Abundance of Cosmological Relics in Low–Temperature Scenarios

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Abstract. We investigate the relic density n_χ of non–relativistic long–lived or stable particles χ in cosmological scenarios in which the temperature T is too low for χ to achieve full chemical equilibrium. The case with a heavier particle decaying into χ is also investigated. We derive approximate solutions for $n_\chi(T)$ which accurately reproduce numerical results when full thermal equilibrium is not achieved. If full equilibrium is reached, our ansatz no longer reproduces the correct temperature dependence of the χ number density. However, it does give the correct final relic density, to an accuracy of about 3% or better, for all cross sections and initial temperatures. This talk is based on our work in Ref. [1].

Introduction. The production of massive, long–lived or stable relic particles χ plays a crucial role in particle cosmology [2]. The perhaps most important example is the production of Massive Weakly–Interacting Particles (WIMPs), which may constitute most of the Dark Matter in the universe. Alternatively, WIMPs may only be meta–stable, and decay into even more weakly interacting particles (e.g. gravitinos or axinos) that form the Dark Matter. It is usually assumed that the WIMPs were in full thermal and chemical equilibrium in the radiation–dominated epoch after the end of inflation. In this "standard" case accurate semi–analytical expressions for $n_\chi(T \ll T_F)$ have been derived [3, 4]. For typical WIMP scenarios, the freeze-out temperature is $T_F \simeq m_\chi/20$. The standard treatment can work only if the reheat temperature T_R, is larger than T_F. On the other hand, we have direct observational evidence only for temperatures $T \lesssim$ MeV, which is well below T_F for most current WIMP candidates. It is therefore legitimate to investigate scenarios with $T_R \lesssim T_F$. Existing treatments of thermal WIMP production assume that n_χ had either achieved full equilibrium, or was completely out of equilibrium. Here we provide an approximate analytic treatment that also works in the intermediate region, where both thermal production and annihilation of χ particles were important.

Standard cosmological scenario. We briefly review the calculation of the relic density of long–lived or stable particles χ in the standard cosmological scenario [3], which assumes that the relic particles were in thermal equilibrium in the early universe and decoupled when they were non–relativistic. The relic density can be calculated by solving the Boltzmann equation which describes the time evolution of the number density n_χ in the expanding universe [2],

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle(n_\chi^2 - n_{\chi,\mathrm{eq}}^2) , \tag{1}$$

with $n_{\chi,\mathrm{eq}}$ being the equilibrium number density of the relic particles, H the Hubble parameter and $\langle\sigma v\rangle$ the thermal average of the annihilation cross section σ multiplied with the relative velocity v of the two annihilating χ particles. In most cases the cross section is well approximated by a nonrelativistic expansion: $\langle\sigma v\rangle = a + 6b/x$. The Boltzmann equation (1) can be rewritten by introducing the new variables $x = m/T$, $Y_\chi = n_\chi/s$ and $Y_{\chi,\mathrm{eq}} = n_{\chi,\mathrm{eq}}/s$, where the entropy density $s = (2\pi^2/45)g_* T^3$ with g_* being the number of the relativistic degrees of freedom:

$$\frac{dY_\chi}{dx} = -1.32\, m_\chi M_{\mathrm{Pl}}\sqrt{g_*}\langle\sigma v\rangle x^{-2}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2)\,. \tag{2}$$

It is useful to express the energy density as $\Omega_\chi = \rho_\chi/\rho_c$, where $\rho_c = 3H_0^2 M_{\mathrm{Pl}}^2$ is the critical density of the universe. The present energy density of the relic particle is given by $\rho_\chi = m_\chi n_{\chi,\infty} = m_\chi s_0 Y_{\chi,\infty}$, with $s_0 \simeq 2900\ \mathrm{cm}^{-3}$ being the present entropy density. It is known that in this standard scenario the following approximate formula can give the correct relic density:

$$\Omega_\chi h^2 \simeq \frac{8.7\times 10^{-11}\, x_F\ \mathrm{GeV}^{-2}}{\sqrt{g_*(x_F)}(a + 3b/x_F)}\,, \tag{3}$$

where h is the scaled Hubble constant, $h \simeq 0.7$, and $x_F = m/T_F$.

Relic abundance in a low–temperature scenario. In the following we attempt to find a convenient analytic formula applicable even to low temperature scenarios. As zeroth order solution of Eq.(2) we consider the case where χ annihilation is negligible,

$$\frac{dY_0}{dx} = 0.028\, g_\chi^2 g_*^{-3/2} m_\chi M_{\mathrm{Pl}} \mathrm{e}^{-2x} x\left(a + \frac{6b}{x}\right). \tag{4}$$

For $Y_\chi(x_0) = 0$, this equation gives

$$Y_0(x \gg x_0) \simeq 0.014\, g_\chi^2 g_*^{-3/2} m_\chi M_{\mathrm{Pl}} \mathrm{e}^{-2x_0} x_0\left(a + \frac{6b}{x_0}\right). \tag{5}$$

The most natural extension is to add a correction term which describes the effect of annihilation on the solution for the pure production case: $Y_1 = Y_0 + \delta$. As long as $|\delta|$ is small compared to Y_0, the evolution equation for δ is given by

$$\frac{d\delta}{dx} = -1.3\, \sqrt{g_*} m_\chi M_{\mathrm{PL}}\left(a + \frac{6b}{x}\right)\frac{Y_0(x)^2}{x^2}\,. \tag{6}$$

At late times, $x \to \infty$, the solution simplifies to

$$\delta(x \gg x_0) \simeq -2.5\times 10^{-4}\, g_\chi^2 g_*^{-5/2} m^3 M_{\mathrm{Pl}}^3 \mathrm{e}^{-4x_0} x_0\left(a + \frac{3b}{x_0}\right)\left(a + \frac{6b}{x_0}\right)^2. \tag{7}$$

Since, for vanishing initial abundance, Y_0 is proportional to σ, δ is proportional to σ^3. On the other hand, for sufficiently large cross section we want to recover the standard expression. This suggests to rewrite our ansatz as

$$Y = Y_0 + \delta = Y_0\left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}\,. \tag{8}$$

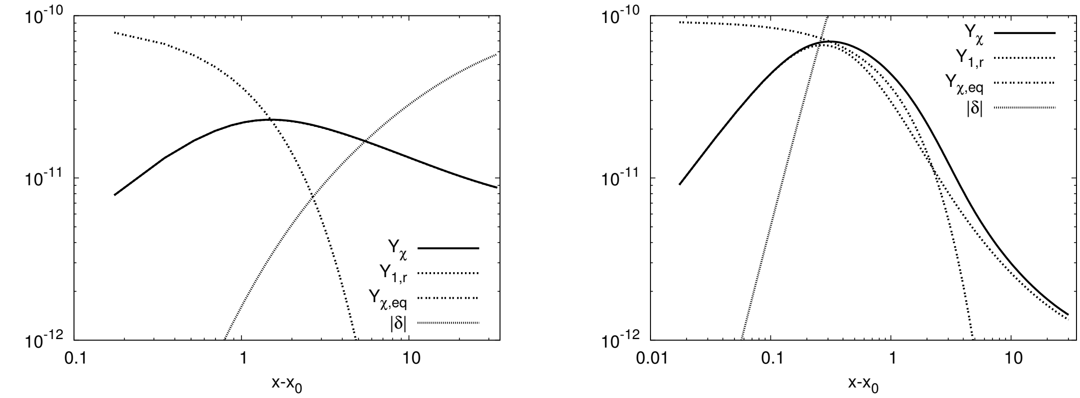

FIGURE 1. Evolution of the exact solution Y_χ (solid curves), $Y_{1,r}$ of Eq.(8) (dotted), the equilibrium density $Y_{\chi,\mathrm{eq}}$ (double–dotted), and $|\delta|$ (short–dashed) as function of $x - x_0$ for $m_\chi = 100$ GeV, $g_\chi = 2$, $g_* = 90$ and $Y_\chi(x_0 = 22) = 0$. We take $a = 10^{-9}$ GeV^{-2}, $b = 0$ (left frame) and $a = 10^{-8}$ GeV^{-2}, $b = 0$ (right). In the left frame the curve for $Y_{1,r}$ practically coincides with the solid line.

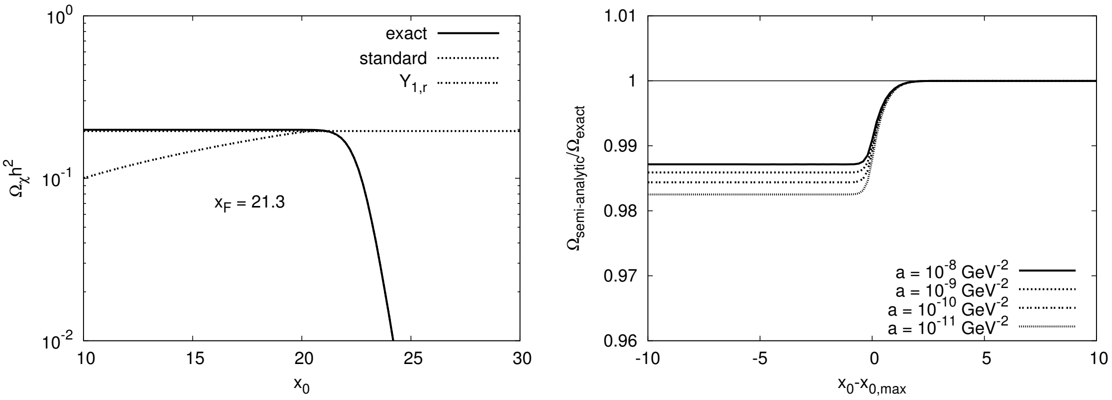

FIGURE 2. (left frame) Present relic density evaluated numerically (solid curve), the old standard approximation (dotted) and our new approximation (double–dotted) as function of x_0 for $a = 10^{-9}$ GeV^{-2} and $b = 0$. (right) Ratio of the semi–analytic result $\Omega_{\mathrm{semi-analytic}}$ to the exact value Ω_{exact} as function of $x_0 - x_{0,\mathrm{max}}$ for $b = 0$. The other parameter are the same as in Fig. 1.

Although the final approximate equality in Eq.(8) only holds for $|\delta| \ll Y_0$, we note that the resulting expression has the right behavior, $Y_{1,r} \propto 1/\sigma$, for large cross section. It is also noted that this ansatz solves the Boltzmann equation (3) exactly in the simple case where thermal χ production can be ignored, but $Y_\chi(x_0)$ is sizable.

In Fig. 1 we present the evolution of the solutions as function of $x - x_0$. Clearly the first order approximation Y_1 fails to reproduce the exact result once $|\delta|$ becomes comparable to Y_0. On the contrary, it is shown that the re–summed ansatz $Y_{1,r}$ of Eq.(8) reproduces the numerical solution very well for all $x > x_0$ if $a \lesssim 10^{-9}$ GeV^{-2}. However, for intermediate values of $x - x_0$, the disagreement between $Y_{1,r}$ and the exact solution becomes large as the cross section increases. Sizable deviations from the exact value are observed at $x - x_0 \sim 1$ for $a = 10^{-8}$ GeV^{-2}. For larger x the deviation becomes smaller again, and for $x \gg x_0$ the difference is insignificant even for these large cross sections.

In the left frame of Fig. 2 we plot the present relic density evaluated numerically (solid curve), the old standard approximation (dotted) and our new approximation (double–dotted) as function of x_0. We find that our approximation agrees with the exact result very well for $x_0 > x_F$. On the other hand, for $x_0 < x_F$, our approximation gives too small an abundance while the old approximation works very well. This figure shows that $Y_{1,r}(x_0, x \to \infty)$ has a well defined maximum when x_0 is varied. This maximum occurs at a value $x_{0,\max}$ which is close to the decoupling temperature x_F.

Since the actual relic density is already practically independent of x_0 for $x_0 < x_{0,\max}$ we can construct a new semi-analytic solution which describes the relic density for the whole range of x_0: for $x_0 > x_{0,\max}$, compute the relic density from $Y_{1,r}(x_0)$, but for $x_0 < x_{0,\max}$, use $Y_{1,r}(x_{0,\max})$ instead. The ratio of this semi–analytic result $\Omega_{\mathrm{semi-analytic}}$ to the exact value Ω_{exact} is depicted in the right frame of Fig. 2. As noted earlier, our approximation becomes exact for $x_0 \gtrsim x_F$. For smaller x_0 the new approximation still slightly under–estimates the correct answer, but the deviation is at most 1.7% for $b = 0$, and 3.0% for $a = 0$.

Relic abundance including the decay of heavier particles. Here we consider a scenario where unstable heavy particles ϕ decay into long–lived or stable particles χ. We assume that ϕ decays out of thermal equilibrium, so that ϕ production is negligible; however, we include both thermal and non–thermal production of χ particles. We assume that ϕ does not dominate the total energy density, so that the co–moving entropy density remains approximately constant throughout. Following the same procedure developed in the previous section, we can obtain Y_0, δ and $Y_{1,r}$ in this scenario. We find that the resummed ansatz describes scenarios with nonthermal χ production from ϕ decay as well as the thermal case.

Summary. In summary, we found analytical or semi–analytical solutions of the Boltzmann equation describing the density of non–relativistic relics which are valid for a wide range of initial conditions. In particular, they allow a complete description of the temperature dependence for small or moderate cross sections, and correctly reproduce the final relic density for all combinations of initial temperature and cross section. This should be a powerful tool for exploring the physics of non–relativistic relics, especially in scenarios with low reheat temperature.

ACKNOWLEDGMENTS

I thank my collaborators, Manuel Drees and Hoernisa Iminniyaz. This work is supported in part by the Japan Society for the Promotion of Science.

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