Learning from WIMPs

Manuel Drees

Bonn University
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Introduction: WIMPs as Dark Matter

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Galactic rotation curves imply $\Omega_{DM} h^2 \geq 0.05$.

$\Omega$: Mass density in units of critical density; $\Omega = 1$ means flat Universe.

$h$: Scaled Hubble constant. Observation: $h = 0.72 \pm 0.07 (?)$
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- Models of structure formation, X ray temperature of clusters of galaxies, . . .

- Cosmic Microwave Background anisotropies (WMAP) imply $\Omega_{DM} h^2 = 0.105^{+0.007}_{-0.013}$ Spergel et al., astro–ph/0603449
Weakly Interacting Massive Particles (WIMPs)

- Exist in well–motivated extensions of the SM: SUSY, (Little Higgs with $T$–Parity), ((Universal Extra Dimension))
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- Can also (trivially) write down “tailor-made” WIMP models
- In standard cosmology, roughly weak cross section automatically gives roughly right relic density for thermal WIMPs! (On logarithmic scale)
- Roughly weak interactions may allow both direct and indirect detection of WIMPs
Let $\chi$ be a generic DM particle, $n_\chi$ its number density (unit: GeV$^3$). Assume $\chi = \bar{\chi}$, i.e. $\chi \chi \leftrightarrow$ SM particles is possible, but single production of $\chi$ is forbidden by some symmetry.
WIMP production

Let $\chi$ be a generic DM particle, $n_\chi$ its number density (unit: GeV$^3$). Assume $\chi = \bar{\chi}$, i.e. $\chi\chi \leftrightarrow$ SM particles is possible, but single production of $\chi$ is forbidden by some symmetry.

Evolution of $n_\chi$ determined by Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\text{ann}}v \rangle (n^2_\chi - n^2_\chi,\text{eq})$$

$H = \dot{R}/R$ : Hubble parameter
$\langle \ldots \rangle$ : Thermal averaging
$\sigma_{\text{ann}} = \sigma(\chi\chi \rightarrow \text{SM particles})$
$v$ : relative velocity between $\chi$’s in their cms
$n_\chi,\text{eq}$ : $\chi$ density in full equilibrium
Thermal WIMP

Assume $\chi$ was in full thermal equilibrium after inflation.
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$$n_\chi \langle \sigma_{\text{ann}} v \rangle > H$$
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For $T < m_\chi$: $n_\chi \simeq n_{\chi, \text{eq}} \propto T^{3/2} e^{-m_\chi/T}$, $H \propto T^2$
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Gives

$$\Omega_\chi h^2 \propto \frac{1}{\langle v \sigma_{\text{ann}} \rangle} \sim 0.1 \text{ for } \sigma_{\text{ann}} \sim \text{pb}$$
Thermal WIMPs: Assumptions

- $\chi$ is effectively stable, $\tau_\chi \gg \tau_U$: partly testable at colliders
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- Universe must have been sufficiently hot:
  
  $$T_R > T_F \simeq m_\chi/20$$
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Can we test these assumptions, if $\Omega_\chi$ and “all” particle physics properties of $\chi$ are known?
Low temperature scenario

Assume $T_0 \lesssim T_F$, $n_\chi(T_0) = 0$ ($T_0$: Initial temperature)
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Introduce dimensionless variables

$$Y_\chi \equiv \frac{n_\chi}{s}, \quad x \equiv \frac{m_\chi}{T} \quad (s: \text{entropy density})$$
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$$Y_\chi \equiv \frac{n_\chi}{s}, \quad x \equiv \frac{m_\chi}{T}$$

($s$: entropy density).

Use non–relativistic expansion of cross section:

$$\sigma_{\text{ann}} = a + b v^2 + O(v^4) \implies \langle \sigma_{\text{ann}} v \rangle = a + 6b/x$$
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![Graph showing dependence of $\Omega_\chi h^2$ on $a$ with $x_0 = 22$ and WMAP constraints]
Using explicit form of $H$, $Y_{\chi,eq}$, Boltzmann eq. becomes

\[
\frac{dY_{\chi}}{dx} = -f \left( a + \frac{6b}{x} \right) x^{-2} \left( Y_{\chi}^2 - cx^3 e^{-2x} \right) .
\]

\[
f = 1.32 \, m_\chi M_{P1} \sqrt{g_*}, \quad c = 0.0210 \, g_\chi^2 / g_*^2.
\]
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$f = 1.32 \, m_{\chi} M_{P1} \sqrt{g_{*}}$, $c = 0.0210 \, g_{\chi}^{2} / g_{*}^{2}$

For $T_{0} \ll T_{F}$: Annihilation term $\propto Y_{\chi}^{2}$ negligible: defines 0–th order solution $Y_{0}(x)$, with

$$Y_{0}(x \to \infty) = f c \left[ \frac{a}{2} x R e^{-2xR} + \left( \frac{a}{4} + 3b \right) e^{-2xR} \right].$$

Note: $\Omega_{\chi} h^{2} \propto \sigma_{\text{ann}}$ in this case!
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For intermediate temperatures, $T_0 \lesssim T_F$: Define 1st-order solution

$$Y_1 = Y_0 + \delta.$$

$\delta < 0$ describes pure annihilation:

$$\frac{d\delta}{dx} = -f \left( a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}.$$

$\delta(x)$ can be calculated analytically: $\delta \propto \sigma_{\text{ann}}^3$
Get good results for $\Omega_\chi h^2$ for all $T_0 \leq T_F$ through “resummation”:

$$Y_1 = Y_0 \left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$

$Y_{1,r} \propto 1/\sigma_{\text{ann}}$ for $|\delta| \gg Y_0$  

MD, Imminniyaz, Kakizaki, hep-ph/0603165
Numerical comparison: $b = 0$

\[ a = 10^{-8} \text{ GeV}^{-2} \]

\[ a = 10^{-9} \text{ GeV}^{-2} \]
Numerical comparison: $b = 0$

Can extend validity of new solution to all $T$, including $T \gg T_0$, by using $\Omega_{\chi}(T_{\text{max}})$ if $T_0 > T_{\text{max}} \simeq T_F$
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Can extend validity of new solution to all $T$, including $T \gg T_0$, by using $\Omega_\chi(T_{\text{max}})$ if $T_0 > T_{\text{max}} \simeq T_F$

Note: $\Omega_\chi(T_0) \leq \Omega_\chi(T_0 \gg T_F)$

Learning from WIMPs – p. 11/29
Application: lower bound on $T_0$ for thermal WIMP

If $n_\chi(T_0) = 0$, demanding $\Omega_\chi h^2 \sim 0.1$ imposes lower bound on $T_0$:
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If $n_\chi(T_0) = 0$, demanding $\Omega_\chi h^2 \sim 0.1$ imposes lower bound on $T_0$: 

![Graph showing the relationship between $\Omega_\chi h^2$ and $x_0$ for different values of $a$ and $b$.]
Application: lower bound on $T_0$ for thermal WIMP

If $n_\chi(T_0) = 0$, demanding $\Omega_\chi h^2 \simeq 0.1$ imposes lower bound on $T_0$:

$$\Rightarrow T_0 \geq \frac{m_\chi}{23}$$

Holds independent of $\sigma_{\text{ann}}$!
Application: lower bound on $T_0$ for thermal WIMP

If $n_\chi(T_0) = 0$, demanding $\Omega_\chi h^2 \sim 0.1$ imposes lower bound on $T_0$:

$$\implies T_0 \geq \frac{m_\chi}{23} \quad \text{Holds independent of } \sigma_{\text{ann}}!$$

If $T_0 \simeq m_\chi/22$: Get right $\Omega_\chi h^2$ for wide range of cross sections!
Constraining $H(T)$

- Assumptions
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- $\Omega_{\chi} h^2$ is known (see below)
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  - Only thermal $\chi$ production (otherwise no constraint)
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  - $\Omega_\chi h^2$ is known (see below)
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- **Parameterize modified expansion history:**

  $$A(z) = \frac{H_{st}(z)}{H(z)} , \ z = T/m_\chi$$
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- **Around decoupling:** $z \ll 1 \implies$ use Taylor expansion

  $$A(z) = A(z_{F, st}) + (z - z_{F, st}) A'(z_{F, st}) + (z - z_{F, st})^2 A''(z_{F, st}) / 2$$
Constraining $H(T)$

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**Successful BBN** $\implies k \equiv A(z \to 0) = 1.0 \pm 0.2$
Constraining $H(T)$ (cont.d)

Assume $T_0 \gg T_F \implies \Omega \chi h^2 \propto \frac{1}{\int_0^{z_F} A(z)(a+6bz)\,dz}$
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The case $A''(z_{F, st}) = 0$
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Relative constraint on $A(z_{F, st})$ weaker than that on $\Omega_\chi h^2$. 
Direct WIMP detection

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- Can elastically scatter on nucleus in detector:

$$\chi + N \rightarrow \chi + N$$

Measured quantity: recoil energy of $N$
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- Detection needs ultrapure materials in deep–underground location; way to distinguish recoils from \( \beta, \gamma \) events; neutron screening; \ldots

Learning from WIMPs – p. 16/29
Direct WIMP detection

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  Measured quantity: recoil energy of \( N \)
- Detection needs ultrapure materials in deep–underground location; way to distinguish recoils from \( \beta, \gamma \) events; neutron screening; . . .
- Is being pursued vigorously around the world!
Direct WIMP detection: theory

Counting rate given by

$$\frac{dR}{dQ} = AF^2(Q) \int_{v_{\text{min}}}^{v_{\text{esc}}} \frac{f_1(v)}{v} dv$$

$Q$: recoil energy
$A = \rho \sigma_0 / (2m_\chi m_r) = \text{const.}: \text{encodes particle physics}$
$F(Q)$: nuclear form factor
$v$: WIMP velocity in lab frame
$v_{\text{min}}^2 = m_N Q / (2m_r^2)$
$v_{\text{esc}}$: Escape velocity from galaxy
$f_1(v)$: normalized one–dimensional WIMP velocity distribution
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\( f_1(v) \): normalized one–dimensional WIMP velocity distribution

In principle, can invert this relation to measure \( f_1(v) \)!
Direct reconstruction of $f_1$

$$f_1(v) = N \left\{ -2Q \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \frac{dR}{dQ} \right] \right\}_{Q=2m_r v^2 / m_N}$$
Direct reconstruction of $f_1$

$$f_1(v) = \mathcal{N} \left\{ -2Q \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \frac{dR}{dQ} \right] \right\}_{Q=2m_r v^2 / m_N}$$

$\mathcal{N}$: Normalization ($\int_{0}^{\infty} f_1(v) dv = 1$).
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Need to know $m_\chi$, but do not need $\sigma_0, \rho$. 
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Need to know $m_\chi$, but do not need $\sigma_0, \rho$.
Need to know *slope* of recoil spectrum!
Direct reconstruction of \( f_1 \)

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Need to know \( m_\chi \), but do not need \( \sigma_0, \rho \).

Need to know slope of recoil spectrum!

\( dR/dQ \) is approximately exponential: better work with logarithmic slope
Determining the logarithmic slope of \( dR/dQ \)

- Good local observable: Average energy transfer \( \langle Q \rangle_i \) in \( i \)–th bin
Determining the logarithmic slope of $dR/dQ$

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Determining the logarithmic slope of $dR/dQ$

- Good local observable: Average energy transfer $\langle Q \rangle_i$ in $i$–th bin
- Stat. error on slope $\propto (\text{bin width})^{-1.5} \implies \text{need large bins}$
- To maximize information: use overlapping bins ("windows")
Recoil spectrum: prediction and simulated measurement

500 events, 5 bins, up to 3 bins per window

$\chi^2$/dof = 0.73

input distribution
Recoil spectrum: prediction and simulated measurement

\[ f_1(v) \text{ [s/km]} \]

\[ v \text{ [km/s]} \]

5,000 events, 10 bins, up to 4 bins per window

\[ \chi^2/\text{dof} = 0.98 \]
Statistical exclusion of constant $f_1$

Average over 1,000 experiments

- Probability
- $N_{ev}$

Mean and median curves are shown.
Statistical exclusion of constant $f_1$

Need several hundred events to begin direct reconstruction!
Determining moments of $f_1$

\[
\langle v^n \rangle \equiv \int_0^\infty v^n f_1(v) \, dv
\]
Determining moments of $f_1$

$$\langle v^n \rangle \equiv \int_0^\infty v^n f_1(v) dv$$

$$\propto \int_0^\infty Q^{(n-1)/2} \frac{1}{F^2(Q)} \frac{dR}{dQ} dQ$$
Determining moments of $f_1$

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$$\propto \int_0^\infty Q^{(n-1)/2} \frac{1}{F^2(Q)} \frac{dR}{dQ} dQ$$
$$\rightarrow \sum \text{events } a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}$$
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Can incorporate finite energy (hence velocity) threshold
Determining moments of $f_1$

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\rightarrow \sum_{\text{events}} a \frac{Q^{(n-1)/2}}{F^2(Q_a)}
\]

Can incorporate finite energy (hence velocity) threshold

Moments are strongly correlated!
Determining moments of $f_1$

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Can incorporate finite energy (hence velocity) threshold

Moments are strongly correlated!

High moments, and their errors, are underestimated in “typical” experiment: get large contribution from large $Q$
Determination of first 10 moments

![Graph showing the determination of first 10 moments with 100 events. The graph plots \( \langle v^n \rangle / \langle v^n \rangle_{\text{exact}} \) against \( n \) from 0 to 10. The y-axis ranges from 0 to 1.5, and the x-axis ranges from 0 to 10. There are error bars at each point, indicating the variability of the measurements.]}
Constraining a “late infall” component

25 events, fit moments $n = -1, 1, 2$

$\Delta \chi^2 = 1$
$\Delta \chi^2 = 4$

$v_{esc}$ [km/s]

$N_1$
Constraining a “late infall” component

100 events, fit moments n = -1, 1, 2, 3

$\Delta \chi^2 = 1$

$\Delta \chi^2 = 4$

$N_1$ vs. $v_{esc}$ [km/s]
Determining the WIMP mass

Can determine \( m_\chi \) from requirement that different targets yield \textit{same} moments of \( f_1 \)

MD & C.L. Shan, in progress
Range of WIMP mass from simulation

Preliminary!

50 + 50 events, Si and Ge, standard halo

$\mu_{\text{rec, hi, lo}}$ [GeV]

$m_{\text{in}}$ [GeV]

$n = -1$

$n = 1$

$n = 2$

$\sigma$

$\text{tot}$

Learning from WIMPs – p. 28/29
Range of WIMP mass from simulation

Preliminary!

500+ 500 events, Si and Ge, standard halo

![Graph showing the range of WIMP mass from simulation with different n values and total (tot).]
Range of WIMP mass from simulation
Preliminary!

500+ 500 events, Si and Ge, standard halo plus 10% late infall

\[ m_{\text{rec, hi, lo}} \] [GeV]

\[ m_{\text{in}} \] [GeV]

- \( n = -1 \)
- \( n = 1 \)
- \( n = 2 \)
- \( \sigma \)
Range of WIMP mass from simulation

Preliminary!

500+ 500 events, Si and Xe, standard halo
Summary

- Learning about the Early Universe:
Learning about the Early Universe:

- If all DM is thermal WIMPs: $T_0 \geq m_\chi/23 \sim 10^4 T_{\text{BBN}}$
Summary

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- **Learning about WIMPs:** Can determine \( m_\chi \) from moments of \( f_1 \) measured with two different targets.