Investigating thermal abundance of semi-relativistic particles

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Introduction
The process of freeze out
Limitations of existing treatment

Mathematical formalism
Modifications required
Treating the thermal average
Solution of the Boltzmann equation

Physical scenarios (toy models)
Feasibility of relic densities
Entropy production

Conclusion
Thermal freeze out

- Once upon a time in distant past....
  - $\Gamma \gtrsim H \Rightarrow A + B \leftrightarrow C + D$
  - $\Gamma < H \Rightarrow$ Freeze out

- Boltzmann Equation $\hat{L}[f] = C[f]$

$$\frac{dY}{dx} = -\frac{x\langle\sigma v\rangle s}{H(m)}(Y^2 - Y_{eq}^2)$$

where:- $s = \text{entropy}$

$Y = \frac{n}{s}$

$x = \frac{m}{T}$

- Relic density $\Omega h^2 = 2.8 \times 10^8 Y_\infty (m/GeV)$
Limiting cases

- **Relativistic treatment \((m \ll T)\)**
  - \(\Omega h^2 \propto m\), independent of \(\langle \sigma v \rangle\)

- **Non-relativistic treatment \((m \gg T)\)**
  - Expansion of thermal average of cross-section in terms of velocity \(\langle \sigma v \rangle = a + \frac{6b}{x}\)

- \[Y_\infty = \frac{\sqrt{90}}{4\pi m M_{Pl} \sqrt{g_*}(x_f) \left( \frac{a}{x_f} + \frac{3b}{x_f^2} \right)} \propto \frac{1}{\langle \sigma v \rangle}\]

- \[\Omega h^2 \propto m Y_\infty \propto \frac{1}{\langle \sigma v \rangle}\]

- No known analytical solution in intermediate range \((m \simeq T)\)
We need to...

- Modify the expression for abundance
  - Assuming Maxwell-Boltzmann distribution
    \[ Y_{eq} \equiv \frac{n_{eq}}{s} = 0.115 \frac{g}{g^*} x^2 K_2(x) \]

  \[ K_n(x) = \text{Modified Bessel function} \]

- New treatment for thermal averaging of cross-section
  \[ \langle \sigma v \rangle = \frac{1}{8m^4TK_2^2(m/T)} \int_{4m^2}^{\infty} ds \sigma (s - 4m^2) \sqrt{s} K_1(\sqrt{s}/T) \]

  Note :- s here is the Mandelstam variable.
Treat the thermal average

- Scenario: Stable neutrinos
- Annihilation cross-section form
  - $\sigma v = \frac{G^2 s}{16\pi}$
    (Dirac type, S-wave)
  - $\sigma v = \frac{G^2 s}{16\pi} \left(1 - \frac{4m^2}{s}\right)$
    (Majorana type, P-wave)
- We do not take into account resonance

- Thermally averaged annihilation cross-section
  - $\langle \sigma v \rangle_{\text{app}} = \frac{G^2 m^2}{16\pi} \left(\frac{12}{x^2} + \frac{5+4x}{1+x}\right)$
    (Dirac type, S-wave)
  - $\langle \sigma v \rangle_{\text{exact}}$

**Ratio of approximate to exact cross sections**

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<th>P-wave</th>
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Solution of the Boltzmann equation

- Freeze out temperature
  \[ \Gamma(x_F) = H(x_F) \]
  where: \[ \Gamma(x_F) = \langle \sigma v \rangle n_{eq}(x_F) \]
  (Different from standard definition of \( x_F \))

- Leads to semi analytical expression for \( x_f \) hence computing \( \Omega h^2 \) possible

- Assume that the comoving relic abundance does not change after decoupling

  - Constant \( g_s \)
  - \( g = 2 \)
Feasibility of relic densities

- Decoupling at $x_F = 1.8$ and $g_{*s} = 10$ with $\Omega_{DM} h^2 = 0.13$
  $\Rightarrow m \sim eV$
  - Too light

- Coupling $G > 1 GeV^{-2}$
  - Not a very promising scenario
Entropy production

- Out of equilibrium decay produces entropy
  \[ \frac{s_f}{s_i} = 1.7 \, g_*^{1/4} \frac{m_Y \sqrt{\tau}}{\sqrt{M_{pl}}} \propto \Omega h^2 \]

- Sterile neutrinos decay through mixing with standard model neutrinos
  \[ \Gamma = \frac{1}{\tau} = \frac{G^2 m^2}{192\pi^3} \sin^2 \theta \]

- Large pair annihilation rate via mediation of new U(1) gauge boson
Final results

- Region to the left of the bold line allowed ($\tau < 1$ Sec BBN constraint)
- Possible to produce large entropy

Entropy production $s_f/s_i$
Conclusion:-

- Found semi-analytical method to compute density of semi-relativistic relics \( T_F \sim m \)
- Semi-relativistic particles as a stable dark matter relic is not a very promising scenario
- They can be used to produce considerable amount of entropy in the early Universe