Model-Independent Data Analyses of the WIMP-Nucleon Cross Sections in Direct Dark Matter Detection

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What can we do with direct detection data

- Differential event rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\rm min}}^{v_{\rm esc}} \left[\frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\rm min} = \alpha \sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy Q in the detector.

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{\rm r,N}^2} \qquad \alpha \equiv \sqrt{\frac{m_{\rm N}}{2 m_{\rm r,N}^2}} \qquad m_{\rm r,N} = \frac{m_\chi m_{\rm N}}{m_\chi + m_{\rm N}}$$

ρ_0: WIMP density near the Earth

σ_0: total cross section ignoring the form factor suppression

$F(Q)$: elastic nuclear form factor

$f_1(v)$: one-dimensional velocity distribution of halo WIMPs

What can we do with direct detection data

- Determining the moments of the velocity distribution of halo WIMPs

$$\langle v^n \rangle = \alpha^n \left[\frac{2Q_{\rm thre}^{1/2} r_{\rm thre}}{F^2(Q_{\rm thre})} + I_0 \right]^{-1} \left[\frac{2Q_{\rm thre}^{(n+1)/2} r_{\rm thre}}{F^2(Q_{\rm thre})} + (n+1) I_n \right]$$

$$I_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)} \qquad r_{\rm thre} = \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}}$$

[M. Drees and CLS, JCAP 0706, 011]

- Determining the WIMP mass

$$m_\chi = \frac{\sqrt{m_X m_Y} - m_X \mathcal{R}_n}{\mathcal{R}_n - \sqrt{m_X/m_Y}}$$

$$\mathcal{R}_n \equiv \frac{\alpha_Y}{\alpha_X}$$

$$= \left[\frac{2Q_{{\rm thre},X}^{(n+1)/2} r_{{\rm thre},X} + (n+1) I_{n,X} F_X^2(Q_{{\rm thre},X})}{2Q_{{\rm thre},X}^{1/2} r_{{\rm thre},X} + I_{0,X} F_X^2(Q_{{\rm thre},X})} \right]^{1/n} \Big(X \longrightarrow Y \Big)^{-1} \qquad (n \neq 0)$$

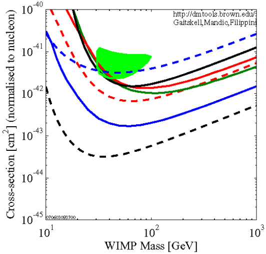
[CLS and M. Drees, arXiv:0710.4296]

Motivation

- Determining the nature of halo WIMPs?

- (Neutralino) LSP or LKP?
  e.g., G. Bertone *et al.*, PRL 99, 151301 (2007)

- Without knowing the WIMP mass?



[http://dmtools.berkeley.edu/limitplots/]

- Determining the local WIMP density?

Ratio of two WIMP-nucleus cross sections

- −1-st moment of the WIMP velocity distribution

$$\left(\frac{dR}{dQ}\right)_{Q=Q_{\rm thre}} = \mathcal{E}\mathcal{A}F^2(Q_{\rm thre})\int_{v_{\rm min}(Q_{\rm thre})}^{v_{\rm esc}}\left[\frac{f_1(v)}{v}\right]dv$$

$$= \mathcal{E}\left(\frac{\rho_0\sigma_0}{2m_\chi m_{\rm r,N}^2}\right)F^2(Q_{\rm thre})\cdot\frac{1}{\alpha}\left[\frac{2r_{\rm thre}}{2Q_{\rm thre}^{1/2}r_{\rm thre}+I_0F^2(Q_{\rm thre})}\right]$$

- Determining the local WIMP density (or the total cross section)

$$\rho_0\sigma_0 = \left(\frac{1}{\mathcal{E}}\right)m_\chi m_{\rm r,N}\sqrt{\frac{m_{\rm N}}{2}}\left[\frac{2Q_{\rm thre}^{1/2}r_{\rm thre}}{F^2(Q_{\rm thre})}+I_0\right]$$

- Ratio of two WIMP-nucleus cross sections

$$\frac{\sigma_{0,X}}{\sigma_{0,Y}} = \left(\frac{\mathcal{E}_Y}{\mathcal{E}_X}\right)\frac{m_{{\rm r},X}\sqrt{m_X}}{m_{{\rm r},Y}\sqrt{m_Y}}\left[\frac{2Q_{{\rm thre},X}^{1/2}r_{{\rm thre},X}+I_{0,X}F_X^2(Q_{{\rm thre},X})}{2Q_{{\rm thre},Y}^{1/2}r_{{\rm thre},Y}+I_{0,Y}F_Y^2(Q_{{\rm thre},Y})}\right]\left[\frac{F_Y^2(Q_{{\rm thre},Y})}{F_X^2(Q_{{\rm thre},X})}\right]$$

Only the SI cross section

- Spin-independent (SI) WIMP-nucleus cross section (neutralino)

$$\sigma_0^{\rm SI} = \left(\frac{4}{\pi}\right) m_{\rm r,N}^2 \Big[Z f_{\rm p} + (A-Z) f_{\rm n}\Big]^2 \simeq A^2 \left(\frac{m_{\rm r,N}}{m_{\rm r,p}}\right)^2 \sigma_{\chi{\rm p}}^{\rm SI}$$

$$\sigma_{\chi{\rm p}}^{\rm SI} \equiv \left(\frac{4}{\pi}\right) m_{\rm r,p}^2 f_{\rm p}^2$$

$f_{\rm p}$, $f_{\rm n}$: effective WIMP-proton/neutron SI coupling

- Determining the WIMP mass

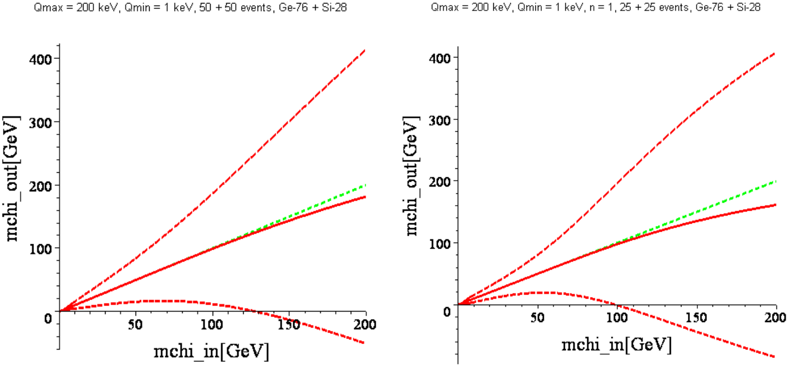
$$m_\chi^{\rm SI} = \frac{\sqrt{m_X m_Y} - m_X \mathcal{R}_0^{\rm SI}}{\mathcal{R}_0^{\rm SI} - \sqrt{m_X/m_Y}}$$

$$\mathcal{R}_0^{\rm SI} \equiv \left(\frac{m_Y}{m_X}\right)^2 \mathcal{R}_0$$

$$\mathcal{R}_0 \equiv \left[\frac{2 Q_{{\rm thre},X}^{1/2} r_{{\rm thre},X} + I_{0,X} F_X^2(Q_{{\rm thre},X})}{\mathcal{E}_X F_X^2(Q_{{\rm thre},X})}\right] (X \longrightarrow Y)^{-1}$$

Only the SI cross section

- Reproduced WIMP mass $m_\chi^{\rm SI}$ / m_χ
  (1 – 200 keV, ^{76}Ge + ^{28}Si, 50 + 50 / 25 + 25 events)



[CLS and M. Drees, arXiv:0710.4296]

➢ A smaller deviation, but a larger statistical error!

Only the SD cross section

- Spin-dependent (SD) WIMP-nucleus cross section

$$\sigma_0^{\rm SD} = \left(\frac{32}{\pi}\right) G_F^2\, m_{\rm r,N}^2 \left(\frac{J+1}{J}\right) \Big[a_{\rm p}\langle S_{\rm p}\rangle + a_{\rm n}\langle S_{\rm n}\rangle\Big]^2$$

$$\sigma_{\chi{\rm p/n}}^{\rm SD} = \left(\frac{32}{\pi}\right) G_F^2\, m_{\rm r,p/n}^2 \cdot \left(\frac{3}{4}\right) a_{\rm p/n}^2$$

J: total nuclear spin

$\langle S_{\rm p}\rangle$, $\langle S_{\rm n}\rangle$: expectation value of the proton/neutron group spin

$a_{\rm p}$, $a_{\rm n}$: effective WIMP-proton/neutron SD coupling

- $m_\chi^{\rm SD} = m_\chi$

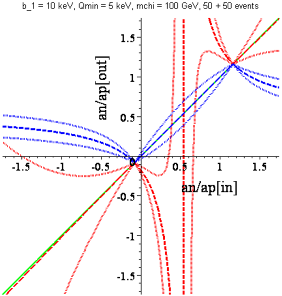
$$\mathcal{R}_0^{\rm SD} \equiv \left(\frac{J_X}{J_X+1}\right)\left(\frac{J_Y+1}{J_Y}\right)\left[\frac{a_{\rm p}\langle S_{\rm p}\rangle_Y + a_{\rm n}\langle S_{\rm n}\rangle_Y}{a_{\rm p}\langle S_{\rm p}\rangle_X + a_{\rm n}\langle S_{\rm n}\rangle_X}\right]^2 \mathcal{R}_0 = \mathcal{R}_n$$

- Ratio of two SD WIMP-nucleon couplings

$$\left(\frac{a_{\rm n}}{a_{\rm p}}\right)_\pm^{\rm SD} = -\frac{\langle S_{\rm p}\rangle_X \pm \langle S_{\rm p}\rangle_Y \mathcal{R}_J}{\langle S_{\rm n}\rangle_X \pm \langle S_{\rm n}\rangle_Y \mathcal{R}_J} \qquad \mathcal{R}_J \equiv \left[\left(\frac{J_X}{J_X+1}\right)\left(\frac{J_Y+1}{J_Y}\right)\frac{\mathcal{R}_0}{\mathcal{R}_n}\right]^{1/2}$$

Only the SD cross section

- Reproduced $(a_n/a_p)^{SD}_{\pm}$
  (5 – 15 keV, $^{73}Ge + ^{37}Cl$, 50 + 50 events, $m_\chi = 100\ GeV/c^2$)



- Two intersections: $-\langle S_p\rangle_X/\langle S_n\rangle_X$, $-\langle S_p\rangle_Y/\langle S_n\rangle_Y$
- $(a_n/a_p)^{SD}_{+}$ or $(a_n/a_p)^{SD}_{-}$: depends on $\langle S_n\rangle_X \pm \langle S_n\rangle_Y \mathcal{R}_J$
- $\sigma(a_n/a_p)^{SD}_{\pm}$ is independent of m_χ (for $m_\chi \geq 30\ GeV/c^2$)
- Need only events in low energy range!

Combining the SI and SD cross sections

- Differential rate for the combination of the SI and SD cross sections

$$\frac{dR}{dQ} = \mathcal{A}' \mathcal{F}(Q) \int_{v_{\rm min}}^{v_{\rm esc}} \left[\frac{f_1(v)}{v} \right] dv$$

with

$$\mathcal{A}' \equiv \frac{\rho_0}{2 m_\chi m_{\rm r,N}^2} \qquad \mathcal{F}(Q) \equiv \sigma_0^{\rm SI} F_{\rm SI}^2(Q) + \sigma_0^{\rm SD} F_{\rm SD}^2(Q)$$

- Determining the local WIMP density

$$\rho_0 = \left(\frac{1}{\mathcal{E}} \right) m_\chi m_{\rm r,N} \sqrt{\frac{m_{\rm N}}{2}} \left[\frac{2 Q_{\rm thre}^{1/2} r_{\rm thre}}{\mathcal{F}(Q_{\rm thre})} + I_0 \right] \qquad I_n = \sum_a \frac{Q_a^{(n-1)/2}}{\mathcal{F}(Q_a)}$$

- Eliminating I_0

$$\frac{\mathcal{F}_X(Q_{{\rm thre},X})}{\mathcal{F}_Y(Q_{{\rm thre},Y})} = \left(\frac{\mathcal{E}_Y}{\mathcal{E}_X} \right) \frac{m_{{\rm r},X} \sqrt{m_X}}{m_{{\rm r},Y} \sqrt{m_Y}} \left[\frac{2 Q_{{\rm thre},X}^{1/2} r_{{\rm thre},X} + I_{0,X} \mathcal{F}_X(Q_{{\rm thre},X})}{2 Q_{{\rm thre},Y}^{1/2} r_{{\rm thre},Y} + I_{0,Y} \mathcal{F}_Y(Q_{{\rm thre},Y})} \right]$$

$$= \left(\frac{\mathcal{E}_Y}{\mathcal{E}_X} \right) \frac{m_{{\rm r},X} \sqrt{m_X}}{m_{{\rm r},Y} \sqrt{m_Y}} \left(\frac{r_{{\rm thre},X}}{r_{{\rm thre},Y}} \right) \mathcal{R}_{-1} = \left(\frac{r_{{\rm thre},X}}{\mathcal{E}_X} \right) \left(\frac{\mathcal{E}_Y}{r_{{\rm thre},Y}} \right) \left(\frac{m_{{\rm r},X}}{m_{{\rm r},Y}} \right)^2$$

Combining the SI and SD cross sections

- Ratio of two WIMP-nucleon cross sections

$$\frac{\sigma_{\chi\mathrm{p}}^{\mathrm{SD}}}{\sigma_{\chi\mathrm{p}}^{\mathrm{SI}}} = \frac{F_{\mathrm{SI},Y}^2(Q_{\mathrm{thre},Y})\mathcal{R}_{m,XY} - F_{\mathrm{SI},X}^2(Q_{\mathrm{thre},X})}{\mathcal{C}_{\mathrm{p},X} F_{\mathrm{SD},X}^2(Q_{\mathrm{thre},X}) - \mathcal{C}_{\mathrm{p},Y} F_{\mathrm{SD},Y}^2(Q_{\mathrm{thre},Y})\mathcal{R}_{m,XY}}$$

$$\mathcal{C}_{\mathrm{p}} \equiv \frac{4}{3}\left(\frac{J+1}{J}\right)\left[\frac{\langle S_{\mathrm{p}}\rangle + (a_{\mathrm{n}}/a_{\mathrm{p}})\langle S_{\mathrm{n}}\rangle}{A}\right]^2 \qquad \mathcal{R}_{m,XY} \equiv \left(\frac{r_{\mathrm{thre},X}}{\mathcal{E}_X}\right)\left(\frac{\mathcal{E}_Y}{r_{\mathrm{thre},Y}}\right)\left(\frac{m_Y}{m_X}\right)^2$$

- Ratio of two SD WIMP-nucleon couplings (3 nuclei, $\langle S_{\mathrm{p/n}}\rangle_Z = 0$)

$$\left(\frac{a_{\mathrm{n}}}{a_{\mathrm{p}}}\right)_{\pm}^{\mathrm{SI+SD}} = \frac{-\left(c_{\mathrm{p},X} s_{\mathrm{n/p},X} - c_{\mathrm{p},Y} s_{\mathrm{n/p},Y}\right) \pm \sqrt{c_{\mathrm{p},X} c_{\mathrm{p},Y}}\left|s_{\mathrm{n/p},X} - s_{\mathrm{n/p},Y}\right|}{c_{\mathrm{p},X} s_{\mathrm{n/p},X}^2 - c_{\mathrm{p},Y} s_{\mathrm{n/p},Y}^2}$$

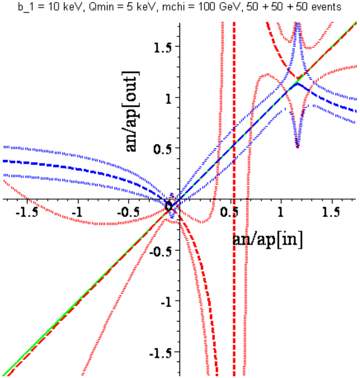
$$= -\frac{\sqrt{c_{\mathrm{p},X}} \mp \sqrt{c_{\mathrm{p},Y}}}{\sqrt{c_{\mathrm{p},X}}\, s_{\mathrm{n/p},X} \mp \sqrt{c_{\mathrm{p},Y}}\, s_{\mathrm{n/p},Y}} \qquad (s_{\mathrm{n/p},X} > s_{\mathrm{n/p},Y})$$

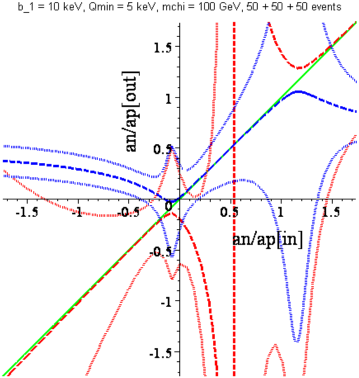
$$c_{\mathrm{p},X} \equiv \frac{4}{3}\left(\frac{J_X+1}{J_X}\right)\left[\frac{\langle S_{\mathrm{p}}\rangle_X}{A_X}\right]^2 \left[F_{\mathrm{SI},Z}^2(Q_{\mathrm{thre},Z})\mathcal{R}_{m,YZ} - F_{\mathrm{SI},Y}^2(Q_{\mathrm{thre},Y})\right] F_{\mathrm{SD},X}^2(Q_{\mathrm{thre},X})$$

$$s_{\mathrm{n/p}} \equiv \frac{\langle S_{\mathrm{n}}\rangle}{\langle S_{\mathrm{p}}\rangle}$$

Combining the SI and SD cross sections

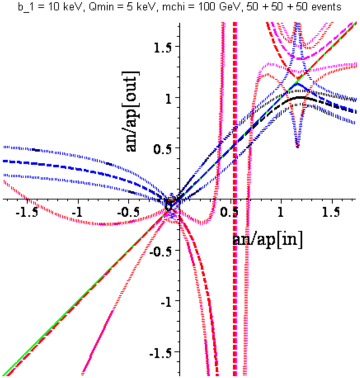
- Reproduced $(a_n/a_p)^{SI+SD}_{\pm}$
  (5 – 15 keV, ^{73}Ge + ^{37}Cl + ^{28}Si, 50 + 50 + 50 events,
  $\sigma^{SI}_{\chi p} = 5 \times 10^{-10}$ pb / 10^{-8} pb, $a_p = 0.1$, $m_\chi = 100$ GeV/c^2)





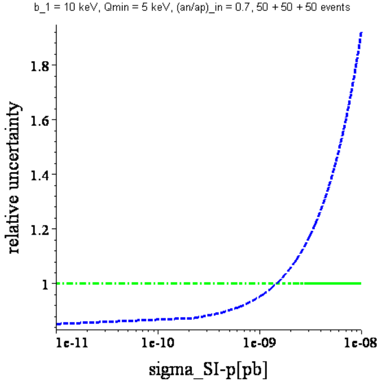
Combining the SI and SD cross sections

- Reproduced $(a_n/a_p)^{SI+SD}_{\pm}$ and $(a_n/a_p)^{SD}_{\pm}$
  (5 – 15 keV, $^{73}Ge + {}^{37}Cl + {}^{28}Si$, 50 + 50 + 50 events,
  $\sigma^{SI}_{\chi p} = 5 \times 10^{-10}$ pb / 10^{-8} pb, $a_p = 0.1$, $m_\chi = 100$ GeV/c^2)



Combining the SI and SD cross sections

- Reproduced $\sigma\left(\sigma_{\chi p}^{SD}/\sigma_{\chi p}^{SI}\right)/\left(\sigma_{\chi p}^{SD}/\sigma_{\chi p}^{SI}\right)$ / $\sigma\left(\sigma_{\chi n}^{SD}/\sigma_{\chi p}^{SI}\right)/\left(\sigma_{\chi n}^{SD}/\sigma_{\chi p}^{SI}\right)$
  (5 – 15 keV, ^{73}Ge + ^{37}Cl + ^{28}Si, 50 + 50 + 50 events,
  $a_p = 0.1$, $a_n/a_p = 0.7$, $m_\chi = 100$ GeV/c^2)



Combining the SI and SD cross sections

- Ratio of two WIMP-nucleon cross sections

$$\frac{\sigma_{\chi p}^{\rm SD}}{\sigma_{\chi p}^{\rm SI}} = \frac{F_{{\rm SI},Y}^2(Q_{{\rm thre},Y})\mathcal{R}_{m,XY} - F_{{\rm SI},X}^2(Q_{{\rm thre},X})}{\mathcal{C}_{{\rm p},X} F_{{\rm SD},X}^2(Q_{{\rm thre},X}) - \mathcal{C}_{{\rm p},Y} F_{{\rm SD},Y}^2(Q_{{\rm thre},Y})\mathcal{R}_{m,XY}}$$

with

$$\mathcal{C}_{\rm p} \equiv \frac{4}{3}\left(\frac{J+1}{J}\right)\left[\frac{\langle S_{\rm p}\rangle + (a_{\rm n}/a_{\rm p})\langle S_{\rm n}\rangle}{A}\right]^2$$

$$\mathcal{R}_{m,XY} \equiv \left(\frac{r_{{\rm thre},X}}{\mathcal{E}_X}\right)\left(\frac{\mathcal{E}_Y}{r_{{\rm thre},Y}}\right)\left(\frac{m_Y}{m_X}\right)^2$$

- Reducing the uncertainty:
  - ➢ Choosing $\langle S_{\rm p/n}\rangle_Y = 0$

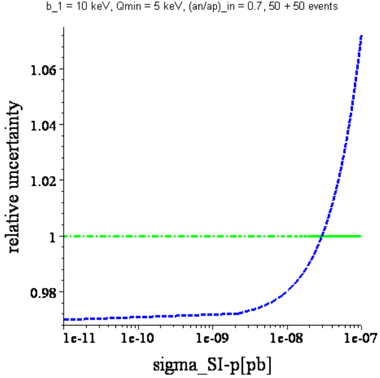
    $$\mathcal{C}_{{\rm p},Y} = 0$$

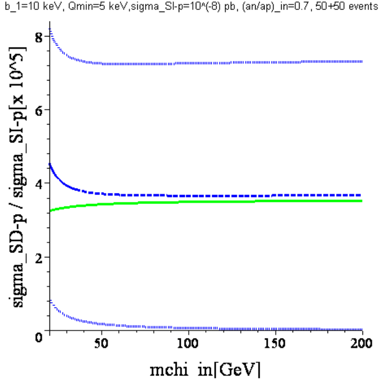
  - ➢ Choosing $\langle S_{\rm p}\rangle_X \gg \langle S_{\rm n}\rangle_X \simeq 0$ or $\langle S_{\rm n}\rangle_X \gg \langle S_{\rm p}\rangle_X \simeq 0$

    $$\mathcal{C}_{{\rm p},X} \simeq \frac{4}{3}\left(\frac{J_X+1}{J_X}\right)\left[\frac{\langle S_{\rm p}\rangle_X}{A_X}\right]^2 \qquad \mathcal{C}_{{\rm n},X} \simeq \frac{4}{3}\left(\frac{J_X+1}{J_X}\right)\left[\frac{\langle S_{\rm n}\rangle_X}{A_X}\right]^2$$

Combining the SI and SD cross sections

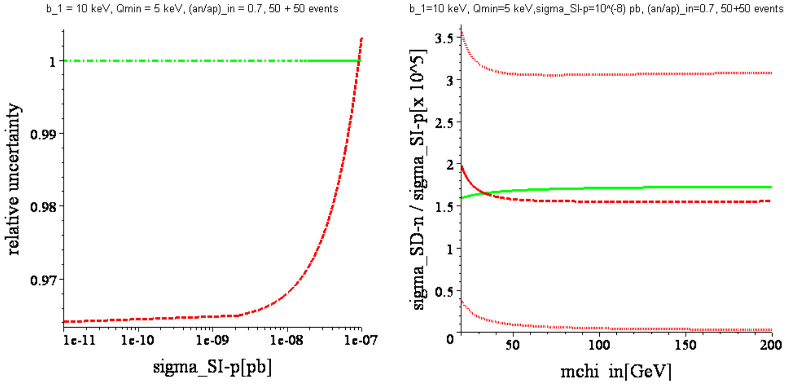
- Reproduced $\sigma\left(\sigma_{\chi p}^{SD}/\sigma_{\chi p}^{SI}\right) / \left(\sigma_{\chi p}^{SD}/\sigma_{\chi p}^{SI}\right)$ / $\sigma_{\chi p}^{SD}/\sigma_{\chi p}^{SI}$
  (5 – 15 keV, ^{76}Ge + ^{23}Na $\left(\langle S_p \rangle = 0.248,\ \langle S_n \rangle = 0.020\right)$, 50 + 50 events,
  $a_p = 0.1,\ a_n/a_p = 0.7,\ m_\chi = 100\ \mathrm{GeV}/c^2$)





Combining the SI and SD cross sections

- Reproduced $\sigma\left(\sigma_{\chi\rm n}^{\rm SD}/\sigma_{\chi\rm p}^{\rm SI}\right)/\left(\sigma_{\chi\rm n}^{\rm SD}/\sigma_{\chi\rm p}^{\rm SI}\right)$ / $\sigma_{\chi\rm n}^{\rm SD}/\sigma_{\chi\rm p}^{\rm SI}$
  (5 – 15 keV, ^{76}Ge + ^{17}O ($\langle S_{\rm p}\rangle = 0$, $\langle S_{\rm n}\rangle = 0.495$), 50 + 50 events,
  $a_{\rm p} = 0.1$, $a_{\rm n}/a_{\rm p} = 0.7$, $m_\chi = 100\ {\rm GeV}/c^2$)



Summary

- Assuming only the SI cross section, we have a second expression for determining the WIMP mass.
- Assuming only the SD cross section, we can determine a_n/a_p.
- Combining the SI and SD cross sections, we can determine a_n/a_p and $\sigma_{\chi p/n}^{SD}/\sigma_{\chi p}^{SI}$.
- Our method is independent of the halo model as well as the WIMP mass.
- We need only $\mathcal{O}(50)$ measured recoil energies from each experiment in low energy range.

Thank you very much for your attention