Embedding MSSM Inflation into the Minimal Left-Right Symmetric Model

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Work in progress, with M. Drees
Slow-roll Inflation

- Basic picture
  - The universe dominated by a scalar field ("inflaton"), $\phi$:
    \[ \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \]
  - Exponential expanding: $R(t) \propto e^{Ht}$.
  - $\epsilon \equiv \frac{1}{2} M_P^2 \left( \frac{V'}{V} \right)^2 \ll 1$; $\dot{\phi} = -\frac{V'}{3H}$ (Or, $|\eta| \equiv M_P^2 \frac{V''}{V} | \ll 1$)
  - "Reheating": After inflation, the inflaton oscillates around the global minimum and produces the entropy density.

- Examples of Inflationary Models
  - Chaotic inflation: $V(\phi) = \frac{1}{2} m^2 \phi^2$.
  - Hybrid inflation:
    \[ V(\phi, \psi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} (\psi^2 - M^2)^2 + \frac{1}{2} \lambda' \psi^2 \phi^2. \]
The scalar potential:

\[ V = |F_i|^2 + \frac{1}{2} D^a D^a, \]

where

\[ F_i = -\frac{\partial W^+}{\partial \phi_i^+}, \quad D^a = -g\phi_i^+ T^a_{ij} \phi_j \]

Flat directions: The field configuration such that the renormalizable scalar potential vanishes identically.
(Lifted) Flat Directions in Supersymmetric Theories

The superpotential:

$$W = W_{\text{renorm}} + \sum_{n>3} \frac{\lambda}{M^{n-3}} \phi^n.$$  

⇒ Flat directions in MSSM are lifted by soft SUSY-breaking terms and by non-renormalizable terms. [Gherghetta, Kolda, Martin]

⇒ The scalar potential:

$$V = \frac{1}{2} m^2 \phi^2 + A \cos(n\theta + \theta_A) \frac{\lambda_n}{nM^3_P} \phi^n + \frac{\lambda^2_n}{M^2_P(n-3)} \phi^{2(n-1)}.$$
Only n=6 (LLe, udd) flat directions can be inflaton candidates. Parametrized by, e.g.,

\[ L_i = \frac{1}{\sqrt{3}}(0 \phi) \]
\[ L_j = \frac{1}{\sqrt{3}}(\phi 0) \]
\[ e_k = \frac{1}{\sqrt{3}} \phi, \]

The scalar potential is:

\[ V = \frac{1}{2} m_\phi^2 \phi^2 - \frac{A \lambda_6}{6 M_P^3} \phi^6 + \frac{\lambda_6^2}{M_P^6} \phi^{10}. \]

Tuning \( A^2 = 40 m_\phi^2 \), at the saddle point,

\[ \phi_0 = \left( \frac{m_\phi M_P^3}{\sqrt{10} \lambda_6} \right)^{1/4} ; \quad V(\phi_0) = \frac{4}{15} m_\phi^2 \phi_0^2. \]
With \( m_\phi \sim 1 \text{TeV}, \lambda_6 \sim 1, \)

\[
\phi_0 \sim 10^{14} \text{GeV}; \quad H_{\text{inf}} \sim \frac{m_\phi \phi_0}{M_P} \sim (1 - 10) \text{GeV};
\]

\[
n_s \sim 1 - \frac{4}{N_{\text{COBE}}} \approx 0.92; \quad \delta \sim \frac{m_\phi M_P}{\phi_0^2} N_{\text{COBE}}^2 \sim 10^{-5}.
\]
MSSM Inflation: Summary

- Gauge-invariant combination of squarks and/or sleptons as inflaton. $\Rightarrow$ No ad-hoc singlet.

- Needs fine-tuning condition: $A^2 = 40m_\phi^2$.

- Testible in laboratory experiments with mild assumptions.

- Low-scale inflation: $H_{inf} \sim (1 - 10)\text{GeV}$
The Minimal Left-Right (LR) Symmetric Model [Aulakh et. al.]

- $U(1)_{B-L} \times SU(2)_R \rightarrow U(1)_Y$ by $SU(2)_R$ triplet Higgs.

- Heavy right-handed neutrino is naturally included. ($\Rightarrow m_\nu \sim m_D^2/M_R$)

- Parity is broken spontaneously.

- Subgroup of SO(10).
The chiral superfields \((SU(3) \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R)\):

\[
Q = (3, 1/3, 2, 1), \quad Q_c = (3^*, -1/3, 1, 2), \quad L = (1, -1, 2, 1)
\]
\[
L_c = (1, 1, 1, 2), \quad H = (1, 0, 2, 2^*), \quad \Sigma = (1, 2, 3, 1)
\]
\[
\Sigma = (1, -2, 3, 1), \quad \Sigma_c = (1, -2, 1, 3), \quad \Sigma_c = (1, 2, 1, 3)
\]

The renormalizable superpotential: (i, j: family index)

\[
W_{ren} = m_\Sigma (\Sigma \Sigma + \Sigma_c \Sigma_c) + Y_q H Q_i Q_j + Y_l H L_i L_j + \frac{i}{2} Y_N (L_c \Sigma_c L_c + L_i \Sigma L_j).
\]

The symmetry is broken by nonrenormalizable terms:

\[
W_{nr} \ni \lambda_\sigma \frac{\Sigma_c \Sigma_c}{4 M_P}.
\]

The LR symmetry breaking scale:

\[
10^{13} \text{GeV} \lesssim M_R (= \sqrt{\frac{m_\Sigma M_P}{\lambda_\sigma}}) \lesssim 10^{16} \text{GeV}.
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The chiral superfields \((SU(3) \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R)\):

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\]

The symmetry is broken by nonrenormalizable terms:
\[
W_{nr} \ni \lambda_\sigma \frac{4}{M_P} (\Sigma_c \Sigma_c)^2.
\]

The LR symmetry breaking scale:
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10^{13}\,\text{GeV} \lesssim M_R = \sqrt{\frac{m_\Sigma M_P}{\lambda_\sigma}} \lesssim 10^{16}\,\text{GeV}.
\]
The Set-up

- $Q_cQ_cQ_cL_c$ flat direction: Lift by $n=4$ $Q_cQ_cQ_cL_c$. Parametrizing the fields such as

\[ Q_{ci} = e^{i\theta}\begin{pmatrix} \phi \\ 0 \end{pmatrix}^T, \quad Q_{cj} = e^{i\theta}\begin{pmatrix} 0 \\ \phi \end{pmatrix}^T, \quad Q_{ck} = e^{i\theta}\begin{pmatrix} 0 \\ \phi \end{pmatrix}^T, \quad L_{cj} = c_je^{i\theta}\begin{pmatrix} \psi \\ 0 \end{pmatrix}^T, \quad L_{ck} = c_ke^{i\theta}\begin{pmatrix} \psi \\ 0 \end{pmatrix}^T, \quad \ldots \quad (j \neq k; c_j^2 + c_k^2 = 1; c_j, c_k \in \mathbb{R}), \]

flat directions:

- $\psi = \phi; \quad \bar{\sigma} = \sigma = 0; \quad \cos(2\theta_j - 2\theta_k) = 1 - \frac{1}{2c_j^2c_k^2}$. 

("LR-symmetric" flat direction)

- $\psi = 0; \quad \bar{\sigma} = \sqrt{-\frac{\phi^2}{4} + \frac{1}{4} \sqrt{\frac{64m^2_{\Sigma}M^2_P}{\lambda^2_\sigma} + \phi^4}}, \quad \sigma = \sqrt{\frac{\phi^2}{4} + \frac{1}{4} \sqrt{\ldots}}$. 

("MSSM-like" flat direction)

- $W_{nr} = \frac{\lambda_\sigma}{4M_P}(\Sigma_c\Sigma_c)^2 + \frac{\lambda_{4j}}{3M_P} \Phi^3L_{cj} + \frac{\lambda_{4k}}{3M_P} \Phi^3L_{ck}$. 

The Dynamics

(i) "LR-symmetric" direction ($\bar{\sigma} = \sigma = 0$)

- Constraints:
  - Nucleon (non-)decay $\Rightarrow \lambda_4 \lesssim 10^{-8}$.
  - LR Symmetry breaking $\Rightarrow m_\Sigma \sim m_{\text{soft}}$.

- $V = V_\sigma + V_\phi + V_c$, where

  \[ V_\sigma = \left( m_\Sigma - \frac{\lambda_\sigma}{2M_P} \sigma \bar{\sigma} \right)^2 (\sigma^2 + \bar{\sigma}^2) + \frac{1}{2} m^2 (\sigma^2 + \bar{\sigma}^2) \]

  \[ - \mu^2 (\sigma \bar{\sigma} + \text{h.c.}) - \frac{\lambda_\sigma A_\sigma}{4M_P} (\sigma \bar{\sigma})^2, \]

  \[ V_\phi = \frac{1}{2} m_\phi^2 \phi^2 - \frac{\lambda_4 A_4}{4M_P} \phi^4 + \frac{\lambda_4^2}{M_P^2} \phi^6. \]

$\Rightarrow \phi_0 = \sqrt{\frac{m_\phi M_P}{\lambda_4}}$

$\Rightarrow$ Consistent with observation.
(ii) "MSSM-like" direction ($\bar{\sigma} \neq 0, \sigma \neq 0$)

We "integrate out" the $\tilde{L}_c$ first: $\tilde{L}_c \simeq -\frac{\lambda_4 \sigma^* \phi^3}{3M_P (\bar{\sigma}^2 + \lambda_4 \phi^4 M_P^2)}$

- $\phi \ll M_R : V \simeq \phi^2 \left(m^2 - \frac{\lambda^2 A \phi^4}{M_P^2 M_R} + \frac{\lambda^4 \phi^8}{M_P^4 M_R^2}\right)$
  \Rightarrow V \rightarrow V_{MSSM}, \text{ with } \frac{\lambda^2 M_P}{M_R} \rightarrow \lambda.
  \Rightarrow \text{Smaller } \phi_0.$

- $\phi \gg M_R$
  
  - $\bar{\sigma} \gg \frac{\phi^2}{M_P} : V \simeq \phi^2 \left(m^2 - \frac{\lambda^2 A \phi^5}{M_P^2 M_R^2} + \frac{\lambda^4 \phi^{14}}{M_P^4 M_R^8}\right)$
    \Rightarrow \text{Very complicated fine-tuning needed.}

  - $\bar{\sigma} \ll \frac{\phi^2}{M_P} : V \simeq m^2 \phi^2 - A M_R^2 \phi + \frac{\lambda^2}{M_P^2} \phi^6$
    \Rightarrow \text{No flat potential.}

\Rightarrow \text{Works only for } \phi_0 < M_R.$
(ii') Assumption: \( \exists \) A symmetry suppressing the \( Q_cQ_cQ_cL_c \).
\[ \Rightarrow W_{nr} = \frac{\lambda_\sigma}{4M_P}(\Sigma_c \Sigma \Sigma_c)^2 + \frac{\lambda_7}{6M_P^4} \Phi^6 \Sigma_c. \]

- \( \phi \ll \sqrt{\frac{8m_\Sigma M_P}{\lambda}} : V \simeq \phi^2 \left( m^2 - \frac{A\lambda M_R}{M_P^4} \phi^4 + \frac{\lambda^2 M_R^2}{M_P^8} \phi^8 \right) \)
  \[ \Rightarrow V \rightarrow V_{MSSM}, \text{ with } \lambda \frac{M_R}{M_P} \rightarrow \lambda \]

- \( \phi \gg \sqrt{\frac{8m_\Sigma M_P}{\lambda}} : V \simeq \phi^2 \left( m^2 - \frac{A\lambda}{M_P^4} \phi^5 + \frac{\lambda^2}{M_P^8} \phi^{10} \right) \)
  \[ \Rightarrow \phi_0 \simeq \left( \frac{M_P^4 m}{\lambda_7 M_R} \right)^{1/4} \]

\[ \Rightarrow \text{Slightly larger } \phi_0 \text{ (compared to that in MSSM inflation), but works.} \]
Preheating

- **Basic picture** [Kofman, Linde, Starobinsky]
  
  Assuming $V = \frac{m^2}{2} \phi^2 + \frac{1}{2} g^2 \phi^2 \chi^2$, EOM for quantum fluctuations of the scalar field $\chi$:
  \[
  \ddot{\chi}_k + 3H\dot{\chi}_k + \left( \frac{k^2}{a^2(t)} + g^2 \Phi(t)^2 \sin^2(m\phi t) \right) \chi_k = 0.
  \]
  ($a$: scale factor, $\Phi$: amplitude of oscillations)
  ⇒ Parametric resonance can happen!
  ⇒ Particle production: $n_k = \frac{\omega_k}{2} \left( \frac{\left| \dot{\chi}_k \right|^2}{\omega_k^2} + |\chi_k|^2 \right) - \frac{1}{2}$.

- **Post-MSSM inflation** [Allahverdi, Enqvist, Garcia-Bellido, Jokinen, Mazumdar]
  
  The gauge bosons and gauginos are produced when the inflaton passes through the origin.
  ⇒ Get "fatten"s when the inflaton oscillates.
  ⇒ Decays to the matter fields.
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  $\Rightarrow$ Get "fatten"s when the inflaton oscillates.

  $\Rightarrow$ Decays to the matter fields.
Post-"MLRSM" inflation - "LR symmetric" direction. 

\[ SU(2)_R \times U(1)_{B-L} \] 
Symmetry breaking 

\[ \exists \delta_0 \left( \sim \lambda \frac{M_R^2}{M_P} \right) \] [Aulakh et. al.] 

\[ \Rightarrow \text{All } \phi \left( \sim 10^{14} \text{GeV} \right), \tilde{L}_c \left( \sim 10^{14} \text{GeV} \right), \delta_0 \left( \sim 10^{14} \text{GeV} \right) \text{ start to oscillate.} \]

\[ \Rightarrow \delta_0 \text{ slowly changing, } \tilde{L}_c \text{ rapidly fixed at the minimum.} \]
Both $n = 4$ and $n = 7$ operator in the $Q_c Q_c Q_c L_c$ direction can provide us the slow-roll inflation, either by tuning the nonrenormalizable coupling or the initial conditions.

- "LR-symmetric" direction: OK, with suppressed nonrenormalizable couplings.
- "MSSM-like" direction: $\phi_0$ should lie below $M_R$.

The post-inflation cosmology is very different along each branch:

- "LR-symmetric" direction: Neutral $SU(2)_R$ triplet Higgs ($m \sim \mathcal{O}(\text{TeV})$) is produced.
- "MSSM-like" direction: All vacuum energy is transferred to the radiation.
Summary Prospects

- Both $n = 4$ and $n = 7$ operator in the $Q_cQ_cQ_cL_c$ direction can provide us the slow-roll inflation, either by tuning the nonrenormalizable coupling or the initial conditions.
  - "LR-symmetric" direction: OK, with suppressed nonrenormalizable couplings.
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- The post-inflation cosmology is very different along each branch:
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Combining the information from cosmological observation with the collider signal, the model can be strongly constrained.

Implications on Baryogenesis will be explored.