Abundance of Cosmological Relics in Low–Temperature Scenarios

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June 16, 2006

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1. Motivation

- Production of massive, long-lived or stable relic particles $\chi$ plays a crucial role in particle cosmology
  - E.g.: weakly interacting massive particles (WIMPs)
    - may constitute most of the dark matter in the universe
    - may produce dark matter particles by the decays
  - Standard picture of thermal WIMP production:
    - WIMPs were in chemical equilibrium in the radiation-dominated (RD) universe after inflation
    - The freeze-out temperature: $T_F \simeq m_\chi/20 \simeq \mathcal{O}(10)$ GeV
    - The reheat temperature $T_R$ is larger than $T_F$
  - Cosmological observations establish the thermal history only for $1T \lesssim \mathcal{O}(1)$ MeV
  - Scenarios with low reheat temperature ($T_R \lesssim T_F$) lowers the $\chi$ abundance and reopens the parameter space
This work

- **Existing treatments of thermal WIMP production:**
  - Full chemical equilibrium (**Standard**): \( n_\chi \propto 1/\langle \sigma v \rangle \)
  - Completely out of equilibrium (**\( Y_0 \)**): \( n_\chi \propto \langle \sigma v \rangle \)

  [Scherrer and Turner (1986); Giudice, Kolb and Riotto (2001), · · ·]

- **We provide an approximate analytic treatment that is also applicable to the in–between case (**\( Y_{1,r} \)**)**
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2. Relic abundance in the standard cosmological scenario

- Let us consider a generic WIMP $\chi$ ($\chi = \bar{\chi}$, single production of $\chi$ is forbidden)
- The number density $n_\chi \Leftarrow$ the Boltzmann equation:

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle (n^2_\chi - n^2_{\chi, eq})$$

- $n_{\chi, eq}$: The number density of $\chi$ in equilibrium
- $H = \dot{R}/R$: The Hubble parameter
- $\langle \sigma v \rangle$: The thermal average of the annihilation cross section $\sigma(\chi\chi \rightarrow \text{SM particles})$ multiplied by relative velocity $v$
- Kinetic equilibrium is assumed to be maintained

$$\Gamma(\chi f \rightarrow \chi f)/\Gamma(\chi\chi \rightarrow f\bar{f}) \sim 1/Y_\chi = s/n_\chi \gg O(1)$$
(f: some SM particle, s: Entopy density)
Standard cosmological scenario

[Scherrer and Turner (1986)]

- Let us introduce $Y_{\chi,\text{eq}} = n_{\chi,\text{eq}}/s$ and $x = m_{\chi}/T$
- For adiabatic expansion, $sR^3 = \text{const.}$
- In the RD era, $H = \pi T^2/M_{Pl} \sqrt{g_*/90}$ ($g_*$: Rel. dof),
  \[
  \frac{dY_{\chi}}{dx} = -1.3 \frac{m_{\chi} M_{Pl} \sqrt{g_*} \langle \sigma v \rangle x^{-2}(Y_{\chi}^2 - Y_{\chi,\text{eq}}^2)}{sR^3}
  \]
- $\chi$ is assumed to be in chemical equilibrium and decoupled when nonrelativistic:
  - $n_{\chi,\text{eq}} = g_{\chi} (m_{\chi} T/2\pi)^{3/2} e^{-m_{\chi}/T}$
  - $\langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2)$
- The relic abundance is inversely proportional to the cross section, and does not depend on $T_R$ if $T_R > T_F$:
  \[
  \Omega_{\chi} h^2 \sim \frac{8.7 \times 10^{-11} x_F \text{ GeV}^{-2}}{\sqrt{g_*(x_F)(a + 3b/x_F)}}, \quad x_F \sim 22
  \]
3. Relic abundance in a low–temperature scenario

- \( T_0 \): The highest temperature of the RD universe
The initial abundance is assumed to be negligible:
\( Y_\chi(x_0) = 0 \)

- Zeroth order approximation:
\( T_0 < T_F \Rightarrow \chi \) annihilation is negligible:
\[
\frac{dY_0}{dx} = 0.028 \ g_\chi^2 g_*^{-3/2} m_\chi M_{\text{Pl}} e^{-2x} x \left( a + \frac{6b}{x} \right)
\]

\( \Rightarrow \) The solution is proportional to the cross section:
\[
Y_0(x \gg x_0) \simeq 0.014 \ g_\chi^2 g_*^{-3/2} m_\chi M_{\text{Pl}} e^{-2x_0} x_0 \left( a + \frac{6b}{x_0} \right)
\]

This solution should be smoothly connected to the standard result
First order approximation

- Add a correction term describing $\chi$ annihilation to $Y_0$: $Y_1 = Y_0 + \delta$ ($\delta < 0$)

- As long as $|\delta| \ll Y_0$, the evolution equation for $\delta$ is

$$\frac{d\delta}{dx} = -1.3 \sqrt{g_* m_\chi} M_{\text{PL}} \left( a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}$$

⇒ The solution is proportional to $\sigma^3$:

$$\delta(x \gg x_0) \simeq -2.5 \times 10^{-4} g_\chi^2 g_*^{5/2} m^3 M_{\text{Pl}}^3$$

$$\times e^{-4x_0} x_0 \left( a + \frac{3b}{x_0} \right) \left( a + \frac{6b}{x_0} \right)^2$$

- $|\delta|$ dominates over $Y_0$ for not very small cross section

⇒ $Y_1$ soon fails to track the exact solution
Resummed ansatz

- $Y_0 \propto \sigma > 0, \delta \propto \sigma^3 < 0$
- For large cross section, $Y_\chi(x \to \infty)$ should be $\propto 1/\langle \sigma v \rangle$

⇒ This observation suggests the resummed ansatz:

$$Y = Y_0 + \delta = Y_0 \left(1 + \frac{\delta}{Y_0}\right) \approx \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$

- For $|\delta| \gg Y_0$,

$$Y_{1,r} \approx -\frac{Y_0^2}{\delta} \approx \frac{x_0}{1.3 \sqrt{g^* m_\chi M_{Pl}} (a + 3b/x_0)} \propto \frac{1}{\sigma}$$

$x_0 \to x_F \Rightarrow$ The standard formula

- When $\chi$ production is negligible but the initial abundance is sizable, $Y_{1,r}$ is exact
Evolution of solutions

\[ Y_\chi: \text{Exact result, } Y_{1,r}: \text{Resummed ansatz, } b=0, \ Y_\chi(x_0=22)=0 \]

- The ansatz \( Y_{1,r} \) describes the full temperature dependence of the abundance when equilibrium is not reached.

- For larger cross section the deviation becomes sizable for \( x-x_0 \sim 1 \), but the deviation becomes smaller for \( x \gg x_0 \).
Semi–analytic solution

- $Y_1, r(x_0, x \to \infty) \propto \Omega_{1, r} h^2$ has a maximum (left).

- A new semi–analytic solution can be constructed (right):
  - For $x_0 > x_{0, \text{max}}$, use $Y_1, r(x_0)$;
  - For $x_0 < x_{0, \text{max}}$, use $Y_1, r(x_{0, \text{max}})$

The semi–analytic solution $\Omega_{\text{new}}$ reproduces the correct final relic density $\Omega_{\text{exact}}$ to an accuracy of a few percent.
4. Relic abundance including the decay of heavier particles

- Consider production of long–lived or stable particles $\chi$ from out–of–equilibrium decay of unstable particles $\phi$.
- Assumption: $\phi$ does not dominate the energy density, the comoving entropy remains constant:

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,eq}^2) + N\Gamma_\phi n_\phi$$
$$\dot{n}_\phi + 3Hn_\phi = -\Gamma_\phi n_\phi$$

$$\Rightarrow \frac{dY_\chi}{dx} = -\frac{\langle \sigma v \rangle s}{Hx} (Y_{\chi}^2 - Y_{\chi,eq}^2) + Nr_x Y_\phi(x_0) \exp \left( -\frac{r}{2}(x^2 - x_0^2) \right)$$

$$r = \frac{\Gamma_\phi}{Hx^2} = \left( \frac{\Gamma_\phi M_{Pl}}{\pi m_\chi^2} \right)$$
is constant

- Following the same procedure we can obtain $Y_0$, $\delta$ and $Y_{1,r}$.
Motivation
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Summary

Evolution of solutions

$Y_\chi$: Exact result, $Y_{1,r}$: Resummed ansatz, $a = 10^{-8} \text{ GeV}^{-2}$, $b = 0$, $Y_\chi(x_0 = 22) = 0$, $r = 0.1$, $N = 1$

- Most difficult situation: thermal and nonthermal production occur simultaneously ($r x_0^2 \sim x_0$) and contribute effectively ($Y_\chi(x \sim x_0) \sim Y_\phi(x \sim x_0)$)
- The resummed ansatz describes scenarios with nonthermal $\chi$ production as well as the thermal case
5. Summary

- We investigated the relic abundance of nonrelativistic long–lived or stable particles $\chi$ in low–temperature scenarios.
- The case with a heavier particle decaying into $\chi$ is also investigated.
- Our approximate solutions for the number density accurately reproduce exact results when full thermal equilibrium is not achieved.
- Even if full equilibrium is reached, our semi–analytic solution reproduces the correct final relic density to an accuracy of a few percent.