Physikalisches Institut
Universität Bonn
Theoretical Physics

Exercise Sheet 4
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Advanced Quantum Theory

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http://www.th.physik.uni-bonn.de/klemm/advancedqm/index.php

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–Exercises–

4.1 Optical Theorem for Central Potentials

For spherical symmetric potentials $V(r)$ (which vanish sufficiently fast for $r \to +\infty$) we derived in the lecture the scattering amplitude

$$f_k(\theta) = \frac{1}{2ik} \sum_{\ell=0}^{+\infty} (2\ell+1) \left(e^{2i\delta_\ell(k)} - 1 \right) P_\ell(\cos\theta) \ , \tag{1}$$

in terms of the polar angle θ, the (real) phase shifts $\delta_\ell(k)$, the wave number k, and the Legendre polynomials P_ℓ.

a) Determine the total cross section $\sigma_{\text{tot}}(k)$ in terms of the phase shifts $\delta_\ell(k)$. *(2 Pts)*

b) Verify for the scattering amplitudes $f_k(\theta)$ the optical theorem. *(4 Pts)*

c) Show that — in the absence of non-generic selection rules — the total cross section $\sigma_{\text{tot}}(k)$ becomes in the low energy limit $k \to 0$

$$\lim_{k\to 0} \sigma_{\text{tot}}(k) = 4\pi a_s^2 \ , \tag{2}$$

in terms of the scattering length $-\frac{1}{a_s} = \lim_{k\to 0} k \cot \delta_0(k)$. *(2 Pts)*

4.2 s-Wave Phase Shift

Derive a formula for $\tan \delta_0(k)$ of the s-wave phase shift $\delta_0(k)$ for the central potential

$$V(r) = \begin{cases} -V_0 & \text{for } r < R \ , \\ 0 & \text{for } r \geq R \ , \end{cases} \tag{3}$$

for all energies $E > 0$ and in all orders in $V_0 > 0$. *(6 Pts)*

4.3 Scattering Length for a Finite Range Central Potential

We want to study in the low energy limit the s-wave ($\ell = 0$ mode) scattering of a particle with mass m at a spherical symmetric potential with a finite range $R > 0$, i.e., $V(r) = 0$ for $r > R$ and $\lim_{r\to 0} r^2 V(r) = \text{const.}$

a) Use the first order Born approximation to compute a formula for the scattering length a_s in the low energy limit $kR \ll 1$ as a function of $\kappa(V) := \int_0^R dr\, r^2 V(r)$. *(4 Pts)*

b) Apply the result of a) together with the optical theorem to calculate the imaginary part of the forward scattering amplitude to *second* order in $\kappa(V)$. *(2 Pts)*