Implications of Compressed Supersymmetry for Collider and Dark Matter Searches

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in collaboration with

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Outline

• Introduction
  - Neutralino Dark matter
  - Review of mSUGRA

• SUSY models without universality in SSB terms
  - Non-universal scalar mass model
  - Non-universal gaugino mass model

• NUGM models at Colliders

• Compressed SUSY model

• Summary and Conclusion
Dark Matter

- Properties of Dark Matter
  - not detected visibly
  - inferred from gravitational effects
  - dominant composition of matter in our universe
  - no DM candidate in the SM

- Evidence for Dark Matter
  - Galactic Clustering
  - Rotation Curves

\[ \text{http://map.gsfc.nasa.gov} \]

\[ \text{Mon. Not. R. Astron. Soc. 249 (1991) 523} \]

- Gravitational Lensing
- Cosmic Microwave Background
- ...
Dark Matter Candidates

- Baryonic dark matter (MACHOs): small fraction of total DM
- Non-baryonic dark matter
  - Hot dark matter: ultra relativistic
  - Warm dark matter: relativistic
  - Cold dark matter: non-relativistic
    * Axion
    * WIMPs (Weakly Interacting Massive Particles): Neutralino (SUSY), KK-photon (extra dim. th.), branon (large extra dim. th.), ...
    * SuperWIMPs: gravitino
    * many other possibilities
Neutralino Dark Matter

- Dark Matter should be non-baryonic (no candidate in the SM), non-relativistic (cold), stable (or long-lived), weakly (or super-weakly) matter.

- Flat universes in the $\Lambda CDM$ cosmological model are characterized by baryon density, matter density, vacuum density, expansion rate ($h$).

- From the WMAP results, the cold dark matter density of the universe is $\Omega_{CDM} h^2 = 0.111^{+0.011}_{-0.015}$ (upper bound is a tight constraint on SUSY models containing DM candidates: DM may consist of several components).

- In SUSY models with $R$-parity conservation
  $\Rightarrow$ the Lightest Supersymmetric Particle (LSP) is absolutely stable
  $\Rightarrow$ lightest neutralino $\tilde{\chi}_1$ is the LSP in most of MSSM parameter space
  $\implies$ $\tilde{\chi}_1$ is a good candidate for Cold Dark Matter (CDM).

- Number density is governed by Boltzmann equation,
  \[ \frac{dn}{dt} = -3Hn - \langle \sigma v_{\text{rel}} \rangle (n^2 - n_0^2) \]
  $\Rightarrow$ requires evaluating many thousands Feynman diagrams
  $\implies$ high (co-)annihilation cross section implies low relic abundance.
Review of mSUGRA

- **Parameter Space:**
  \[ m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu) \]

- **WMAP allowed Regions:**
  
  *Region 1.* \( \tilde{\tau} \) co-annihilation region at low \( m_0 \)
  
  *Region 2.* bulk region at low \( m_0 \) and \( m_{1/2} \)
    - light sleptons (LEP2 excluded)
  
  *Region 3.* \( A \)-funnel
    - \( H, A \) resonance annihilation
  
  *Region 4.* FP/HB region at large \( m_0 \), small \( \mu \)
    - mixed higgsino dark matter (MHDM)

- In most of the parameter space of the mSUGRA model, a value of neutralino relic density is beyond the WMAP bound
  \[ \Omega_{CDM} h^2 = 0.111^{+0.011}_{-0.015} \]
SUSY models without universality

- Non-universal scalar mass models
  - Generation non-universality: Normal scalar mass hierarchy (NMH)
  - Non-universal Higgs mass: one extra parameter case
    (NUHM1_μ, NUHM1_A)
  - Non-universal Higgs mass: two extra parameter case (HS-Higgs Splitting)

- Non-universal gaugino mass models
  - Mixed Wino Dark Matter (MWDM)
  - Bino-Wino Co-Annihilation Scenario (BWCA)
  - Low |M_3| Dark Matter: Compressed SUSY (LM3DM)
  - High |M_2| Dark Matter: left-right split SUSY (HM2DM)

- Some benchmark cases with
  \[ m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu) = 300 \text{ GeV}, 300 \text{ GeV}, 0, 10, +1 \]
  and \[ m_t = 171.4 \text{ GeV} \]

for more details, see Baer, Mustafayev, EKP and Tata, arXiv:0802.3384
Parameter space of SUSY models without universality

- Non-universal scalar mass models
  - NMH: $m_0(1,2), m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$
    $m_0(1,2)$: first/second generation, $m_0(3) = m_{H_u} = m_{H_d} \equiv m_0$: remaining
dial $m_0(1,2)$ to low enough to bulk (co-)annihilation via light sleptons
  - NUHM1$_\mu$, NUHM1$_A$: $m_0, \delta_\phi, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$
    $m_\phi = m_0(1 + \delta_\phi), m_{H_u}^2 = m_{H_d}^2 \equiv \text{sign}(m_\phi)|m_\phi|^2$
    $m_\phi >> m_0$: small $\mu$ and MHDM, $m_\phi < 0$: $m_A \sim 2m_{\tilde{Z}_1}$
  - HS: $m_0, m_{H_u}^2$ (equivalently $\mu$), $m_{H_d}^2$ (equivalently $m_A$), $m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$
    $m_{H_{u,d}}^2 = m_0^2 (1 \mp \delta_H)$
    $\delta_H < 0$: low $\mu$ and low $m_A, \delta_H > 0$: WMAP region via $\tilde{l}_L/\tilde{\nu}$ or $\tilde{u}_R/\tilde{c}_R$
    co-annihilation
- Non-universal gaugino mass models
  - MWDM: $m_0, M_1$ (or $M_2$), $m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$
  - BWCA: same as MWDM but $M_1$ and $M_2$ are in opposite sign
  - LM3DM: $m_0, M_3, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$
  - HM2DM: $m_0, M_2, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$
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m_0 = 300 GeV, m_{1/2} = 300 GeV, \tan beta = 10, A_0 = 0, \mu > 0, m_t = 175 GeV

- Mild evolution of \( m_{H_d}^2 \) due to small Yukawa coupling \( f_b, f_\tau \)
- Lighter squarks and gluinos \( \rightarrow \) reduced effect of \( f_t \) on \( m_{H_u}^2 \)
  \( \Rightarrow \) smaller \( \mu \)
- \( \frac{dm_{H_d}^2}{dt} \propto f_{b,\tau}^2 X_{b,\tau}, \quad \frac{dm_{H_u}^2}{dt} \propto f_t^2 X_t \)
- \( \mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2} \approx -m_{H_u}^2 \)
**NUGM at Colliders**

- **CERN LHC and Fermilab Tevatron**
  - If $\tilde{Z}_2 \rightarrow \tilde{l} \bar{l}$ or $\tilde{Z}_2 \rightarrow \tilde{l}_1 \tilde{l}$ are open ($l = e$ or $\mu$)
    $\Rightarrow$ good prospects for measuring the $\tilde{Z}_2 - \tilde{Z}_1$ mass gap at the CERN LHC and possibly at the Fermilab Tevatron
  - In the mSUGRA case, most of the parameter space has $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} > 90$ GeV,
    $\Rightarrow$ $\tilde{Z}_2 \rightarrow \tilde{Z}_1 Z^0$ or $\tilde{Z}_1 h$ “spoiler” decays dominant
  - When the mass gap is much smaller
    * spoiler decays are closed, 3-body decays are open
    * $\tilde{l} \bar{l}$ mass edge always visible at LHC

- **Linear $e^+ e^-$ collider (ILC)**
  - $m_{\tilde{Z}_2}$, $m_{\tilde{W}_1}$ and $m_{\tilde{Z}_1}$ can be inferred from $\tilde{W}_1^+ \tilde{W}_1^- \rightarrow \tilde{l}_1 \tilde{Z}_1 + q \bar{q} \tilde{Z}_1$
    (dijet events)
  - $\tilde{W}_1^+ \tilde{W}_1^-$, $\tilde{Z}_1 \tilde{Z}_2$, $\tilde{Z}_2 \tilde{Z}_2$ production cross sections can be measured as a function of beam polarization: $P_L(e^-) = f_L - f_R$
    ($f_{L,R}$: fraction of left(right) polarized electron in the beam)
Dilepton Distribution at LHC

- mSUGRA: sharp peak at $m(l^+l^-) \sim M_Z$ from $\tilde{Z}_2 \rightarrow \tilde{Z}_1 Z^0$ decays
- NUGM: $Z^0$ peak from $\tilde{Z}_3, \tilde{Z}_4, \tilde{W}_2$ decays + continuum distribution $m(l^+l^-) < m\tilde{Z}_2 - m\tilde{Z}_1$
Cross Section for $\tilde{W}_1^+\tilde{W}_1^-$ and $\tilde{Z}_i\tilde{Z}_j$ Production at ILC

- $\tilde{W}_1$ and $\tilde{Z}_2$ are mainly wino-like
  $\rightarrow \sigma(\tilde{W}_1\tilde{W}_1)$ and $\sigma(\tilde{Z}_2\tilde{Z}_2)$ are similar to one another
- $\tilde{Z}_1\tilde{Z}_2$ process are quite different
Compressed SUSY*: mass spectrum

- \( M_3 < M_1 \) or \( M_2 \rightarrow \) gluino and squark masses reduced \( \rightarrow \) compressed sparticle mass spectrum

- **Parameter Space at** \( Q = M_{GUT} \)
  - Case A: \( m_0, m_{1/2}, M_3, A_0, \tan \beta, \text{sign}(\mu) \)
    \( (M_1 = M_2 = m_{1/2}, A_0 = -1.5m_{1/2}) \)
  - Case B: \( m_0, M_1, A_0, \tan \beta, \text{sign}(\mu) \)
    \( (1.5M_1 = M_2 = 3M_3, m_t = 175 \text{ GeV}, \)
    \( A_0 = -0.75M_1, \mu > 0, \tan \beta = 10, m_0 = 340 \text{ GeV} ) \)

- **Case B**
  - cut after 1000 GeV: \( \tilde{t}_1 \) LSP \( \rightarrow \) imply upper bound on gluino and squark masses
  - LEP2 bound on chargino mass below \( \sim 160 \) GeV
  - 440 GeV < \( M_1 \) < 1000 GeV: light \( \tilde{t} \) (NLSP),
    \( m_{\tilde{Z}_1} > m_t \) \( \Rightarrow \tilde{Z}_1 \tilde{Z}_1 \rightarrow t\bar{t} \) accessible in the early Universe


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Eun-Kyung Park  Implications of Compressed SUSY for Collider and Dark matter Searches
Compressed SUSY: neutralino relic density

- $M_1$ 400 - 800 GeV: $\tilde{Z}_1 \tilde{Z}_1 \rightarrow t\bar{t}$ dominant ⇒ neutralino relic density is in close accord with WMAP value
- larger $M_1$: $\tilde{t}_1 - \tilde{Z}_1$ mass gap low ⇒ $\tilde{t}_1 \tilde{Z}_1$ co-annihilation rate large ⇒ below WMAP value
- $M_1 < 400$ GeV: annihilation into $t\bar{t}$ not allowed, $\tilde{Z}_1$ dominantly into $WW$ and quarks and leptons ⇒ $h$ and $Z$ poles
Compressed SUSY: direct and indirect DM searches

- a) Direct detection: as $M_1$ decreases, the rate increases due to decreasing $m_{\tilde{q}}$ and $\mu$
  $t\bar{t}$ dominant region $\Rightarrow$ detectable by SuperCDMS or 100-1000 kg noble liquid DM detectors

- b) Detection of $\mu$: neutrinos in the solar core: as $M_1$ decreases, the rate slightly increases due to increasing spin-dependent $\tilde{Z}_1 - N$ cross section
  $M_1 < 400 \text{ GeV}$: rate jumps b/c $\nu\bar{\nu}$ jumps once $t\bar{t}$ turns off
Compressed SUSY: direct and indirect DM searches (cont’d)

• c)d)e) Detection of anti-particle ($e^+, \bar{p}, \bar{D}$): annihilation in the galactic halo
  In the region where $m_{\tilde{Z}_1} > m_t$ so that $\tilde{Z}_1 \tilde{Z}_1 \to t\bar{t}$ occurs, signals increase
  For the less clumpy Burkert halo profile, $\bar{D}$ rate lowered by a factor of 10-15

• Detection of $\gamma$ - ray: from the galactic center
  For the Burkert halo model, scale downwards by over 4 orders
Compressed SUSY: LHC searches

- $\tilde{t}_1$ decay branching fraction
  - at large $M_1$, $m_{\tilde{t}_1} > m_b + M_W + m_{\tilde{Z}_1}$: $\tilde{t}_1 \rightarrow c\tilde{Z}_1$
  - for lower $M_1$: $\tilde{t}_1 \rightarrow bW\tilde{Z}_1$ opens up
  - for $M_1 < 400$ GeV, $m_{\tilde{t}_1} > m_b + m_{\tilde{W}_1}$: $\tilde{t}_1 \rightarrow b\tilde{W}_1$

- muti-isolated-lepton + jet + $E_T^{miss}$
  - signals in all channels observable with $E_T^c = 200$ GeV
  - jet multiplicity $n_{jet} \geq 2$, transverse sphericity $S_T > 0.2$, $E_T(j_1)$, $E_T(j_2) > E_T^c$ and $E_T^{miss} > E_T^c$
  - isolated leptons classified: $p_T > 10$ GeV, $|\eta(\ell)| < 2.5$, visible activity within a cone of $R = 0.3 < E_T(\text{cone}) = 5$ GeV.

Summary and Conclusion

- In most region of mSUGRA parameter space, neutralino relic abundance is too high compared to the WMAP measured result
- Allowing non-universality of gaugino or scalar masses provides the relic density in agreement with WMAP
- Many relic-density-consistent models should lead to observable signals at LHC. For instance, in the models $\tilde{Z}_2 - \tilde{Z}_1$ mass gap is less than $M_Z$, so that at least one dilepton mass edge is likely to be detectable at LHC
- In non-universal models with mixed higgsino or higgsino-wino dark matter, we have enhanced rates for direct and indirect DM searches.
- In models with bino-like dark matter, if we have a mechanism to elevate neutralino annihilation rates such as into top-antitop quark pairs via top squark, we should be able to get enhanced direct and indirect detection rates due to reduced gluino, squark masses and $\mu$ parameter
$BF(b \to s\gamma)$

$m_0 = 340 \text{ GeV}, \quad A_0 = -0.75M_1, \quad \tan\beta = 10, \quad \mu > 0, \quad m_1 = 175 \text{ GeV}$

$1.5M_1 = M_2 = 3M_3$
MSSM RGEs

\[
\frac{dm_{H_u}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{3}{5} g_1^2 M_1^2 - 3 g_2^2 M_2^2 + \frac{3}{10} g_1^2 S + 3 f_t^2 X_t \right)
\]

\[
\frac{dm_{H_d}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{3}{5} g_1^2 M_1^2 - 3 g_2^2 M_2^2 - \frac{3}{10} g_1^2 S + 3 f_b^2 X_b + f_{\tau}^2 X_{\tau} \right)
\]

\[
\frac{dm_{Q_3}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{1}{15} g_1^2 M_1^2 - 3 g_2^2 M_2^2 - \frac{16}{3} g_3^2 M_3^2 + \frac{1}{10} g_1^2 S + f_t^2 X_t + f_b^2 X_b \right)
\]

\[
\frac{dm_{i_R}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{16}{15} g_1^2 M_1^2 - \frac{16}{3} g_3^2 M_3^2 - \frac{2}{5} g_1^2 S + 2 f_t^2 X_t \right)
\]

\[
\frac{dm_{b_R}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{4}{15} g_1^2 M_1^2 - \frac{16}{3} g_3^2 M_3^2 + \frac{1}{5} g_1^2 S + 2 f_b^2 X_b \right)
\]

\[
\frac{dm_{L_3}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{3}{5} g_1^2 M_1^2 - 3 g_2^2 M_2^2 - \frac{3}{10} g_1^2 S + f_{\tau}^2 X_{\tau} \right)
\]

\[
\frac{dm_{\tau_R}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{12}{5} g_1^2 M_1^2 + \frac{3}{5} g_1^2 S + 2 f_{\tau}^2 X_{\tau} \right)
\]

\[
S = m_{H_u}^2 - m_{H_d}^2 + Tr \left[ m_Q^2 - m_L^2 - 2m_U^2 + m_D^2 + m_E^2 \right]
\]
where \( t = \log(Q) \), \( f_{t,b,\tau} \) are the \( t, b \) and \( \tau \) Yukawa couplings, and

\[
\begin{align*}
X_t &= m_{Q3}^2 + m_{tR}^2 + m_{Hu}^2 + A_t^2 \\
X_b &= m_{Q3}^2 + m_{bR}^2 + m_{Hd}^2 + A_b^2 \\
X_\tau &= m_{L3}^2 + m_{\tau R}^2 + m_{Hd}^2 + A_\tau^2
\end{align*}
\]