Making Dark Matter

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Contents

Contents

Contents

Contents

Contents

Contents

Contents

Contents

Contents

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- Models of structure formation, X ray temperature of clusters of galaxies, . . .
- Cosmic Microwave Background anisotropies (WMAP etc.) imply $\Omega_{\rm DM} h^2 = 0.112 \pm 0.006$ **PDG, 2012 edition**

Need for non–baryonic DM

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$\Longrightarrow$ Need non–baryonic DM!

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Possible loophole: primordial black holes; not easy to make in sufficient quantity sufficiently early.

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- Evidence for $\Omega_{\rm DM} \gtrsim 0.2$ much more robust than that! (Does, however, assume standard law of gravitation.)

Particle DM: Possibilities

Theorist's tasks:

- Introduce right kind of particle (stable, neutral, non–relativistic)

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$\Longrightarrow$ Use theoretical "prejudice" as guideline: Only consider candidates that solve (at least) one additional problem!

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Here: focus on WIMP (e.g. Neutralino) and Gravitino.

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  - For WIMPs: Order of magnitude of $\Omega_{\rm DM}$ is understood; $\Omega_{\rm baryon}$ isn't

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- Early Universe was dominated by radiation! (Except in some extreme 'quintessence' or 'brane cosmology' models.)

Thermal DM production

Let χ be a generic DM particle, n_χ its number density (unit: GeV^3). Assume $\chi = \bar{\chi}$, i.e. $\chi\chi \leftrightarrow$SM particles is possible, but single production of χ is forbidden by some symmetry.

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Evolution of n_χ determined by Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\rm ann} v \rangle \left(n_\chi^2 - n_{\chi,\,{\rm eq}}^2 \right)$$

$H = \dot{R}/R$: Hubble parameter
$\langle \ldots \rangle$: Thermal averaging
$\sigma_{\rm ann} = \sigma(\chi\chi \rightarrow {\rm SM\ particles})$
v : relative velocity between χ's in their cms
$n_{\chi,\,{\rm eq}}$: χ density in full equilibrium

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2^nd lhs term: Describes χ dilution by expansion of Universe:

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Check: creation and annihilation balance iff $n_\chi = n_{\chi,\,\rm eq}$.

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If interactions are negligible: $Y_\chi \rightarrow$ const., i.e. χ density in *co–moving* volume is unchanged

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$$\Longrightarrow \dot{x} = -\frac{m_\chi}{T^2}\dot{T} = -\frac{x}{T}\dot{T} = xH$$

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For $T \gtrsim 200$ MeV: $10 \lesssim \frac{4\pi\sqrt{g_*}}{\sqrt{90}} \lesssim 20$ (SM, MSSM)

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(See: freeze–in).

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- For non–renormalizable interactions: easiest to satisfy at maximal temperature, $T \simeq T_R$. (See: $\tilde{G}$)

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$$\langle \sigma_{\rm ann} v \rangle (T \simeq m_\chi) > \frac{1}{m_\chi M_{\rm P}}$$

  (See: freeze–in).

- For non–renormalizable interactions: easiest to satisfy at maximal temperature, $T \simeq T_R$. (See: $\tilde{G}$)

- For $T_R < m_\chi$: Easiest to satisfy for $T \simeq T_R$ (see: WIMP at low T_R).

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Typically, $a, b \lesssim \frac{\alpha^2}{m_\chi^2}, \;\; \alpha^2 \sim 10^{-3}$, unless a is suppressed by some symmetry; e.g. for $\tilde{\chi}\tilde{\chi} \to f\bar{f}: \; a \propto m_f^2$.

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Let T_R be the highest temperature of the radiation–dominated universe (after inflation).

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Example: $\hat{a} = 0,\ \hat{b} = 10^{-4} \Longrightarrow$ need $T_R \simeq 0.04 m_\chi$

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For $T < T_F$: WIMP production negligible, only annihilation relevant in Boltzmann equation.

Thermal WIMP: solution of Boltzmann eq.

Had $\frac{dY_\chi}{dx} = -\frac{4\pi\sqrt{g_*}}{\sqrt{90}}\frac{m_\chi M_P}{x^2}\langle\sigma_{\rm ann}v\rangle\left(Y_\chi^2 - Y_{\chi,\,{\rm eq}}^2\right)$

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To keep $\frac{d\Delta}{dx} = 0$: need

$$\Delta \simeq \frac{x^2}{2.64 m_\chi M_P \sqrt{g_*} \langle \sigma_{\rm ann} v \rangle}$$

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- Smooth transition to previous case ($T_R < T_F$): MD, Iminniyaz, Kakizaki, hep-ph/0603165

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Previous treatment still applies, with replacement:

$\sigma_{\rm ann} \rightarrow \sigma_{\rm eff} \sim \sigma_{\rm ann} + f_B \sigma(\tilde\chi\tilde\chi' \rightarrow {\rm SM}) + f_B^2 \sigma(\tilde\chi'\tilde\chi' \rightarrow {\rm SM})$

f_B : relative Boltzmann factor $= \left(1 + \frac{\delta m}{m_{\tilde\chi}}\right)^{3/2} e^{-\delta m/T}$

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Is important for SUSY scenarios with small mass splitting between LSP and NLSP: $\delta m \equiv m_{\tilde{\chi}'} - m_{\tilde{\chi}} \ll m_{\tilde{\chi}}$

Rate ($\tilde{\chi} + f \leftrightarrow \tilde{\chi}' + f'$) $\gg$ Rate($\tilde{\chi}\tilde{\chi} \leftrightarrow f f$), by factor $\propto e^{(2m_{\tilde{\chi}} - m_{\tilde{\chi}'})/T}$ $(f,\ f'$: SM particles)

$\tilde{\chi},\ \tilde{\chi}'$ retain *relative* equilibrium well after sparticles decouple from SM particles: $n_{\tilde{\chi}'} = n_{\tilde{\chi}} e^{-\delta m/T}$

Previous treatment still applies, with replacement:

$$\sigma_{\rm ann} \rightarrow \sigma_{\rm eff} \sim \sigma_{\rm ann} + f_B \sigma(\tilde{\chi}\tilde{\chi}' \rightarrow {\rm SM}) + f_B^2 \sigma(\tilde{\chi}'\tilde{\chi}' \rightarrow {\rm SM})$$

f_B : relative Boltzmann factor $= \left(1 + \frac{\delta m}{m_{\tilde{\chi}}}\right)^{3/2} e^{-\delta m/T}$

$\sigma(\tilde{\chi}\tilde{\chi}'),\ \sigma(\tilde{\chi}'\tilde{\chi}') \gg \sigma(\tilde{\chi}\tilde{\chi})$ possible!

Case 3: Freeze–in

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- Final relic density proportional to cross section, independent of FIMP mass

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Thermal Gravitino Dark Matter

Each gravitino coupling gives factor $\frac{m_{\text{sparticle}} s}{m_{\tilde{G}} M_P}$ in cross section, if $m_{\tilde{G}} \ll \sqrt{s}$, $m_{\text{sparticle}}$

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$\tilde G$ annihilation can be ignored; write Boltzmann eq. for $\tilde Y_{\tilde G} \equiv n_{\tilde G}/n_\gamma$:

$$\frac{d\tilde Y_{\tilde G}}{dT} = -\frac{n_\gamma \sigma_{\tilde G}}{4 T H(T)}$$

Gravitino DM (cont.'d)

Solution of Boltzmann eq.:

$\tilde{Y}_{\tilde{G},0} = \frac{4 n_\gamma(T_R)\sigma_{\tilde{G}}}{4H(T_R)} \propto T_R$ (assuming $\tilde{Y}_{\tilde{G}}(T_R) = 0$)

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In general, have to add $\Omega_{\rm NLSP} \frac{m_{\tilde{G}}}{m_{\rm NLSP}}$ from (late) decays of NLSPs. (BBN!)

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Inflatons are non–relativistic when they decay.

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If χ production and annihilation at $T < T_R$ is negligible, universe evolves adiabatically:

$$\Longrightarrow \Omega_\chi h^2 = 2.1 \cdot 10^8 \frac{m_\chi}{m_\phi} \frac{T_R}{1 \text{ GeV}} B(\phi \to \chi)$$

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  $$B(\phi \to \chi) \sim \frac{\alpha_\chi^2}{96\pi^3}\left(1 - \frac{4m_\chi^2}{m_\phi^2}\right)^2 \left(1 - \frac{2m_\chi}{m_\phi}\right)^{5/2}$$

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  (Assumes $\sigma(\chi\chi \leftrightarrow f\bar{f}) \sim \frac{\alpha_\chi^2}{m_\chi^2}$, $\phi \to f\bar{f}$ dominates.)
- Can be most important production mechanism for superheavy Dark Matter ($m_\chi \sim 10^{12}$ GeV) in chaotic inflation ($m_\phi \sim 10^{13}$ GeV); for LSP if $T_R \lesssim 0.03 m_\chi$; ...

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- Production from inflaton decay: $\Omega_\chi h^2 \propto \frac{m_\chi T_R B(\phi \rightarrow \chi)}{m_\phi}$.
- Only the thermal WIMP scenario can be tested using collider data and results from WIMP search experiments. Other scenarios can only be tested with additional input to constrain cosmology ($T_R, \ldots$).