Abundance of Cosmological Relics in Low–Temperature Scenarios

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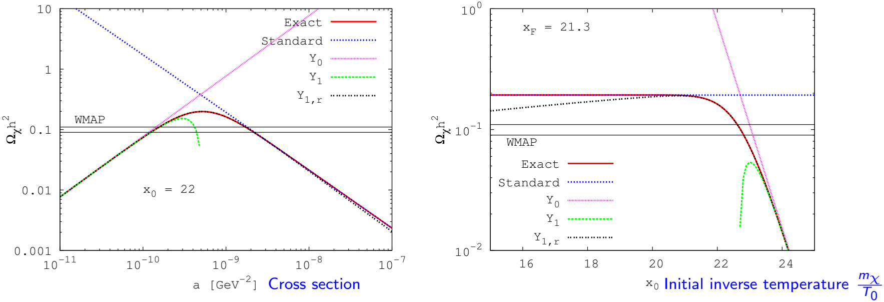
1. Motivation

- Production of massive, long–lived or stable relic particles χ plays a crucial role in particle cosmology
- E.g.: weakly interacting massive particles (WIMPs)
  - may constitute most of the dark matter in the universe
  - may produce dark matter particles by the decays
- Standard picture of thermal WIMP production:
  - WIMPs were in chemical equilibrium in the radiation–dominated (RD) universe after inflation
  - The freeze–out temperature: $T_F \simeq m_\chi/20 \simeq \mathcal{O}(10)$ GeV
  - The reheat temperature T_R is larger than T_F
- Cosmological observations establish the thermal history only for $1T \lesssim \mathcal{O}(1)$ MeV
- Scenarios with low reheat tempereture ($T_R \lesssim T_F$) lowers the χ abundance and reopens the parameter space

This work



Cross section

Initial inverse temperature $\frac{m_\chi}{T_0}$

- Existing treatments of thermal WIMP production:
  - Full chemical equilibrium (Standard): $n_\chi \propto 1/\langle\sigma v\rangle$
  - Complitely out of equilibrium (Y_0): $n_\chi \propto \langle\sigma v\rangle$

  [Scherrer and Turner (1986); Giudice, Kolb and Riotto (2001), $\cdots$]
- We provide an approximate analytic treatment that is also applicable to the in–between case ($Y_{1,r}$)

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2. Relic abundance in the standard cosmological scenario

- Let us consider a generic WIMP χ ($\chi = \bar{\chi}$, single production of χ is forbidden)
- The number density $n_\chi \Leftarrow$ the Boltzmann equation:

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

- $n_{\chi,\mathrm{eq}}$: The number density of χ in equilibrium
- $H = \dot{R}/R$: The Hubble paremeter
- $\langle \sigma v \rangle$: The thermal average of the annihilation cross section $\sigma(\chi\chi \to \mathrm{SM\ particles})$ multiplied by relative velocity v
- Kinetic equilibrium is assumed to be maintained $\Gamma(\chi f \to \chi f)/\Gamma(\chi\chi \to f\bar{f}) \sim 1/Y_\chi = s/n_\chi \gg \mathcal{O}(1)$ (f: some SM particle, s: Entopy density)

Standard cosmological scenario

[Scherrer and Turner (1986)]

- Let us introduce $Y_{\chi(,\mathrm{eq})} = n_{\chi(,\mathrm{eq})}/s$ and $x = m_\chi/T$
- For addiabatic expansion, $sR^3 = \text{const.}$
- In the RD era, $H = \pi T^2/M_{\mathrm{Pl}}\sqrt{g_*/90}$ (g_*: Rel. dof),

$$\frac{dY_\chi}{dx} = -1.3\ m_\chi M_{\mathrm{Pl}}\sqrt{g_*}\langle\sigma v\rangle x^{-2}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2)$$

- χ is assumed to be in chemical equilibrium and decoupled when nonrelativistic:
  - $n_{\chi,\mathrm{eq}} = g_\chi\ (m_\chi T/2\pi)^{3/2}\mathrm{e}^{-m_\chi/T}$
  - $\langle\sigma v\rangle = a + 6b/x + \mathcal{O}(1/x^2)$
- The relic abundance is inversely proportional to the cross section, and does not depend on T_R if $T_R > T_F$:

$$\Omega_\chi h^2 \simeq \frac{8.7\times 10^{-11}\ x_F\ \mathrm{GeV}^{-2}}{\sqrt{g_*(x_F)}(a + 3b/x_F)}, \quad x_F \simeq 22$$

3. Relic abundance in a low–temperature scenario

- T_0: The highest temperature of the RD universe
  The initial abundance is assumed to be negligible:
  $Y_\chi(x_0) = 0$
- Zeroth order approximation:
  $T_0 < T_F \Rightarrow \chi$ annihilation is negligible:

$$\frac{dY_0}{dx} = 0.028\ g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} e^{-2x} x \left(a + \frac{6b}{x}\right)$$

$\Rightarrow$ The solution is proportional to the cross section:

$$Y_0(x \gg x_0) \simeq 0.014\ g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} e^{-2x_0} x_0 \left(a + \frac{6b}{x_0}\right)$$

This solution should be smoothly connected to the standard result

First order approximation

- Add a correction term describing χ annihilation to Y_0: $Y_1 = Y_0 + \delta$ ($\delta < 0$)
- As long as $|\delta| \ll Y_0$, the evolution equation for δ is

$$\frac{d\delta}{dx} = -1.3\sqrt{g_*}m_\chi M_{\mathrm{PL}}\left(a + \frac{6b}{x}\right)\frac{Y_0(x)^2}{x^2}$$

$\Rightarrow$ The solution is proportional to σ^3:

$$\delta(x \gg x_0) \simeq -2.5 \times 10^{-4}\ g_\chi^2 g_*^{-5/2} m^3 M_{\mathrm{Pl}}^3 \times \mathrm{e}^{-4x_0} x_0 \left(a + \frac{3b}{x_0}\right)\left(a + \frac{6b}{x_0}\right)^2$$

- $|\delta|$ dominates over Y_0 for not very small cross section

$\Rightarrow$ Y_1 soon fails to track the exact solution

Resummed ansatz

- $Y_0 \propto \sigma > 0$, $\delta \propto \sigma^3 < 0$
- For large cross section, $Y_\chi(x \to \infty)$ should be $\propto 1/\langle \sigma v \rangle$

$\Rightarrow$ This observation suggests the resummed ansats:

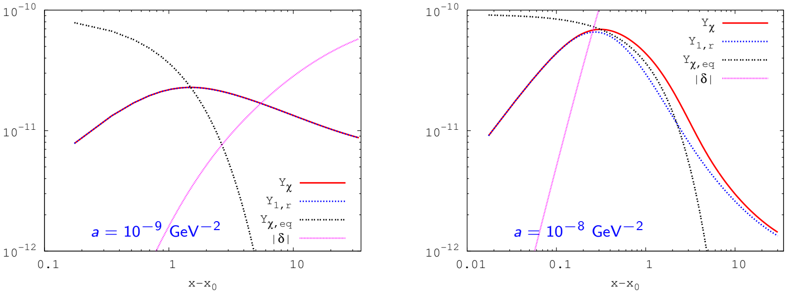
$$Y = Y_0 + \delta = Y_0 \left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$

- For $|\delta| \gg Y_0$,

$$Y_{1,r} \simeq -\frac{Y_0^2}{\delta} \simeq \frac{x_0}{1.3\,\sqrt{g_*}\, m_\chi M_{\rm Pl}(a + 3b/x_0)} \propto 1/\sigma$$

  $x_0 \to x_F \Rightarrow$ The standard formula
- When χ production is negligible but the initial abundace is sizable, $Y_{1,r}$ is exact

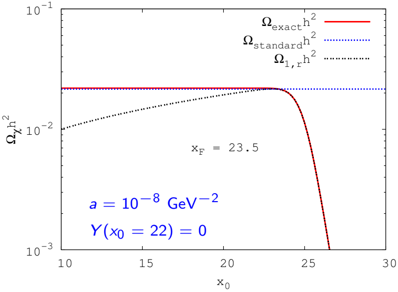
Evolution of solutions

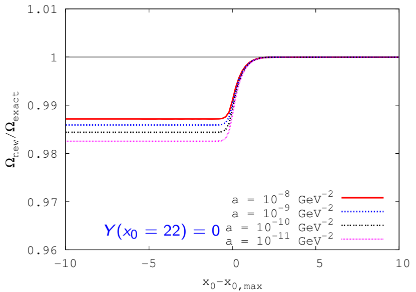


Y_χ: Exact result, $Y_{1,r}$: Resummed ansatz, $b = 0$, $Y_\chi(x_0 = 22) = 0$

- The ansatz $Y_{1,r}$ describes the full temperature dependence of the abundance when equilibrium is not reached
- For larger cross section the deviation becomes sizable for $x - x_0 \sim 1$, but the deviation becomes smaller for $x \gg x_0$

Semi–analytic solution





- $Y_{1,r}(x_0, x \to \infty)$ ($\propto \Omega_{1,r}h^2$) has a maximum (left)
- A new semi–analytic solution can be constructed (right):
  - For $x_0 > x_{0,\mathrm{max}}$, use $Y_{1,r}(x_0)$;
    For $x_0 < x_{0,\mathrm{max}}$, use $Y_{1,r}(x_{0,\mathrm{max}})$

The semi–analytic solution Ω_{new} reproduces the correct final relic density Ω_{exact} to an accuracy of a few percent

4. Relic abundance including the decay of heavier particles

- Consider production of long–lived or stable particles χ from out–of–equilibrium decay of unstable particles ϕ
- Assumption: ϕ does not dominate the energy density, the comoving entropy remains constant:

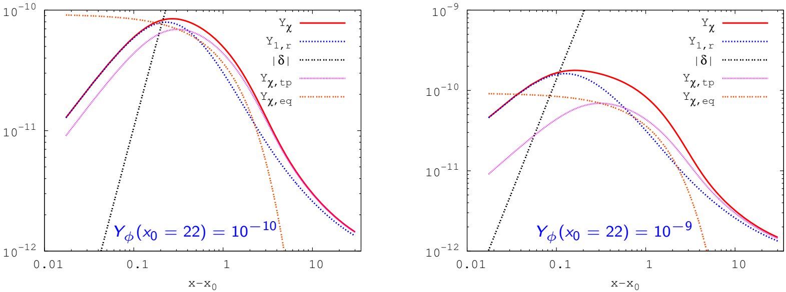
$$\begin{aligned} \dot{n}_\chi + 3Hn_\chi &= -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2) + N\Gamma_\phi n_\phi \\ \dot{n}_\phi + 3Hn_\phi &= -\Gamma_\phi n_\phi \end{aligned}$$

$$\Rightarrow \frac{dY_\chi}{dx} = -\frac{\langle \sigma v \rangle s}{Hx}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2) + NrxY_\phi(x_0)\exp\left(-\frac{r}{2}(x^2 - x_0^2)\right)$$

$r = \Gamma_\phi / Hx^2 = (\Gamma_\phi M_{\mathrm{Pl}}/\pi m_\chi^2)$ is constant

- Following the same procedure we can obtain Y_0, δ and $Y_{1,r}$

Evolution of solutions



Y_χ: Exact result, $Y_{1,r}$: Resummed ansatz, $a = 10^{-8}$ GeV^{-2}, $b = 0$, $Y_\chi(x_0 = 22) = 0$, $r = 0.1$, $N = 1$

- Most difficult situation: thermal and nonthermal production occur simultaneously ($rx_0^2 \sim x_0$) and contribute effectively ($Y_\chi(x \sim x_0) \sim Y_\phi(x \sim x_0)$)
- The resummed ansatz describes scenarios with nonthermal χ prodcution as well as the thermal case

5. Summary

- We investigated the relic abundance of nonrelativistic long–lived or stable particles χ in low–temperature scenarios
- The case with a heavier particle decaying into χ is also investigated
- Our approximate solutions for the number density accurately reproduce exact results when full thermal equilibrium is not achieved
- Even if full equilibrium is reached, our semi–analytic solution reproduces the correct final relic density to an accuracy of a few percent