Relic-density-consistent SUSY models without soft term universality: consequences for collider and neutralino dark matter searches

Eun-Kyung Park
Bonn University

based on JHEP05 (2008) 058
in collaboration with
H. Baer(Florida State U.), A. Mustafayev (U. of Kansas) and X. Tata (U. of Hawaii)

SUSY08, June 16-21, 2008
Outline

• Introduction
  ⋆ Dark Matter
  ⋆ Neutralino
  ⋆ Universal SUSY model : mSUSGRA

• Models without universality in SSB terms
  ⋆ Non-universal scalar mass models
  ⋆ Non-universal gaugino mass models

• Implications for collider searches
• Implications for direct and indirect dark matter detections
• Conclusions
Dark Matter

- Dominant composition of matter in our universe is not detected visibly but inferred from gravitational effects (Galactic Clustering, Rotation Curves, Gravitational Lensing, Cosmic Microwave Background ...)
- Dark Matter should be non-baryonic (no candidate in the SM), non-relativistic (cold), stable (or long-lived), weakly (or super-weakly) interacting matter
- From the WMAP results, the cold dark matter density of the universe is $\Omega_{CDM} h^2 = 0.111^{+0.011}_{-0.015}$: (upper bound is a tight constraint on SUSY models containing DM candidates: DM may consist of several components)

http://map.gsfc.nasa.gov

Neutralino

- In SUSY models with $R$-parity conservation
  $\Rightarrow$ the Lightest Supersymmetric Particle (LSP) is stable
  $\Rightarrow$ lightest neutralino $\tilde{Z}_1$ is the LSP in most of MSSM parameter space

$\Rightarrow$ $\tilde{Z}_1$ is good candidate for Cold Dark Matter (CDM)

$$\tilde{Z}_1 = v_1^{(1)} \psi_{h_u^0} + v_2^{(1)} \psi_{h_d^0} + v_3^{(1)} \lambda_3 + v_4^{(1)} \lambda_0$$

Here, $R_{\tilde{w}} = |v_3^{(1)}|$, $R_{\tilde{B}} = |v_4^{(1)}|$ and $R_{\tilde{H}} = \sqrt{|v_1^{(1)}|^2 + |v_2^{(1)}|^2}$

: $W$-ino, $B$-ino and Higgsino

- We assume,
  - MSSM is an effective theory between the weak and GUT scale
  - $R$-parity is conserved
  - Neutralino LSP

- Number density is governed by Boltzmann equation,

$$dn/dt = -3Hn - \langle \sigma v_{rel} \rangle (n^2 - n_0^2)$$

$\Rightarrow$ requires evaluating many thousands Feynman diagrams

$\Rightarrow$ high (co-)annihilation cross section implies low relic abundance
Universal SUSY model: mSUGRA

- Parameter space: universal Soft Susy Breaking terms at $Q = M_{GUT}$
  $m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$

- WMAP allowed Regions in $m_0$-$m_{1/2}$ space
  1. $\tilde{\tau}$ co-annihilation region at low $m_0$, $m_{\tilde{\tau}_1} \sim m_{\tilde{Z}_1}$
  2. bulk region at low $m_0$ and $m_{1/2}$, light sleptons (LEP2 excluded)
  3. Higgs-funnel $H$, $A$ resonance ($2m_{\tilde{Z}_1} \simeq m_{A,H}$) at large $\tan\beta \sim 50$ or $h$-resonance at low $m_{1/2}$ ($2m_{\tilde{Z}_1} \simeq m_h$)
  4. FP/HB region at large $m_0$, low $\mu \rightarrow$ mixed higgsino dark matter (MHDM)
     * Region 1, 2, 3 $\rightarrow$ Bino-like LSP

- Motivations for models with non-universality
  * all relic-density-consistent regions in mSUGRA are near the edges of theoretically (or LEP2 experiment) excluded regions
  * need to examine how already drawn conclusions from the mSUGRA model are affected by relaxing the universality assumptions
  * within $R$-parity conserved neutralino dark matter assumption, WMAP value provides a strong constraint reducing model parameter space by one unit
Models without universality in SSB terms

- **Relic-density-consistent models** obtained by adjusting
  - composition of neutralino (WTN: Well-Tempered Neutralino*)
  - masses of neutralino or other sparticles

- **Non-universal scalar mass models**
  - Generation non-universality: Normal scalar mass hierarchy (NMH)
  - Non-universal Higgs mass: one extra parameter case (NUHM1$_{\mu}$, NUHM1$_{A}$)
  - non-universal Higgs mass: two extra parameter case (HS-Higgs Splitting)

- **Non-universal gaugino mass models**
  - Mixed Wino Dark Matter (MWDM)
  - Bino-Wino Co-Annihilation Scenario (BWCA)
  - Low $|M_3|$ Dark Matter: Compressed SUSY (LM3DM)
  - High $|M_2|$ Dark Matter: left-right split SUSY (HM2DM)

- Some benchmark cases with mSUGRA parameter space
  $m_0$, $m_{1/2}$, $A_0$, tan$\beta$, sign($\mu$) = 300 GeV, 300 GeV, 0, 10, +1 and $m_t = 171.4$ GeV

Eun-Kyung Park neutralino DM searches in relic-density-consistent models without universality
Non-universal scalar mass models

- generation non-universality: Normal scalar Mass Hierarchy (NMH)
  \[ m_0(1, 2), m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu) \]
  - \( m_0(1, 2) \): first/second generation, \( m_0(3) = m_{H_u} = m_{H_d} \equiv m_0 \): remaining
  - dial \( m_0(1, 2) \) to low enough to bulk (co-)annihilation via light sleptons

- non-universal Higgs mass: one extra parameter case (NUHM1_\mu, NUHM1_A)
  \[ m_0, \delta_\phi, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu) \]
  - \( m_\phi = m_0(1 + \delta_\phi), m_{H_u}^2 = m_{H_d}^2 \equiv \text{sign}(m_\phi)|m_\phi|^2 \)
  - \( m_\phi > m_0 \): small \( \mu \) and MHDM
  - \( m_\phi < 0 \): \( m_A \sim 2m_{\tilde{Z}_1} \rightarrow \) at any \( \tan \beta \)

- non-universal Higgs mass: two extra parameter case (HS-Higgs Splitting)
  \[ m_0, m_{H_u}^2 \text{(equivalently } \mu), m_{H_d}^2 \text{(equivalently } m_A), m_{1/2}, A_0, \tan \beta, \text{sign}(\mu) \]
  - \( m_{H_u,d}^2 = m_0^2 (1 \mp \delta_H) \)
  - \( \delta_H < 0 \): low \( \mu \) and low \( m_A \)
  - \( \delta_H > 0 \): WMAP region via \( \tilde{l}_L/\tilde{\nu} \) or \( \tilde{u}_R/\tilde{c}_R \) co-annihilation
Non-universal gaugino mass models

- Mixed Wino Dark Matter (MWDM1, MWDM2):
  \( m_0, M_1(\text{or } M_2), m_{1/2}, A_0, \tan\beta, \text{sign}(\mu) \)
  - by increasing the wino content of the LSP by reducing the ratio \( M_2/M_1 \)
  - \( M_1 \neq M_2 = M_3 = m_{1/2} \) or \( M_2 \neq M_1 = M_3 = m_{1/2} \)

- Bino-Wino Co-Annihilation Scenario (BWCA1, BWCA2):
  same as MWDM but \( M_1 \) and \( M_2 \) are in opposite sign
  - by allowing co-annihilation between high bino-like and wino-like states

- Low \( |M_3| \) Dark Matter: Compressed SUSY (LM3DM):
  \( m_0, M_3, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu) \)
  - by increasing the higgsino content of the LSP by decreasing the gluino mass
  - \( M_3 \neq M_1 = M_2 = m_{1/2} \)

- High \( |M_2| \) Dark Matter: left-right split SUSY (HM2DM):
  \( m_0, M_2, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu) \)
  - by allowing large \( M_2 \) mass
  - \( M_2 \gg M_1 = M_3 = m_{1/2} \)
Some Benchmark Cases: non-universal scalar mass models

<table>
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<th>parameter</th>
<th>mSUGRA</th>
<th>NMH</th>
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| $\Omega_{\tilde{Z}_1} h^2$ | 1.1 | 0.10 | 0.11 | 0.11 | 0.10 |
| $\sigma_{SI}(\tilde{Z}_1p)$ | $2.1 \times 10^{-9}$ pb | $2.1 \times 10^{-9}$ pb | $7.8 \times 10^{-8}$ pb | $1.2 \times 10^{-9}$ pb | $2.7 \times 10^{-8}$ pb |
| $R_{H}$ | 0.15 | 0.14 | 0.84 | 0.06 | 0.26 |
Some Benchmark Cases: non-universal gaugino mass models

<table>
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<th>parameter</th>
<th>mSUGRA</th>
<th>MWDM</th>
<th>BWCA</th>
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Dark matter at Colliders

- CERN LHC and Fermilab Tevatron
  - If $\tilde{Z}_2 \rightarrow \tilde{\ell}\bar{\ell}, \tilde{\ell} \rightarrow \tilde{Z}_1\ell\bar{\ell}$ or $\tilde{Z}_2 \rightarrow \tilde{\ell}\ell$ are open ($l = e$ or $\mu$)
    $\Rightarrow$ good prospects for measuring the $\tilde{Z}_2 - \tilde{Z}_1$ mass gap at the CERN LHC and possibly at the Fermilab Tevatron
  - In the mSUGRA case, most of the parameter space has $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} > 90$ GeV, $\Rightarrow \tilde{Z}_2 \rightarrow \tilde{Z}_1Z^0$ or $\tilde{Z}_1h$ “spoiler” decays dominant
  - When the mass gap is much smaller
    * spoiler decays are closed, 3-body decays are open
    * $\ell\bar{\ell}$ mass edge always visible at LHC

- Linear $e^+e^-$ collider (ILC)
  - $m_{\tilde{Z}_2}$, $m_{\tilde{W}_1}$ and $m_{\tilde{Z}_1}$ can be inferred from $\tilde{W}_1^+\tilde{W}_1^- \rightarrow \ell\nu_1\tilde{Z}_1 + q\bar{q}\tilde{Z}_1$ (dijet events)
  - $\tilde{W}_1^+\tilde{W}_1^-, \tilde{Z}_1\tilde{Z}_2$, $\tilde{Z}_2\tilde{Z}_2$ production cross sections can be measured as a function of beam polarization

- ISAJET program (H. Baer, F.E. Paige, S.D. Protopopescu, and X. Tata)
Implications for collider searches 1

- with $A_0 = 0$, $m_t = 171.4$ GeV, $\tan \beta = 10$
  (except for the mSUGRA model: $\tan \beta = 10, 30, 45, 50, 52$ and 55)
- non-universal mass dialed to yield $\Omega \tilde{Z}_1 h^2 \approx 0.11$

- $m_{\tilde{g}}$ vs. $m_{\tilde{u}_R}$
  - dotted lines: 100 fb$^{-1}$ reach of CERN LHC
  - dashed line: $m_{\tilde{u}_R} = m_{\tilde{g}}$
- most of models within reach of LHC except HB/FP region of mSUGRA

- $m_{\tilde{W}_1}$ vs. $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$
  - dashed line: $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} = M_Z$
- below the line, 3-body decay like $\tilde{Z}_2 \rightarrow \tilde{Z}_1 l \bar{l}$
  - open
- in most models, $m(l\bar{l})$ mass edge visible at LHC
Dilepton Distribution at LHC

- **mSUGRA**: sharp peak at $m(l^+l^-) \sim M_Z$ from $\tilde{Z}_2 \rightarrow \tilde{Z}_1 Z^0$ decays

- **NUGM**: $Z^0$ peak from $\tilde{Z}_3, \tilde{Z}_4, \tilde{W}_2$ decays + continuum distribution $m(l^+l^-) < m\tilde{Z}_2 - m\tilde{Z}_1$
Implications for collider searches 2

- $m_h$ vs. $m_{\tilde{t}_1}$
  - heavier $\tilde{t}_1$ squarks are correlated with larger values of $m_h$ (due to top-Yukawa radiative corrections to $m_h$)
  - in many models with $m_A \gg M_Z$, then $h \approx H_{SM}$: the LEP2 lower bound of 114.1 GeV applicable

- $m_{\tilde{W}_1}$ vs. $m_{\tilde{\tau}_1}$
  - dashed lines: reach of ILC500 ($\sqrt{s} = 500$ GeV)
  - dotted lines: reach of ILC1000 ($\sqrt{s} = 1000$ GeV)
Implications for $BF(b \to s\gamma)$ and $(g - 2)_\mu$

- $BF(b \to s\gamma)$
  
  - dotted line: combined experimental measurement (CLEO, Belle, BABAR)
  
  $$BF(b \to s\gamma) = (3.55 \pm 0.26) \times 10^{-4}$$

  - dashed line: SM prediction
  
  $$BF(b \to s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$$

- $(g - 2)_\mu$
  
  - positive deviation in $a_\mu \equiv \frac{(g - 2)_\mu}{2}$
  
  $$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 22(10) \times 10^{-10}$$

  - $\Delta a_\mu^{SUSY} \propto \tan \beta$

  * We assume,

  - (near)degeneracy of first and second generation of SSB sfermions $\rightarrow$ FCNC suppressed

  - CP-violating phases in SSB suppressed $\rightarrow$ CP contribution of SUSY is small
Direct and Indirect Dark Matter Detection

• Direct Detection: Spin independent Neutralino-Proton scattering Cross section
  (with current experimental sensitivities: Xenon-10(100, 1000), SuperCDMS, LUX)

• Indirect Detection
  – Detection of $\mu$: Neutrinos from the Sun - IceCube
    $\tilde{Z}_1\tilde{Z}_1 \rightarrow W^+W^-, q\bar{q}, \ldots \rightarrow \pi^- (\pi^+) \rightarrow \bar{\nu}_\mu (\nu_\mu) \rightarrow \mu^- (\mu^+)$
  – Detection of antiparticles: $\tilde{Z}_1\tilde{Z}_1 \rightarrow W^+W^-, q\bar{q}, ZZ, \ldots \rightarrow jets$
    Antiprotons ($jets \ni \bar{p}$): PAMELA, Positrons ($jets \ni e^+$): PAMELA,
    Antideuterons ($jets \ni \bar{D}$): GAPS
  – Detection of Gamma Rays from the galactic center - GLAST

• IsaRES code (Baer-Belyaev-O’Farrill) and DarkSUSY
Implications for direct/indirect (neutrino) DM detection

- models with WTN within reach of next generation of detectors
- models adjusted masses to get WMAP value below sensitivities of detectors
- muon fluxes from neutralino annihilation in the solar core to $\nu_\mu$ states
- main contribution comes from $Z$-exchange $\leftrightarrow$ enhanced if neutralino has high higgsino content
Implications for indirect (γ-ray, antiparticle) DM detection

Gamma-ray Detection : Ad. Contr. N03 HM

Positron Detection : Ad. Contr. N03 HM

Anti-proton Detection : Ad. Contr. N03 HM

Anti-deuteron Detection : Ad. Contr. N03 HM

Eun-Kyung Park neutralino DM searches in relic-density-consistent models without universality
Conclusions

1. ★ WTN occurs only in FP/HB region in mSUGRA (MHDM: \( m_{\tilde{q}} >> m_{\tilde{Z}_1, \tilde{W}_1, \tilde{g}} \)). But, in relic-density-consistent models, easily get WTN with \( m_{\tilde{q}} \sim m_{\tilde{g}} \)
   ★ Higgs funnel enhancement is only for very large \( \tan \beta \) values in mSUGRA.
   But, in non-universal Higgs mass models, we have Higgs funnel for any \( \tan \beta \) value

2. In many relic-density-consistent models, \( \tilde{Z}_2 - \tilde{Z}_1 \) mass gap < \( M_Z \)
   → 2-body decay modes kinematically closed
   → 3-body decay modes open ⇒ at least one dilepton mass edge detectable at LHC
   → location of dilepton mass edge is clean signature of SUSY models

3. ★ \( m_{\tilde{q}} = m_{\tilde{g}}, m_{\tilde{q}, \tilde{g}} < 3100 \) GeV for most relic-density-consistent models
   → implies SUSY signals at LHC
   ★ \( m_{\tilde{\tau}} < 500 \) GeV for LM3DM
   → accessible at ILC with \( \sqrt{s} = 1 \) TeV

4. In WTN models,
   ★ enhanced annihilation rates enhance direct DM detection rates
   ★ in many cases, muon neutrino signals accessible at IceCube
   ★ indirect DM searches in galactic halo into gamma rays and anti-matter elevated; large uncertainties associated with unknown galactic DM density profile
MSSM RGEs

\[
\frac{dm_{H_u}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{3}{5} g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10} g_1^2 S + 3f_t^2 X_t \right)
\]

\[
\frac{dm_{H_d}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{3}{5} g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{3}{10} g_1^2 S + 3f_b^2 X_b + f_{\tau}^2 X_{\tau} \right)
\]

\[
\frac{dm_{Q_3}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{1}{15} g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{16}{3} g_3^2 M_3^2 + \frac{1}{10} g_1^2 S + f_t^2 X_t + f_b^2 X_b \right)
\]

\[
\frac{dm_{t_R}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{16}{15} g_1^2 M_1^2 - \frac{16}{3} g_3^2 M_3^2 - \frac{2}{5} g_1^2 S + 2f_t^2 X_t \right)
\]

\[
\frac{dm_{b_R}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{4}{15} g_1^2 M_1^2 - \frac{16}{3} g_3^2 M_3^2 + \frac{1}{5} g_1^2 S + 2f_b^2 X_b \right)
\]

\[
\frac{dm_{L_3}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{3}{5} g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{3}{10} g_1^2 S + f_{\tau}^2 X_{\tau} \right)
\]

\[
\frac{dm_{\tau_R}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{12}{5} g_1^2 M_1^2 + \frac{3}{5} g_1^2 S + 2f_{\tau}^2 X_{\tau} \right)
\]

\[
S = m_{H_u}^2 - m_{H_d}^2 + Tr \left[ m_{Q}^2 - m_{L}^2 - 2m_{U}^2 + m_{D}^2 + m_{E}^2 \right]
\]
where $t = \log(Q)$, $f_{t,b,\tau}$ are the $t$, $b$ and $\tau$ Yukawa couplings, and

\[
\begin{align*}
X_t &= m_{Q3}^2 + m_{tR}^2 + m_{Hu}^2 + A_t^2 \\
X_b &= m_{Q3}^2 + m_{bR}^2 + m_{Hd}^2 + A_b^2 \\
X_\tau &= m_{L3}^2 + m_{\tau R}^2 + m_{Hd}^2 + A_\tau^2
\end{align*}
\]
Feynman Diagrams Contributing to Neutralino DM Detection

• Direct Detection

• Indirect Detection