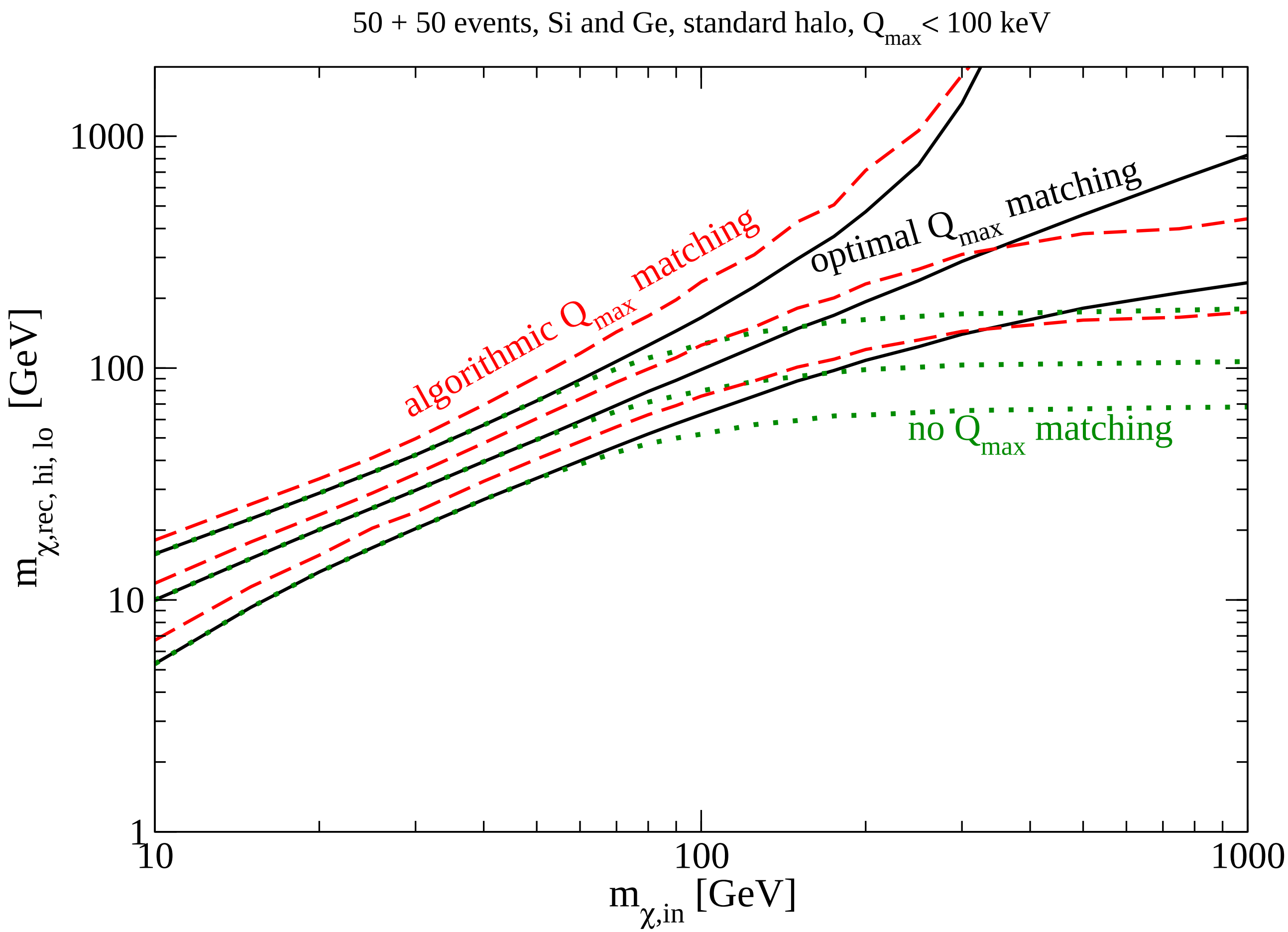
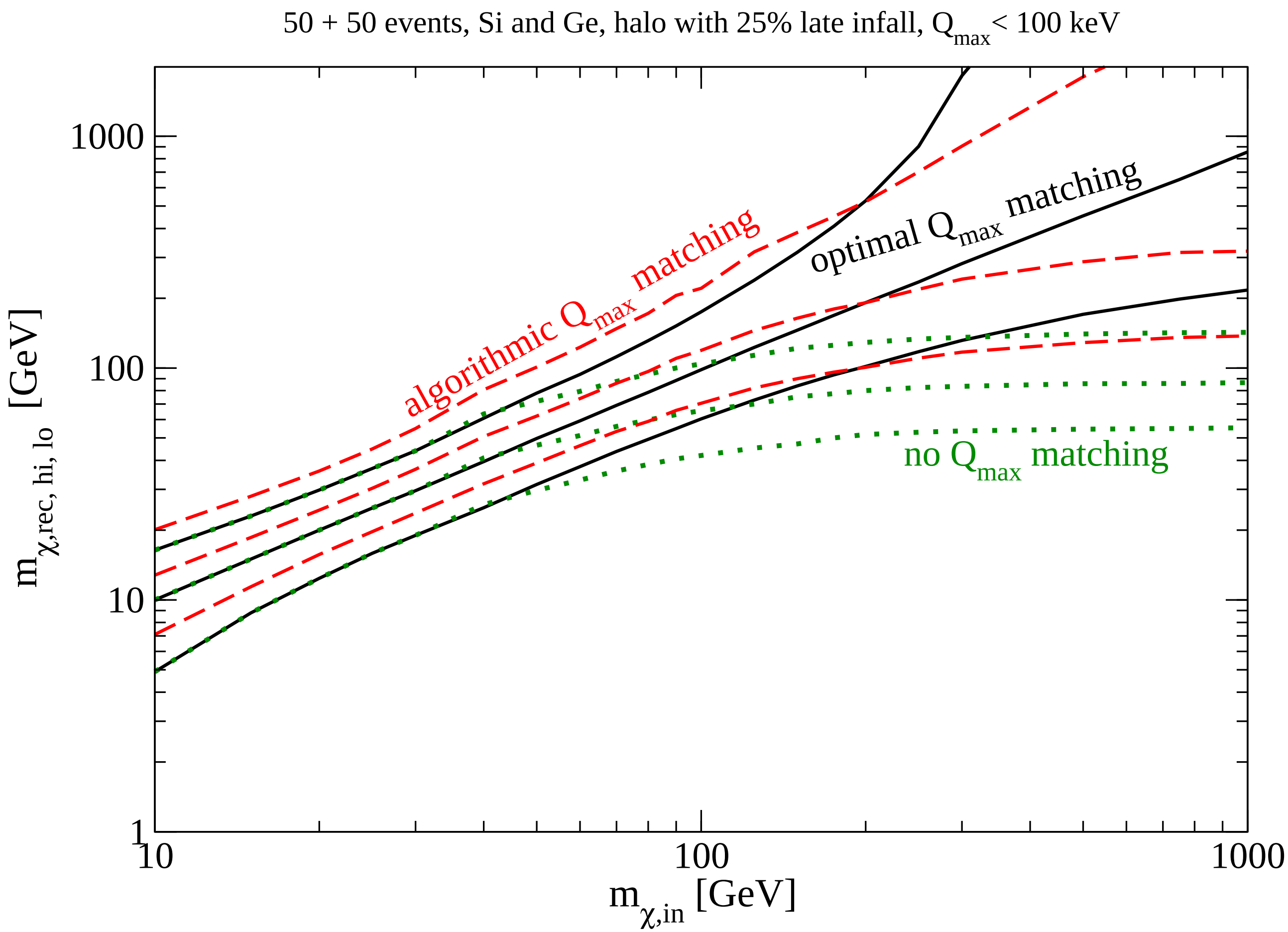
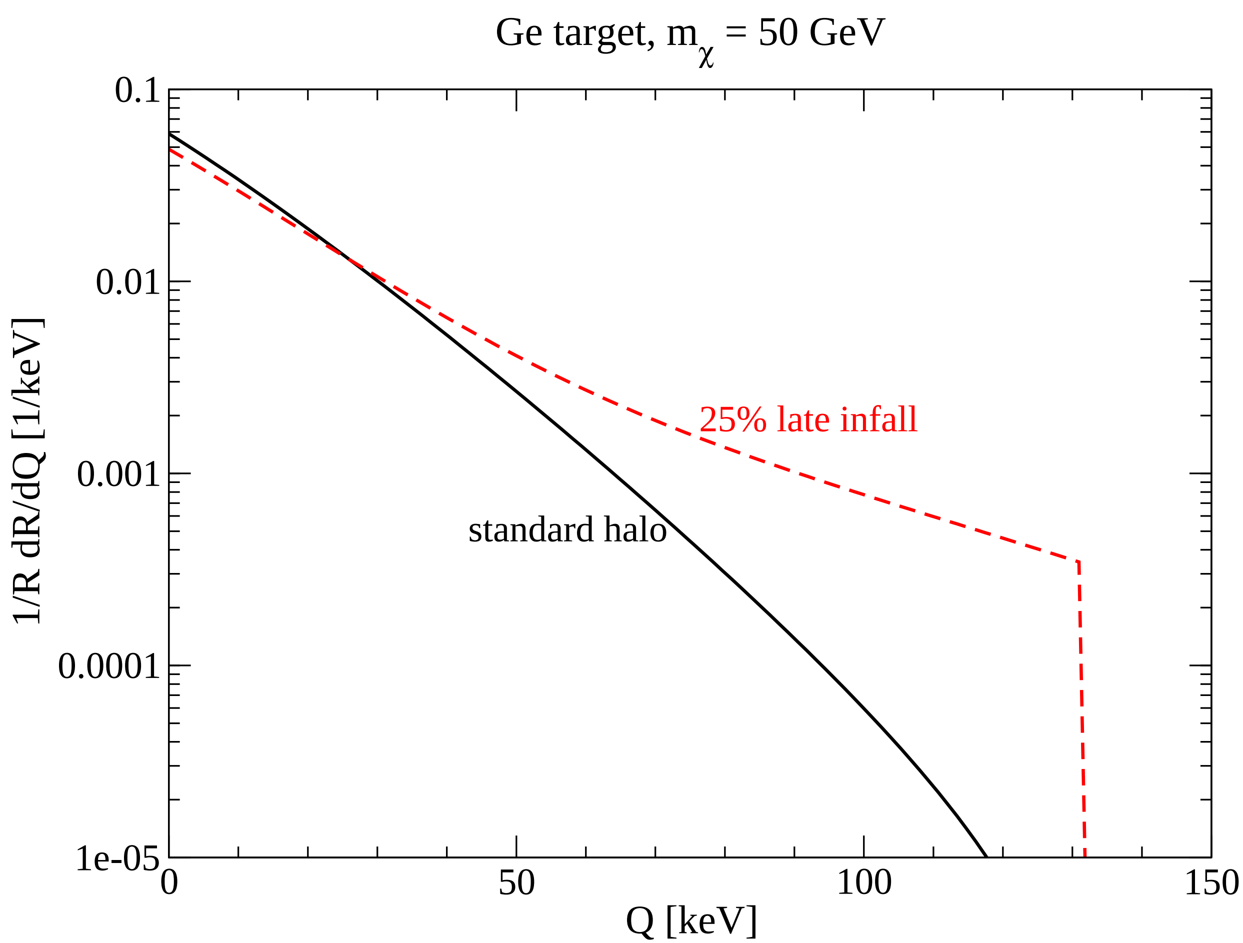


Median reconstructed WIMP mass: non–standard halo

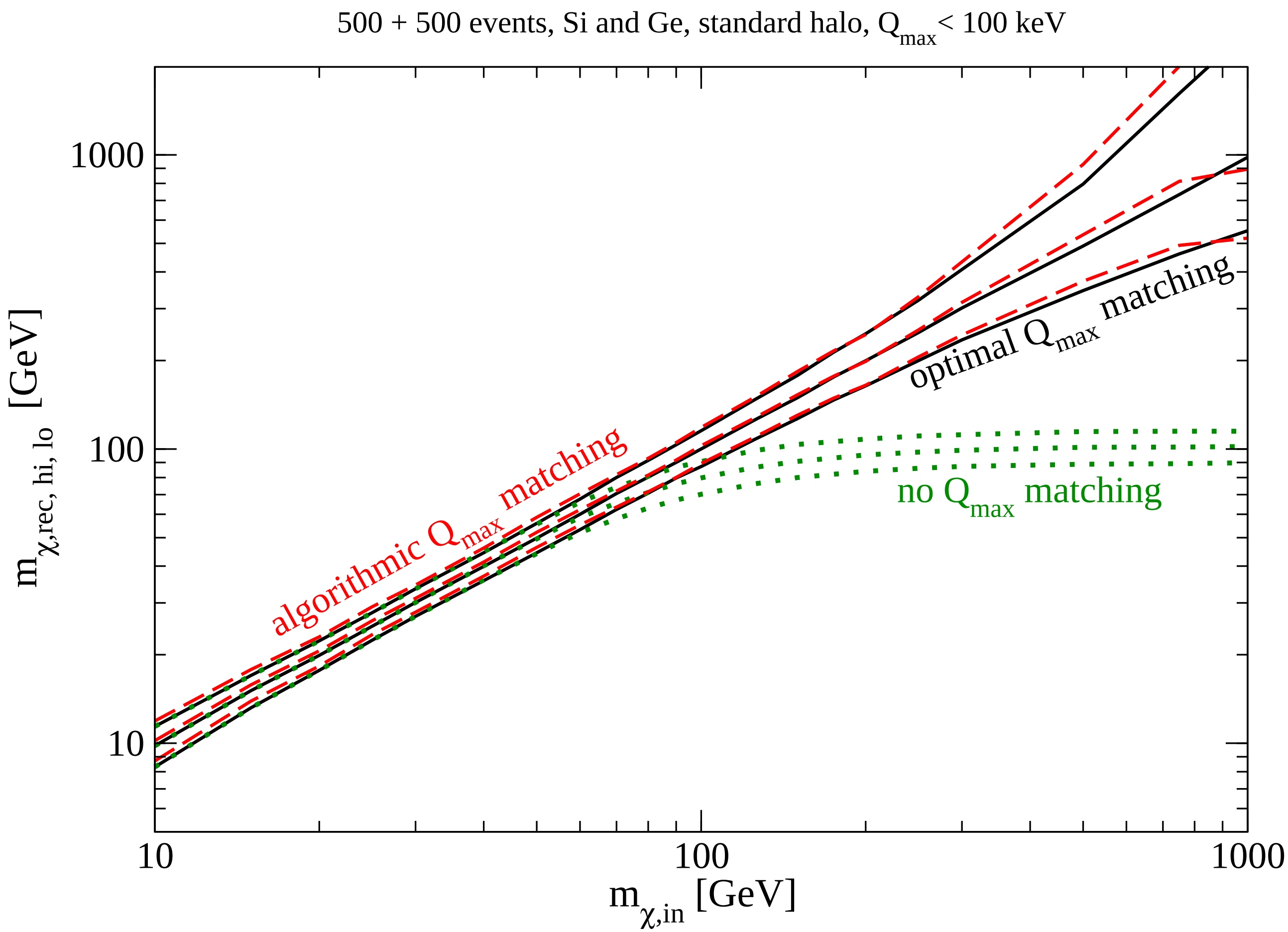


Comparison of corresponding recoil spectra



Difference is smaller for larger m_χ

Median reconstructed WIMP mass



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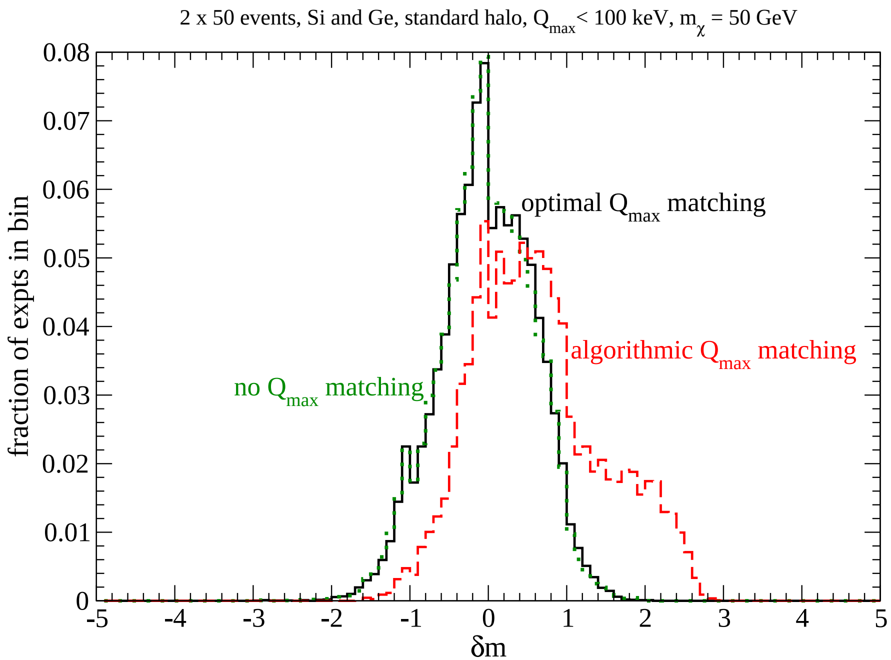

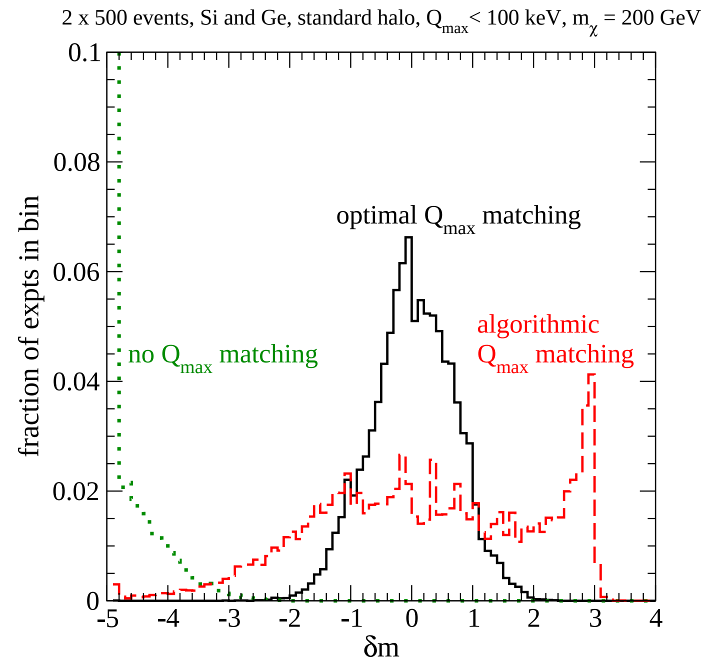

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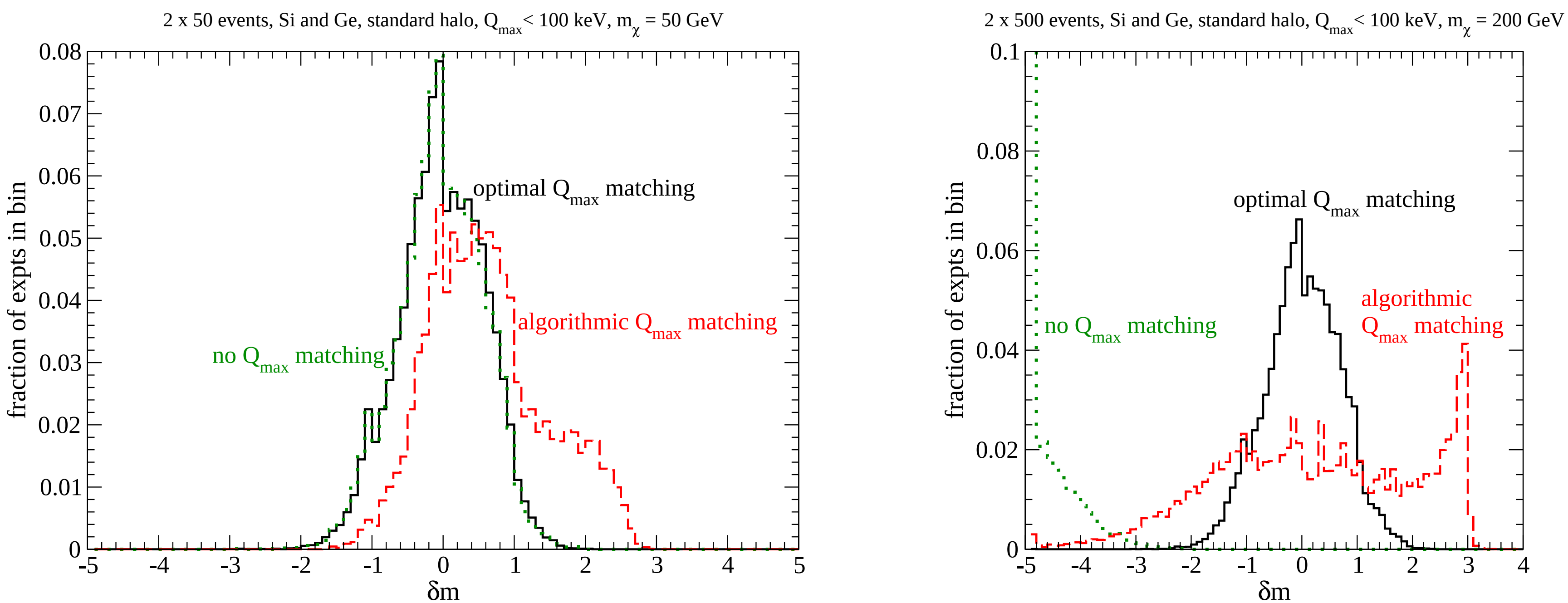

χ^2 matching of $Q_{\rm max}$ values obscures meaning of final error estimate!

Summary: Learning from direct WIMP detection

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  - Direct reconstruction of $f_1(v)$ needs several hundred events

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- Gives motivation to collect lots of direct WIMP scattering events!