

1.) Inner product of descendant states

2 pt

Show that the norm squared of the descendant state $L_{-1}^n |h\rangle$ of the (normalized) primary state $|h\rangle$ of conformal weight h is

$$\langle h | (L_{-1}^n)^\dagger L_{-1}^n |h\rangle = 2^n n! \prod_{\ell=1}^n \left(h + \frac{\ell-1}{2} \right).$$

2.) Gram matrix at level 3

4 pt

Let us consider a (normalized) primary state $|h\rangle$ in a conformal field theory with central charge c .

- a) Determine a basis of descendant states at level 3 and calculate the Gram matrix $M^{(3)}(h, c)$.
- b) Check that the Kac determinant at level 3, i.e., the determinant of $M^{(3)}(h, c)$, is of the right degree in h and that the eigenvalue $h = 0$ appears with the expected multiplicity.

3.) Singular vectors

4 pt

We want to determine the singular vectors at level 3.

- a) Show that a vector $|\chi\rangle$ is singular (i.e., $L_n |\chi\rangle = 0$ for all $n \geq 1$), if $L_1 |\chi\rangle = 0$ and $L_2 |\chi\rangle = 0$.
- b) Determine the two singular vectors $|\chi_{1,3}\rangle$ and $|\chi_{3,1}\rangle$ and at level 3.
Hint: Start with a general ansatz for a descendant state at level 3 of a primary state $|h\rangle$.