Learning from WIMPs

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1 Introduction
Contents

1 Introduction
2 Learning about the early Universe
Contents

1 Introduction
2 Learning about the early Universe
3 Learning about our galaxy
Contents

1 Introduction
2 Learning about the early Universe
3 Learning about our galaxy
4 Learning about WIMPs
5 Summary
Introduction: WIMPs as Dark Matter

Several observations indicate existence of non-luminous Dark Matter (DM) (more exactly: missing force)
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 غالبية التسجيكات تشير إلى وجود ضوء مظلم (DM) (بما هو أكثر دقة: التفاعل المفقود)

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- Cosmic Microwave Background anisotropies (WMAP) imply $\Omega_{DM} h^2 = 0.105^{+0.007}_{-0.013}$  
  Spergel et al., astro-ph/0603449
Weakly Interacting Massive Particles (WIMPs)

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- In standard cosmology, roughly weak cross section automatically gives roughly right relic density for thermal WIMPs! (On logarithmic scale)
- Roughly weak interactions may allow both direct and indirect detection of WIMPs
WIMP production

Let $\chi$ be a generic DM particle, $n_\chi$ its number density (unit: GeV$^3$). Assume $\chi = \bar{\chi}$, i.e. $\chi\chi \leftrightarrow$ SM particles is possible, but single production of $\chi$ is forbidden by some symmetry.
Let $\chi$ be a generic DM particle, $n_\chi$ its number density (unit: GeV$^3$). Assume $\chi = \bar{\chi}$, i.e. $\chi\chi \leftrightarrow$ SM particles is possible, but single production of $\chi$ is forbidden by some symmetry.

Evolution of $n_\chi$ determined by Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3H n_\chi = -\langle \sigma_{\text{ann}} v \rangle (n_\chi^2 - n_{\chi,\text{eq}}^2)$$

$H = \dot{R}/R$: Hubble parameter

$\langle \ldots \rangle$: Thermal averaging

$\sigma_{\text{ann}} = \sigma(\chi\chi \rightarrow \text{SM particles})$

$v$: relative velocity between $\chi$’s in their cms

$n_{\chi,\text{eq}}$: $\chi$ density in full equilibrium
Thermal WIMP

Assume $\chi$ was in full thermal equilibrium after inflation.
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For $T < m_\chi$:

$$n_\chi \simeq n_{\chi, \text{eq}} \propto T^{3/2} e^{-m_\chi / T}, \ H \propto T^2$$
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For \( T < T_F \): WIMP production negligible, only annihilation relevant in Boltzmann equation.
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Gives

$$\Omega_\chi h^2 \propto \frac{1}{\langle v \sigma_{\text{ann}} \rangle} \sim 0.1 \text{ for } \sigma_{\text{ann}} \sim \text{pb}$$
Thermal WIMPs: Assumptions

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- Universe must have been sufficiently hot:
  \[ T_R > T_F \simeq m_\chi / 20 \]

Can we test these assumptions, if $\Omega_\chi$ and “all” particle physics properties of $\chi$ are known?
Assume $T_0 \lesssim T_F$, $n\chi(T_0) = 0$ ($T_0$: Initial temperature)
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Introduce dimensionless variables

\[ Y_\chi \equiv \frac{n_\chi}{s}, \quad x \equiv \frac{m_\chi}{T} \quad (s: \text{entropy density}). \]
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Use non–relativistic expansion of cross section:

$$\sigma_{\text{ann}} = a + bv^2 + \mathcal{O}(v^4) \implies \langle \sigma_{\text{ann}} v \rangle = a + 6b/x$$
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Low temperature scenario (cont.’d)

Using explicit form of $H$, $Y_{\chi, eq}$, Boltzmann eq. becomes

$$\frac{dY_{\chi}}{dx} = -f \left( a + \frac{6b}{x} \right) x^{-2} \left( Y_{\chi}^2 - cx^3 e^{-2x} \right).$$

$$f = 1.32 \ m_{\chi} M_{P1} \sqrt{g_*}, \ c = 0.0210 \ g_{\chi}^2/g_*$$
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f = 1.32 \ m_\chi M_{Pl} \sqrt{g_*}, \ c = 0.0210 \ g_\chi^2 / g_*^2
\]
For $T_0 \ll T_F$: Annihilation term $\propto Y_\chi^2$ negligible: defines 0-th order solution $Y_0(x)$, with
\[
Y_0(x \to \infty) = fc \left[ \frac{a}{2} x R e^{-2xR} + \left( \frac{a}{4} + 3b \right) e^{-2xR} \right].
\]
Note: $\Omega_\chi h^2 \propto \sigma_{ann}$ in this case!
Using explicit form of $H$, $Y_{\chi,eq}$, Boltzmann eq. becomes

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For intermediate temperatures, $T_0 \lesssim T_F$: Define 1st–order solution

$$Y_1 = Y_0 + \delta.$$

$\delta < 0$ describes pure annihilation:

$$\frac{d\delta}{dx} = -f \left( a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}.$$

$\delta(x)$ can be calculated analytically: $\delta \propto \sigma_{\text{ann}}^3$.
Low temperature scenario (cont.’d)

Get good results for $\Omega \chi h^2$ for all $T_0 \leq T_F$ through “resummation”:

$$Y_1 = Y_0 \left(1 + \frac{\delta}{Y_0}\right) \approx \frac{Y_0}{1-\delta/Y_0} \equiv Y_{1,r}$$

$Y_{1,r} \propto 1/\sigma_{\text{ann}}$ for $|\delta| \gg Y_0$
Numerical comparison: \( b = 0 \)

\[
\begin{align*}
\Omega_{\chi^2}^{\text{exact}} &= 10^{-1} \\
\Omega_{\chi^2}^{\text{old}} &= 10^{-2} \\
\Omega_{\chi^2}^{\text{new}} &= 10^{-3}
\end{align*}
\]

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\[a = 10^{-8} \text{ GeV}^{-2}\]

\[a = 10^{-9} \text{ GeV}^{-2}\]
Numerical comparison: $b = 0$

Can extend validity of new solution to all $T$, including $T \gg T_0$, by using $\Omega_{\chi}(T_{\text{max}})$ if $T_0 > T_{\text{max}} \simeq T_F$
Numerical comparison: \( b = 0 \)

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Note: \( \Omega_\chi(T_0) \leq \Omega_\chi(T_0 \gg T_F) \)
If $n_\chi(T_0) = 0$, demanding $\Omega_\chi h^2 \simeq 0.1$ imposes lower bound on $T_0$:
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Application: lower bound on $T_0$ for thermal WIMP

If $n_{\chi}(T_0) = 0$, demanding $\Omega_{\chi} h^2 \sim 0.1$ imposes lower bound on $T_0$:

$$\implies T_0 \geq \frac{m_{\chi}}{23}$$

Holds independent of $\sigma_{\text{ann}}$!
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If $T_0 \approx m_\chi/22$: Get right $\Omega_\chi h^2$ for wide range of cross sections!
Constraining $H(T)$

- Assumptions
Constraining $H(T)$

**Assumptions**

- $\Omega_\chi h^2$ is known (see below)
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  - $\Omega_{\chi} h^2$ is known (see below)
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- **Parameterize modified expansion history:**

  $$A(z) = \frac{H_{st}(z)}{H(z)}, \quad z = \frac{T}{m_{\chi}}$$
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- **Parameterize modified expansion history:**

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A(z) = \frac{H_{st}(z)}{H(z)} , \quad z = \frac{T}{m_\chi}
\]

- **Around decoupling:** $z \ll 1 \implies$ use Taylor expansion

\[
A(z) = A(z_{F, st}) + (z - z_{F, st}) A'(z_{F, st}) + (z - z_{F, st})^2 A''(z_{F, st}) / 2
\]
Constraining $H(T)$

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**Successful BBN**

$$k \equiv A(z \to 0) = 1.0 \pm 0.2$$
Constraining $H(T)$ (cont.d)

Assume $T_0 \gg T_F \implies \Omega \chi h^2 \propto \frac{1}{\int_0^{z_F} A(z)(a+6bz) \, dz}$
Constraining $H(T)$ (cont.d)

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The case $A''(z_{F,st}) = 0$
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Relative constraint on \( A(z_{F,\text{st}}) \) weaker than that on \( \Omega\chi h^2 \).
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Relative constraint on \( A(z_{F, st}) \) weaker than that on \( \Omega_\chi h^2 \).

\( H \gg H_{st} \) can be excluded from WIMP non–observation (Schelke et al.)
Direct WIMP detection

WIMPs are everywhere!
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  \[ \chi + N \rightarrow \chi + N \]
  Measured quantity: recoil energy of \( N \)
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- Detection needs ultrapure materials in deep–underground location; way to distinguish recoils from \( \beta, \gamma \) events; neutron screening; . . .
- Is being pursued vigorously around the world!
Direct WIMP detection: theory

Counting rate given by

\[ \frac{dR}{dQ} = A F^2(Q) \int_{v_{\text{min}}}^{v_{\text{esc}}} \frac{f_1(v)}{v} dv \]

- \( Q \): recoil energy
- \( A = \frac{\rho \sigma_0}{(2m_\chi m_r)} = \text{const.: encodes particle physics} \)
- \( F(Q) \): nuclear form factor
- \( v \): WIMP velocity in lab frame
- \( v_{\text{min}}^2 = m_N Q/(2m_r^2) \)
- \( v_{\text{esc}} \): Escape velocity from galaxy
- \( f_1(v) \): normalized one–dimensional WIMP velocity distribution
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In principle, can invert this relation to measure \(f_1(v)\)!
Direct reconstruction of $f_1$

$$f_1(v) = \mathcal{N} \left\{ -2Q \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \frac{dR}{dQ} \right] \right\}_{Q = 2m_r v^2 / m_N}$$

MD & C.L. Shan, astro-ph/0703651
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$\mathcal{N}$: Normalization ($\int_0^\infty f_1(v) dv = 1$).
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Need to know slope of recoil spectrum!
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$dR/dQ$ is approximately exponential: better work with logarithmic slope.
Determining the logarithmic slope of $dR/dQ$

- Good local observable: Average energy transfer $\langle Q \rangle_i$ in $i$–th bin
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- Stat. error on slope $\propto (\text{bin width})^{-1.5} \implies$ need large bins
**Determining the logarithmic slope of** $dR/dQ$

- Good local observable: Average energy transfer $\langle Q \rangle_i$ in $i$–th bin
- Stat. error on slope $\propto (\text{bin width})^{-1.5} \implies$ need large bins
- To maximize information: use overlapping bins ("windows")
Recoil spectrum: prediction and simulated measurement

500 events, 5 bins, up to 3 bins per window

\[ \chi^2 / \text{dof} = 0.73 \]
Recoil spectrum: prediction and simulated measurement

\[ f_1(v) \text{ [s/km]} \]

5,000 events, 10 bins, up to 4 bins per window

\[ \chi^2/\text{dof} = 0.98 \]

input distribution
Statistical exclusion of constant $f_1$
Statistical exclusion of constant $f_1$

Need several hundred events to begin direct reconstruction!
Determining moments of $f_1$

$$\langle v^n \rangle \equiv \int_{0}^{\infty} v^n f_1(v) \, dv$$
Determining moments of $f_1$

\[
\langle v^n \rangle \equiv \int_0^\infty v^n f_1(v) dv \\
\propto \int_0^\infty Q^{(n-1)/2} \frac{1}{F^2(Q)} \frac{dR}{dQ} dQ
\]
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\]
\[
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\]
\[
\rightarrow \sum_{\text{events}} a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}
\]
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Can incorporate finite energy (hence velocity) threshold
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Moments are strongly correlated!

High moments, and their errors, are underestimated in “typical” experiment: get large contribution from large $Q$
Determination of first 10 moments

\[ \frac{\langle v^n \rangle}{\langle v^n \rangle_{\text{exact}}} \]

100 events

\( n = 0, 2, 4, 6, 8, 10 \)
Constraining a “late infall” component

25 events, fit moments $n = -1, 1, 2$

$\Delta \chi^2 = 1$

$\Delta \chi^2 = 4$
Constraining a “late infall” component

100 events, fit moments $n = -1, 1, 2, 3$

$\Delta \chi^2 = 1$

$\Delta \chi^2 = 4$
Determining the WIMP mass

Can determine $m_\chi$ from requirement that different targets yield *same* moments of $f_1$
Range of WIMP mass from simulation

Preliminary!

50 + 50 events, Si and Ge, standard halo

\( m_{\text{in}} \) vs. \( m_{\text{rec, hi, lo}} \) in [GeV]

Learning from WIMPs – p. 28/32
Range of WIMP mass from simulation
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500+ 500 events, Si and Ge, standard halo

\[
m_{\text{rec, hi, lo}} \quad [\text{GeV}]\n\]

\[
m = -1
\]

\[
m = 1
\]

\[
tot
\]

\[
\sigma
\]

\[
n = 2
\]
Range of WIMP mass from simulation

Preliminary!

500+ 500 events, Si and Ge, standard halo plus 10% late infall

$n = -1$

$n = 1$

$n = 2$

$\sigma$

$\text{tot}$

$m_{\text{reco}, \text{hi, lo}}$ [GeV]

$m_{\text{in}}$ [GeV]
Range of WIMP mass from simulation
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Summary

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- Learning about WIMPs: Can determine $m_\chi$ from moments of $f_1$ measured with two different targets.