Abundance of Thermal Relics in Non-standard Cosmological Scenarios

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September 9, 2008

IPMU seminar

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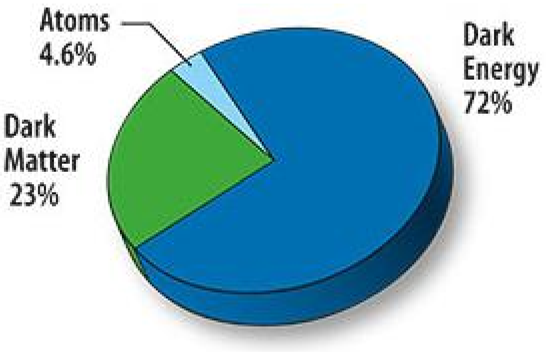
Refs:
- PRD73 123502 (2006)
- PRD76 103524 (2007)
- work in progress

1. Motivation



[http://wmap.gsfc.nasa.gov]

- Observations of
  - cosmic microwave background
  - structure of the universe
  - etc.

→ **Non-baryonic dark matter:** $\Omega_{\rm DM}h^2 = 0.1143 \pm 0.0034$

- Weakly interacting massive particles (WIMPs) χ are good candidates for cold dark matter (CDM)

The predicted thermal relic abundance naturally explains the observed dark matter abundance: $\Omega_{\chi,\rm standard}h^2 \sim 0.1$

- Neutralino (LSP); 1st KK mode of the B boson (LKP); etc.

Investigation of early universe using DM abundance

- The abundance of thermal relics (e.g. DM) is determined by the Boltzmann equation: $\dot{n}_\chi + 3Hn_\chi = -\langle\sigma_{\rm eff} v\rangle(n_\chi^2 - n_{\chi,\rm eq}^2)$

  (and the reheat temperature: T_R)

  Numerical calculation needed in evaluating the relic density in many cases

  ⇒ **Analytic methods should be developed in various scenarios**

- The (effective) cross section $\sigma_{\rm eff}$ can be determined from collider and DM detection experiments

  ⇒ **We can test the standard CDM scenario and investigate conditions of very early universe: $T_R, H, \cdots$**

Outline

This work

- Analytic treatment applicable to low-reheat-temperature scenarios
- Dark matter = thermal WIMPs

  ⇒ constraints on the reheating temperature and on modifications of the Hubble parameter
- Analytic treatment that connects the hot and cold relic solutions

1. Motivation
2. Standard calculation of WIMP relic abundance (review)
3. Low-temperature scenario
4. Constraints on the very early universe from WIMP dark matter
5. Abundance of semi-relativistic relics
6. Summary

2. Standard calculation of the WIMP relic abundance

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- **Conventional assumptions for WIMPs as DM particle:**
  - $\chi = \bar{\chi}$, single production of χ is forbidden
- **WIMP abundance n_χ is determined by the Boltzmann eq.:**

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

$H = \dot{R}/R$: Hubble expansion parameter

$\langle \sigma v \rangle$: thermal average of the annihilation cross section $\sigma(\chi\chi \to \mathrm{SM\ particles})$ times relative velocity v

$n_{\chi,\mathrm{eq}}$: equilibrium number density

- **Introduce** $Y_{\chi(,\mathrm{eq})} = \frac{n_{\chi(,\mathrm{eq})}}{s}, \, x = \frac{m_\chi}{T}$

$$\Rightarrow \quad \frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle \sigma v \rangle s}{Hx}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2)$$

Standard formula

- High temperature $(T \geq T_F)$:
  - Thermal equilibrium was maintained:

$$\Gamma = n_\chi \langle \sigma v \rangle > H = R/\dot{R}$$

- χ decoupled when non-relativistic in RD era:

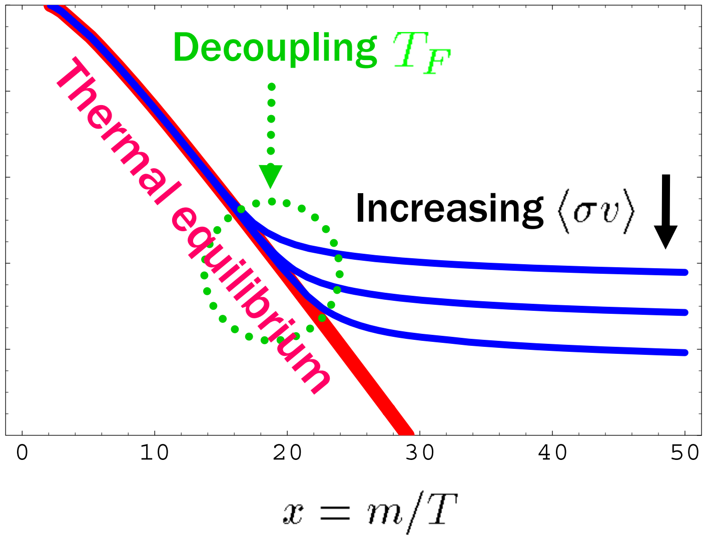
$$H = \frac{\pi T^2}{M_{\rm Pl}} \sqrt{\frac{90}{g_*}}$$

$$\langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2),$$

$$n_{\chi,{\rm eq}} = g_\chi \ (m_\chi T/2\pi)^{3/2} {\rm e}^{-m_\chi/T}$$

Co-moving number density

$$Y_\chi = n_\chi / s$$



$$\Omega_\chi h^2 = 2.7 \times 10^8 \ Y_\chi \left(\frac{m_\chi}{1 \ {\rm GeV}} \right)$$

- Low temperature $(T \leq T_F)$:
  - WIMP production negligible :

$$\Omega_{\chi,{\rm standard}} h^2 \simeq 0.1 \times \left(\frac{a + 3b/x_F}{10^{-9} \ {\rm GeV}^{-2}} \right)^{-1} \left(\frac{x_F}{22} \right) \left(\frac{g_*}{90} \right)^{-1/2} \sim \Omega_{\rm DM} h^2$$

3. Low-temperature scenario

- T_R : Reheat temperature

The initial abundance is assumed to be negligible: $Y_\chi(x_0) = 0\,, \quad x_0 = \frac{m_\chi}{T_R}$

- Zeroth order approximation:

$T_R < T_F$ ⇨ χ annihilation is negligible:

$$\frac{dY_0}{dx} = 0.028\, g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} {\rm e}^{-2x} x \left(a + \frac{6b}{x}\right)$$

⇨ The solution is proportional to the cross section:

At late times,

$$Y_0(x \gg x_0) \simeq 0.014\, g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} {\rm e}^{-2x_0} x_0 \left(a + \frac{6b}{x_0}\right)$$

This solution should be smoothly connected to the standard result

First order approximation

- Add a correction term describing annihilation to Y_0 : $Y_1 = Y_0 + \delta \ (\delta < 0)$
- As long as $|\delta| \ll Y_0$, the evolution equation for δ is

$$\frac{d\delta}{dx} = -1.3 \sqrt{g_*} m_\chi M_{\rm PL} \left(a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}$$

➡ The solution is proportional to σ^3

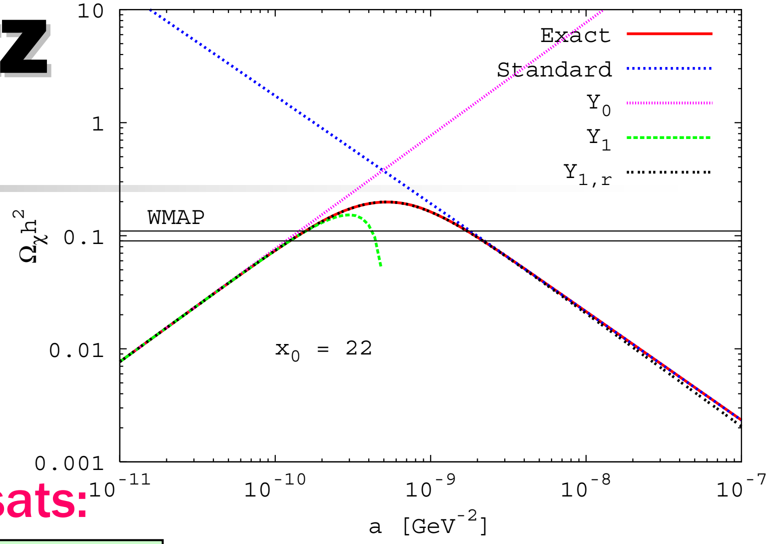
At late times,

$$\delta(x \gg x_0) \simeq -2.5 \times 10^{-4} \ g_\chi^4 g_*^{-5/2} m^3 M_{\rm Pl}^3 e^{-4x_0} x_0 \left(a + \frac{3b}{x_0} \right) \left(a + \frac{6b}{x_0} \right)^2$$

- $|\delta|$ soon dominates over Y_0 for not very small cross section

➡ Y_1 fails to track the exact solution

Re-summed ansatz



- It is noticed that $Y_0 \propto \sigma > 0,\ \delta \propto \sigma^3 < 0$

  For large cross section,

  $Y_\chi(x \to \infty)$ should be $\propto 1/\langle \sigma v \rangle$

- This observation suggests the re-summed ansats:

$$Y(x) = Y_0 + \delta = Y_0\left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$

- For $|\delta| \gg Y_0$, $Y_{1,r}(x) \simeq -\frac{Y_0^2}{\delta} \propto \frac{1}{\sigma}$

$x_0 \to x_F$ Standard formula

At late times, $Y_{1,r}(x \to \infty) = \dfrac{x_0}{1.3\sqrt{g_*}\, m_\chi M_{\rm Pl}(a + 3b/x_0)}$

- In the case where χ production is negligible but the initial abundance is sizable, $Y_{1,r}$ is exact

4. Constraints on the very early universe from WIMP DM

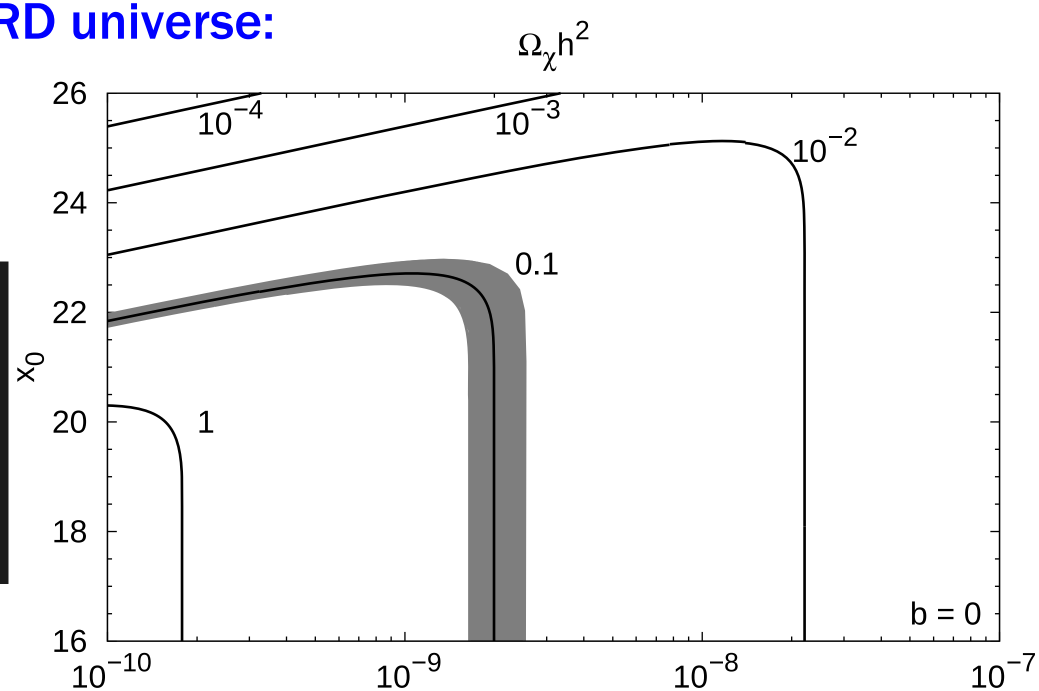
- **Out-of-equilibrium case:** $\sigma \nearrow \Rightarrow \Omega h^2 \nearrow$; $T_0 = m_\chi / x_0 \nearrow \Rightarrow \Omega h^2 \nearrow$

  Equilibrium case: $\sigma \nearrow \Rightarrow \Omega h^2 \searrow$; $\Omega_\chi h^2$ is independent of T_R

- **Thermal relic abundance in the RD universe:**

■ $0.8 < \Omega_{\rm DM} h^2 < 0.12$

> Requirement that $\Omega_\chi h^2 \simeq 0.1$
>
> ⇓
>
> Lower bound on the reheat temperature: $T_R > m_\chi / 23$



Modified expansion rate

- Various cosmological models predict a non-standard early expansion

[e.g. Scherrer et al.,PRD(1985); Salati,PLB(2003); Fernengo et al.,PRD(2003); Chung et al., PRD (1999); ...]

⇨ Predicted WIMP relic abundances are also changed

- When WIMPs were in full thermal equilibrium, in terms of the modification parameter $A(x) = H_{\rm st}(x)/H(x)$ the relic abundance is

$$\Omega_\chi h^2 = 0.1 \left(\frac{I(x_F)}{8.5 \times 10^{-10}\ \mathrm{GeV}^{-2}} \right)^{-1}$$

$$I(x_F) = \int_{x_F}^{\infty} dx \frac{\sqrt{g_*}\langle\sigma v\rangle A(x)}{x^2}; \quad x_F = \ln \left[\sqrt{\frac{45}{\pi^5}} \xi m_\chi M_{\rm Pl} g_\chi \frac{\langle\sigma v\rangle A(x)}{\sqrt{x g_*}} \right] \Bigg|_{x=x_F}$$

If $A(x) = 1$, $x_F = x_{F,\rm st}$ and we recover the standard formula

This formula is capable of predicting the final relic density correctly

Constrains on modifications of the Hubble parameter

- In terms of $z \equiv T/m_\chi = 1/x$

  we need to know $A(z)$ only for $z_{\rm BBN} = 10^{-5} - 10^{-4} \leq z \leq z_F \sim 1/20 \ll \mathcal{O}(1)$

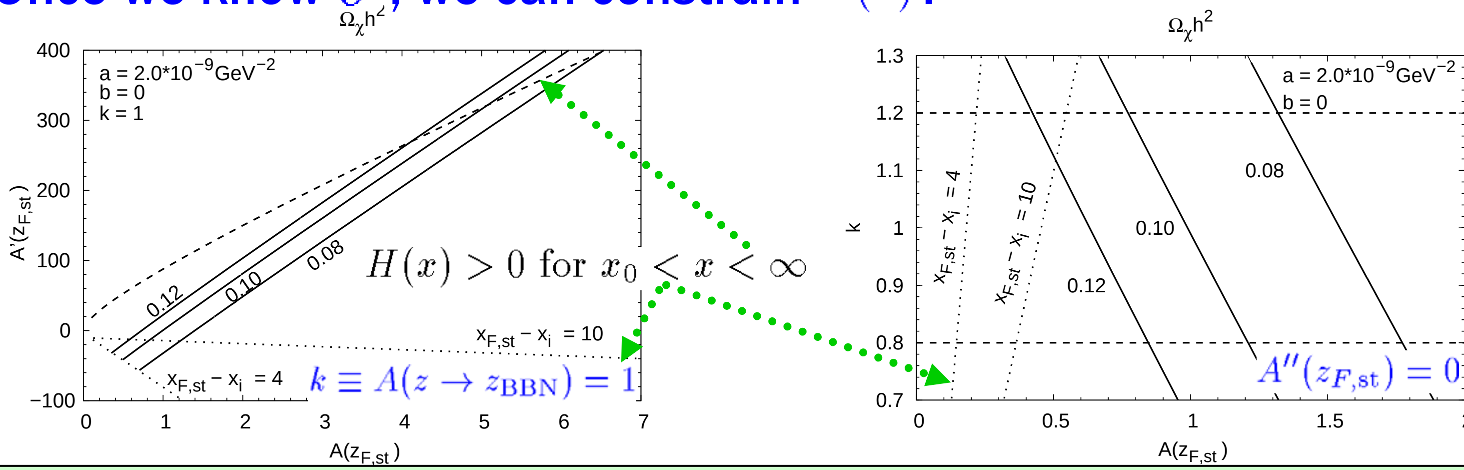
➡ This suggests a parametrization of $A(z)$ in powers of $(z - z_{F,{\rm st}})$:

$$A(z) = A(z_{F,{\rm st}}) + (z - z_{F,{\rm st}})A'(z_{F,{\rm st}}) + \frac{1}{2}(z - z_{F,{\rm st}})^2 A''(z_{F,{\rm st}})$$

subject to the BBN limit: $0.8 \leq k \equiv A(z \to z_{\rm BBN}) \leq 1.2$

x_i :Maximal temperature where

- Once we know σ, we can constrain $A(z)$:



$\Omega_\chi h^2$ depends on all $H(T_{\rm BBN} < T < T_F)$ ➡ Larger allowed region for $H(T_F)$

5. Abundance of semi-relativistic relics

- Precise evaluation of the abundance of particles that freeze out when they are semi-relativistic $(x_F \sim 3)$ is complicated

→ **Goal: simple analytic treatment that describes the transition from non-relativistic to relativistic relics**

- Assume the Maxwell-Boltzmann distribution:

$$Y_{\chi,\mathrm{eq}} \equiv \frac{n_{\chi,\mathrm{eq}}}{s} = 0.115\frac{g_\chi}{g_{*s}} x^2 K_2(x)$$

($K_n(x)$: modified Bessel function)

→ Thermal average of cross section σ :

$$\langle \sigma v \rangle = \frac{1}{8m_\chi^4 T K_2^2(m_\chi/T)} \int_{4m_\chi^2}^{\infty} \mathrm{d}s\ \sigma(s-4m_\chi^2)\sqrt{s}\ K_1(\sqrt{s}/T)$$

Ansatz for approximate cross sections

- Consider neutrinos as stable relic:
- Annihilation cross section:

$$\sigma v^{\mathrm{Dirac}\ \nu} = \frac{G_F^2 s}{16\pi}$$

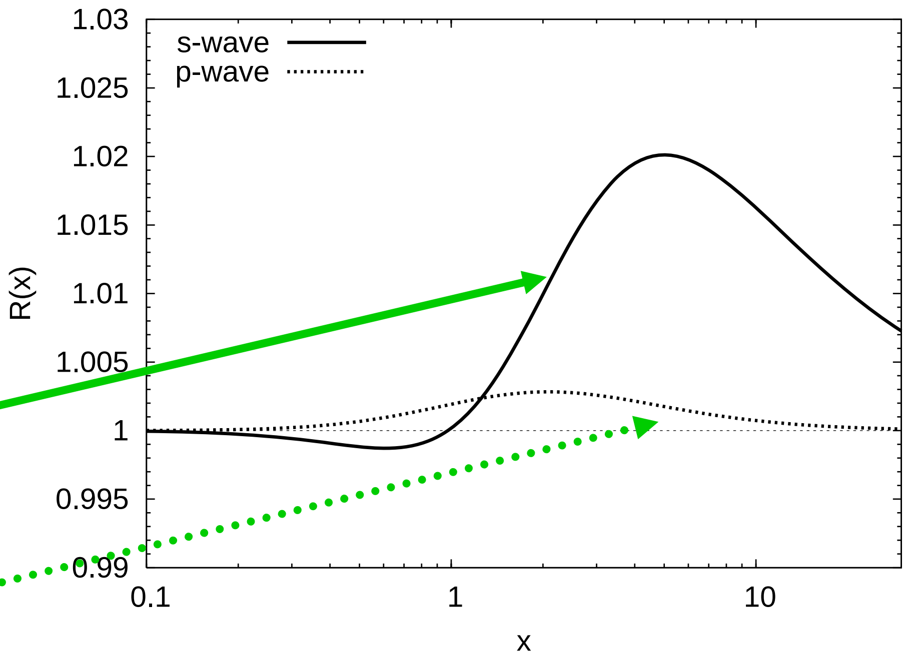
$$\sigma v^{\mathrm{Majorana}\ \nu} = \frac{G_F^2 s v^2}{16\pi}$$

- Ansatz for the thermally-averaged annihilation cross section:

$$\langle \sigma v \rangle_{\mathrm{app}}^{\mathrm{Dirac}} = \frac{G_F^2 m_\chi^2}{16\pi}\left(\frac{12}{x^2} + \frac{5+4x}{1+x}\right)$$

$$\langle \sigma v \rangle_{\mathrm{app}}^{\mathrm{Majorana}} = \frac{G_F^2 m_\chi^2}{16\pi}\left(\frac{12}{x^2} + \frac{3+6x}{(1+x)^2}\right)$$

- $\langle \sigma v \rangle_{\mathrm{app}} / \langle \sigma v \rangle_{\mathrm{exact\ MB}}$:



The approx. cross sections reproduce the exact results with accuracy of a few %

Approximate abundance of semi-relativistic relics

- Define the freeze–out temperature by

$$\Gamma(x_F) = H(x_F)$$

where

$$\Gamma(x_F) = n_{\chi,\mathrm{eq}}(x_F)\langle\sigma v\rangle(x_F)$$

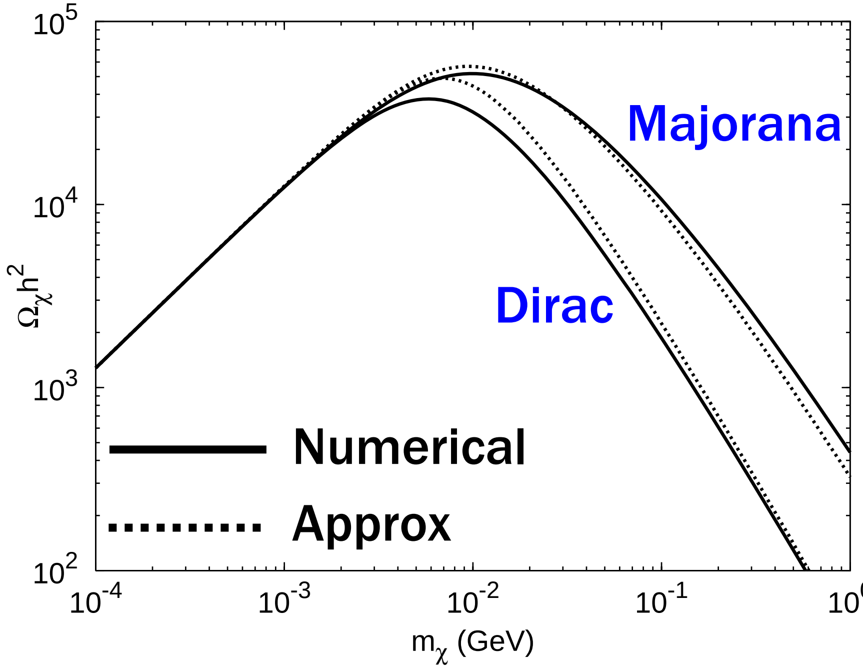
(different from
the standard definition of x_F)

- Assume the relic abundance does not change after decoupling

➡ Final abundance:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F)$$

- Comparison between the numerical and approx solutions



Applications of semi-relativistic relics

- As DM candidates

Hypothetical semi-relativistic relics should decouple before BBN

$$\Rightarrow m_\chi \sim T_F > T_{\rm BBN} \simeq {\rm MeV} \Rightarrow \Omega_\chi h^2 > 10^3$$

The relic abundance is too high!

- As source of large entropy production

Out-of-equilibrium decay of relic particles produces entropy

Ratio of the final to initial entropy: $\frac{S_f}{S_i} = g_*^{1/4} \frac{m_\chi Y_{\chi,i} \tau_\chi^{1/2}}{M_{\rm Pl}^{1/2}} \propto \Omega_\chi h^2$

$\Rightarrow$ **Semi-relativistic relics can produce significant entropy!**

Example: sterile neutrino

- Consider a sterile neutrino mixed with an active neutrino (mixing angle: θ)

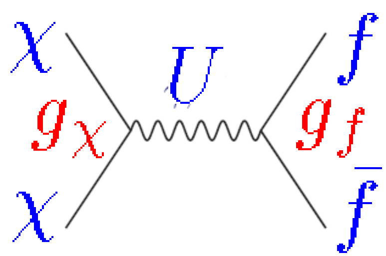
- Decay rate of the sterile neutrino:

$$\Gamma_\chi = \frac{G_F^2 m_\chi^5}{192\pi^3}\sin^2\theta$$

large enough not to spoil BBN

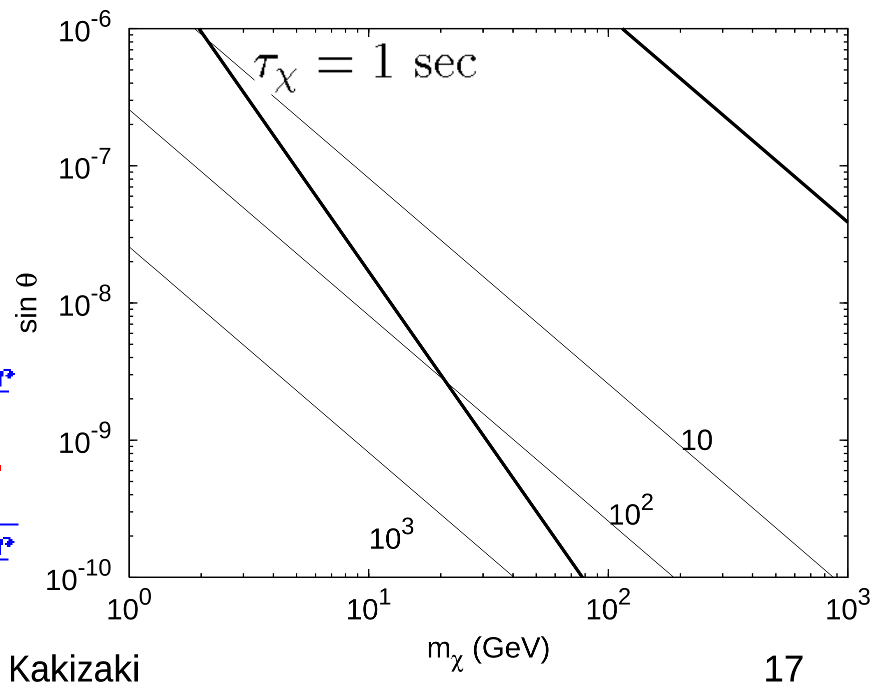
- By introducing a new particle, U, large pair annihilation can be induced:

$$\sigma v = \frac{s v^2}{12\pi}\frac{g_\chi^2 g_f^2}{M_U^4}$$



➡ $x_F \sim 3$ possible

- Entropy production S_f/S_i by the decay of semi-relativistic sterile neutrinos



5. Summary

- Using the DM relic density we can probe very early universe at around $T \sim m_\chi/20 \sim \mathcal{O}(10)\ \mathrm{GeV}$ (well before BBN $T_{\mathrm{BBN}} \sim \mathcal{O}(1)\ \mathrm{MeV}$)

- We find an approximate analytic formula for the WIMP abundance that is valid for all $T_R \leq T_F$

- $\Omega_{\chi,\mathrm{thermal}} h^2 = \Omega_{\mathrm{DM}} h^2$

  ⇨ Lower bound on the reheat temperature: $T_R > m_\chi/23$

- The sensitivity of $\Omega_{\chi,\mathrm{thermal}} h^2$ on $H(T_F)$ is weak

- We find an approximate analytic formula for the abundance of semi-relativistic relics

- Semi-relativistic relics are useful for producing a large amount of entropy

Backup slides

Hot relics

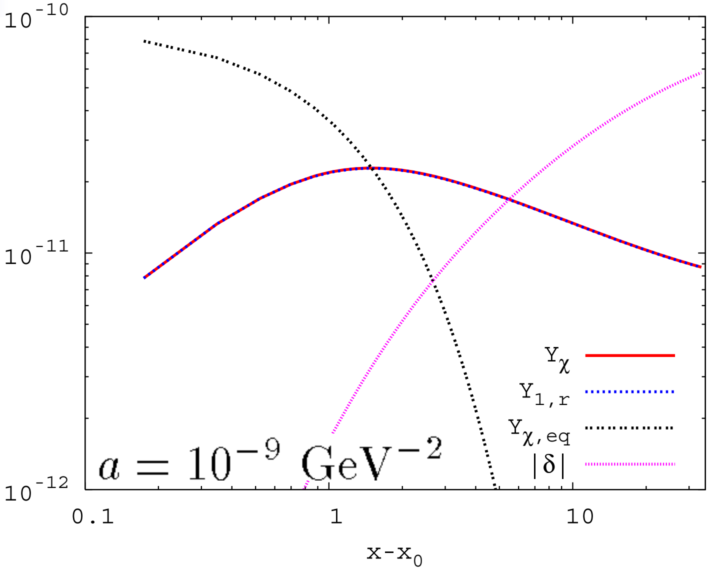
- Hot relics (decouple for $x_F < 3$):

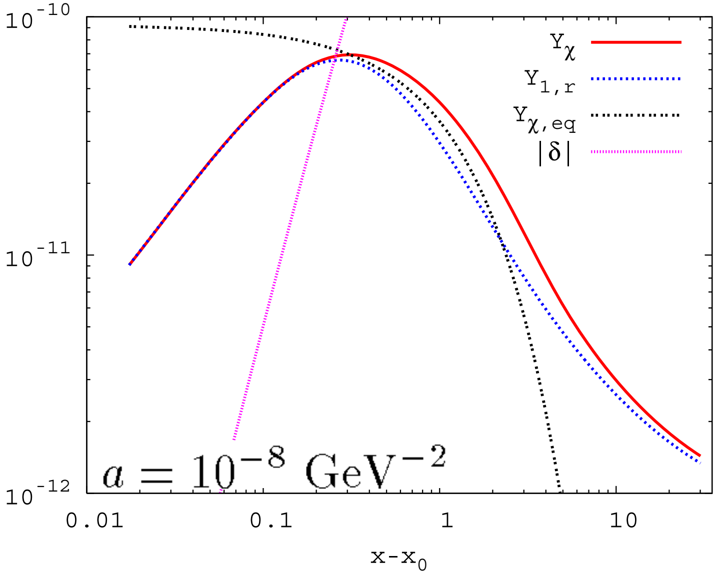
  $Y_{\chi,\mathrm{eq}}(x)$ almost constant

  ⇒ Final abundance is insensitive to the freeze out temperature:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F) = \frac{45}{2\pi^4}\frac{g_\chi}{g_{*s}(x_F)}$$

Evolution of solutions

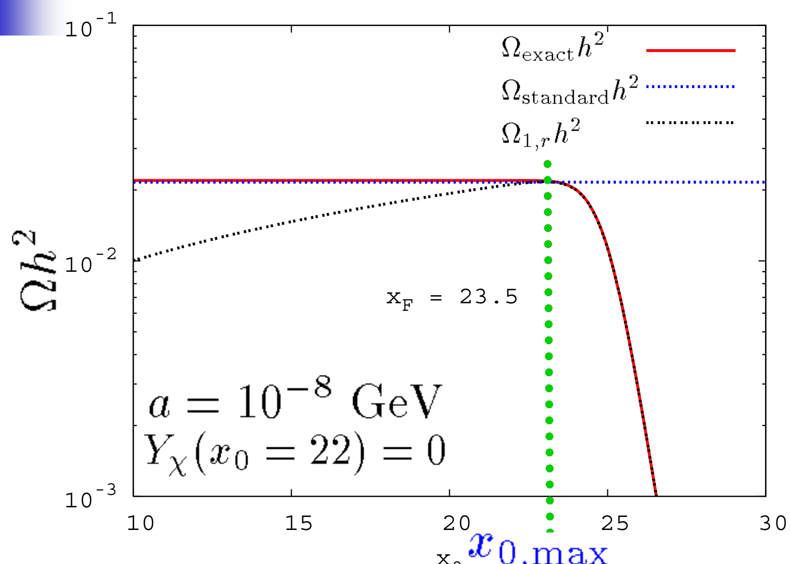


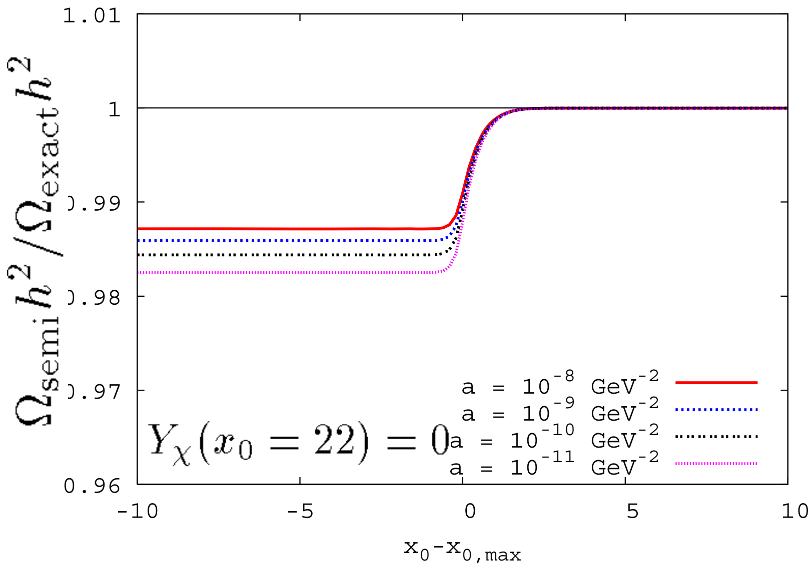


Y_χ : Exact result, $Y_{1,r}$: Re-summed ansatz, $b = 0,\ Y_\chi(x_0 = 22) = 0$

- The re-summed ansatz $Y_{1,r}$ describes the full temperature dependence of the abundance when equilibrium is not reached
- For larger cross section the deviation becomes sizable for $x - x_0 \sim 1$, but the deviation becomes smaller for $x \gg x_0$

Semi-analytic solution





- $Y_{1,r}(x_0, x \to \infty)$ $(\propto \Omega_{1,r} h^2)$ has a maximum (left)
- New semi-analytic solution can be constructed: $\Omega_{\rm semi} h^2$ (right)

For $x_0 > x_{0,\rm max}$, use $Y_{1,r}(x_0)$; for $x_0 < x_{0,\rm max}$, use $Y_{1,r}(x_{0,\rm max})$

The semi-analytic solution $\Omega_{\rm semi} h^2$ reproduces the correct final relic density $\Omega_{\rm exact} h^2$ to an accuracy of a few percent