Application of the Dark Matter Direct Detection with an Annual Modulated Event Rate

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One-dimensional velocity distribution function of WIMPs
- Deriving $f_1(v)$ from a scattering spectrum
- Constructing $f_1(v)$ from experimental data

Considering an annual modulated event rate
- Extension of our earlier work
- Amplitude of the annual modulation of $f_1(v)$

Summary

Deriving $f_1(v)$ from a scattering spectrum

- WIMP-nucleus elastic-scattering rate equation

$$\frac{dR}{dQ} = \mathcal{A}F^2(Q) \int_{v_{\rm min}}^{\infty} \left[\frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\rm min} = \alpha\sqrt{Q}$$

is the minimal incoming velocity with which the incident WIMPs can deposit the energy Q in the detector.

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{\rm r}^2} \qquad \alpha \equiv \sqrt{\frac{m_{\rm N}}{2 m_{\rm r}^2}} \qquad m_{\rm r} = \frac{m_\chi m_{\rm N}}{m_\chi + m_{\rm N}}$$

ρ_0: the WIMP density near the Earth
σ_0: the total cross section ignoring form factor effect
$F(Q)$: a nuclear form factor

Deriving $f_1(v)$ from a scattering spectrum

- A simple Maxwellian halo

$$f_{1,\text{Gaussian}}(v) = \frac{4}{\sqrt{\pi}} \left(\frac{v^2}{v_0^3} \right) e^{-v^2/v_0^2}$$

$$\left(\frac{dR}{dQ} \right)_{\text{Gaussian}} = \mathcal{A} F^2(Q) \left(\frac{2}{\sqrt{\pi} v_0} \right) e^{-\alpha^2 Q/v_0^2}$$

v_0: the orbital speed of the Sun around the Galactic center.

- Taking into account the orbital motion of the Earth around the Sun

$$f_{1,\text{shifted}}(v, v_e) = \frac{1}{\sqrt{\pi}} \left(\frac{v}{v_e v_0} \right) \left[e^{-(v-v_e)^2/v_0^2} - e^{-(v+v_e)^2/v_0^2} \right]$$

$$\left(\frac{dR}{dQ} \right)_{\text{shifted}} = \mathcal{A} F^2(Q) \left(\frac{1}{2v_e} \right) \left[\text{erf}\left(\frac{\alpha\sqrt{Q}+v_e}{v_0} \right) - \text{erf}\left(\frac{\alpha\sqrt{Q}-v_e}{v_0} \right) \right]$$

$$v_e(t) = v_0 \left[1.05 + 0.07 \cos\left(\frac{2\pi(t-t_p)}{1 \text{ yr}} \right) \right] \qquad t_p \simeq \text{June 2nd}$$

is the velocity of the Earth relative to the WIMP halo.

Deriving $f_1(v)$ from a scattering spectrum

- One-dimensional velocity distribution function

$$f_1(v) = \frac{\mathcal{N}}{\mathcal{A}} \left\{ -2Q \cdot \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2}$$

$$= \frac{4}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1} \left\{ -Q \cdot \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2}$$

$$\mathcal{N} = \mathcal{A} \left(\frac{2}{\alpha} \right) \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$

- Mean velocity and velocity dispersion

$$\langle v \rangle = 2\alpha \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1} \left\{ \int_0^\infty \frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) dQ \right\}$$

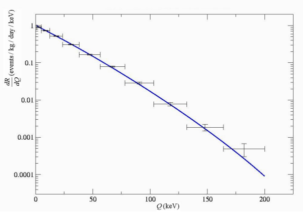
$$\bar{v}^2 = 3\alpha^2 \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1} \left\{ \int_0^\infty \sqrt{Q} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}$$

Constructing $f_1(v)$ from experimental data

- Experimental data

$$Q_n - \frac{b_n}{2} \le Q_{n,i} \le Q_n + \frac{b_n}{2} \qquad i = 1,\ 2,\ \cdots,\ N_n, \quad n = 1,\ 2,\ \cdots,\ N$$

- Theoretical predicted scattering spectrum



- Ansatz: in nth Q-bin

$$\left(\frac{dR}{dQ}\right)_n = r_n e^{k_n (Q - Q_n)} \qquad r_n \equiv \left(\frac{dR}{dQ}\right)_{Q=Q_n}$$

Constructing $f_1(v)$ from experimental data

- Event number at $Q = Q_n$

$$r_n = \frac{N_n}{b_n}\left(\frac{\widetilde{k}_n}{\sinh \widetilde{k}_n}\right) \qquad \widetilde{k}_n \equiv \left(\frac{b_n}{2}\right) k_n$$

- Logarithmic slope in the nth Q-bin

$$k_n = \frac{8\,\overline{Q - Q_n}|_n}{b_n^2 - 4\overline{(Q - Q_n)^2}|_n} \qquad \overline{(Q - Q_n)^\mu}|_n \equiv \frac{1}{N_n}\sum_{i=1}^{N_n}(Q_{n,i} - Q_n)^\mu$$

- Rewriting the differential

$$\left\{-Q \cdot \frac{d}{dQ}\left[\frac{1}{F^2(Q)}\left(\frac{dR}{dQ}\right)\right]\right\}_{Q=Q_n}$$

$$= \frac{Q_n}{F^2(Q_n)}\left[\frac{N_n}{b_n}\left(\frac{\widetilde{k}_n}{\sinh \widetilde{k}_n}\right)\right]\left[\frac{2}{F(Q_n)}\left(\frac{dF}{dQ}\right)_{Q=Q_n} - k_n\right] \equiv \tilde{f}_{1,n}$$

Constructing $f_1(v)$ from experimental data

- Rewriting the integrals

$$\frac{1}{N_n}\int_{Q_n-b_n/2}^{Q_n+b_n/2}\frac{Q^\mu}{F^2(Q)}\left(\frac{dR}{dQ}\right)dQ=\frac{1}{N_n}\sum_{i=1}^{N_n}\frac{Q_{n,i}^\mu}{F^2(Q_{n,i})}\equiv\overline{S_{2,\mu,n}}$$

$$\int_0^\infty\frac{Q^\mu}{F^2(Q)}\left(\frac{dR}{dQ}\right)dQ\simeq\sum_{n=1}^{N}\sum_{i=1}^{N_n}\frac{Q_{n,i}^\mu}{F^2(Q_{n,i})}=\sum_{n=1}^{N}N_n\overline{S_{2,\mu,n}}\equiv S_{2,\mu,\mathrm{tot}}$$

$$\mu=0,\pm\frac{1}{2}$$

- Constructed one-dimensional velocity distribution function

$$f_1\left(v_n=\alpha\sqrt{Q_n}\right)=\frac{4}{\alpha}\left(\frac{\tilde{f}_{1,n}}{S_{2,-1/2,\mathrm{tot}}}\right)$$

- Constructed mean velocity and velocity dispersion

$$\langle v\rangle=2\alpha\left(\frac{S_{2,0,\mathrm{tot}}}{S_{2,-1/2,\mathrm{tot}}}\right)\qquad\bar{v}^2=3\alpha^2\left(\frac{S_{2,1/2,\mathrm{tot}}}{S_{2,-1/2,\mathrm{tot}}}\right)$$

Extension of our earlier work

- Fourier cosine series ($\omega \equiv 2\pi/365$)

$$\left(\frac{dR}{dQ}\right)_t = \left(\frac{dR}{dQ}\right)_{(0)} + \left(\frac{dR}{dQ}\right)_{(1)} \cos(\omega t) + \left(\frac{dR}{dQ}\right)_{(2)} \cos(2\omega t) + \cdots$$

$$f_1(v,t) = f_{1,(0)}(v) + f_{1,(1)}(v)\cos(\omega t) + f_{1,(2)}(v)\cos(2\omega t) + \cdots$$

$$\left(\frac{dR}{dQ}\right)_{(m)} = \mathcal{A} F^2(Q) \int_{v_{\min}}^{\infty} \left[\frac{f_{1,(m)}(v)}{v}\right] dv \qquad m = 0,\ 1,\ 2,\ \cdots$$

- Redefining N_n, $\overline{(Q-Q_n)^\mu}|_n$, and $\overline{S_{2,\mu,n}}$

$$N_n = \int_{Q_n - b_n/2}^{Q_n + b_n/2} \left(\frac{dR}{dQ}\right)_{(0)} dQ = \frac{1}{365} \int_0^{365} \int_{Q_n - b_n/2}^{Q_n + b_n/2} \left(\frac{dR}{dQ}\right)_t dQ\, dt = \frac{N_{n,1\ \rm yr}}{365}$$

$$\overline{(Q-Q_n)^\mu}|_n = \frac{1}{N_{n,1\ \rm yr}} \sum_{i=1}^{N_{n,1\ \rm yr}} (Q_{n,i} - Q_n)^\mu \qquad \overline{S_{2,\mu,n}} = \frac{1}{N_{n,1\ \rm yr}} \sum_{i=1}^{N_{n,1\ \rm yr}} \frac{Q_{n,i}^\mu}{F^2(Q_{n,i})}$$

Amplitude of the annual modulation of $f_1(v)$

- Considering $(dR/dQ)_{(1)}$

$$\frac{1}{N_n}\int_{Q_n-b_n/2}^{Q_n+b_n/2}(Q-Q_n)^\mu\left(\frac{dR}{dQ}\right)_{(1)}dQ = \frac{2}{N_{n,1\ \rm yr}}\sum_{i=1}^{N_{n,1\ \rm yr}}(Q_{n,i}-Q_n)^\mu\cos(\omega t_{n,i})$$

$$\equiv 2\,\overline{(Q-Q_n)^\mu\cos(\omega t)}|_n$$

- Ansatz: in nth Q-bin

$$\left(\frac{dR}{dQ}\right)_{(1),n} = \left(\frac{dR}{dQ}\right)_{(0)}\cdot\left[l_n(Q-Q_n)+h_n\right]$$

- Solving the l_n and h_n

$$l_n = \frac{2\,\overline{(Q-Q_n)\cos(\omega t)}|_n - 2\,\overline{\cos(\omega t)}|_n\,\overline{Q-Q_n}|_n}{\overline{(Q-Q_n)^2}|_n - \overline{Q-Q_n}|_n^2}$$

$$h_n = \frac{2\,\overline{\cos(\omega t)}|_n\,\overline{(Q-Q_n)^2}|_n - 2\,\overline{(Q-Q_n)\cos(\omega t)}|_n\,\overline{Q-Q_n}|_n}{\overline{(Q-Q_n)^2}|_n - \overline{Q-Q_n}|_n^2}$$

Amplitude of the annual modulation of $f_1(v)$

- Unnormalized velocity distribution function

$$f_{1,\mathrm{un},(m)}(Q) = -Q \cdot \frac{d}{dQ}\left[\frac{1}{F^2(Q)}\left(\frac{dR}{dQ}\right)_{(m)}\right] \qquad m = 0,\ 1,\ 2,\ \cdots$$

- Ratio of $f_{1,\mathrm{un},(1)}(Q)$ to $f_{1,\mathrm{un},(0)}(Q)$ at $Q = Q_n$

$$\eta_n \equiv \frac{f_{1,\mathrm{un},(1),n}(Q_n)}{f_{1,\mathrm{un},(0),n}(Q_n)} = h_n - l_n\left[\frac{2}{F(Q_n)}\left(\frac{dF}{dQ}\right)_{Q=Q_n} - k_n\right]^{-1}$$

- Three advantages:
  - Each one of these η's is independent of the other ones.
  - We can calculate the η's even if we can not obtain enough data in the high energy range.
  - The η's are independent of α.

Summary

- From a dR/dQ curve of the WIMP-nucleus elastic scattering, we can derive $f_1(v)$, $\langle v \rangle$, and $\bar{v}$ of WIMPs.
- From experimental data $Q_{n,i}$, $i = 1, 2, \cdots, N_n$, $n = 1, 2, \cdots, N$, of the WIMP direct detection over some whole years, we can construct $f_1(v)$, $\langle v \rangle$, and $\bar{v}$ directly.
- Our constructions of $f_1(v)$, $\langle v \rangle$, and $\bar{v}$ are independently of each other and of some as yet unknown quantities, e.g. WIMP density.
- We can also construct the amplitude of the annual modulation of $f_1(v)$ without knowing the WIMP mass.