Physikalisches Institut
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Theoretische Physik

Exercise 6
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Exercises on Conformal Field Theory

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http://www.th.physik.uni-bonn.de/klemm/cftss16/

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Due on June 3rd, 2016

H 6.1 Four-point correlation function of the free boson *(4 points)*

Let us consider the two-dimensional free conformal field theory of a real boson.

a) Use Wick's theorem to calculate the four-point correlation function

$$\langle \partial_{z_1}\phi(z_1,\bar{z}_1)\partial_{z_2}\phi(z_2,\bar{z}_2)\partial_{z_3}\phi(z_3,\bar{z}_3)\partial_{z_4}\phi(z_4,\bar{z}_4)\rangle$$

of the primary field $\partial_z\phi(z,\bar{z})$.

b) Compare the result with the general expression for four-point correlators

$$\langle \phi_1(z_1,\bar{z}_1)\phi_2(z_2,\bar{z}_2)\phi_3(z_3,\bar{z}_3)\phi_4(z_4,\bar{z}_4)\rangle = f(\eta,\bar{\eta}) \prod_{1\leq k<l\leq 4} (z_k - z_l)^{h/3-h_k-h_l} (\bar{z}_k - \bar{z}_l)^{\bar{h}/3-\bar{h}_k-\bar{h}_l}$$

of quasi-primaries $\phi_k(z,\bar{z})$ of conformal weight h_k and $\bar{h}_k$ with

$$h = \sum_{k=1}^{4} h_k \ , \quad \bar{h} = \sum_{k=1}^{4} \bar{h}_k \ , \quad \eta = \frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_3)(z_2-z_4)} \ .$$

Determine the function $f(\eta,\bar{\eta})$.

H 6.2 Generalized ghost system *(3 points)*

Consider the two-dimensional free conformal field theory of a pair of anti-commuting ghost fields $\mathbf{b}(z)$ and $\mathbf{c}(z)$ with the OPE

$$\mathbf{c}(z)\mathbf{b}(w) \sim \frac{1}{z-w} \ ,$$

and the holomorphic energy momentum tensor

$$T(z) = (1-\lambda) :(\partial_z \mathbf{b}(z))\mathbf{c}(z): - \lambda :\mathbf{b}(z)(\partial_z \mathbf{c}(z)): \ .$$

a) Show that the ghost fields $\mathbf{b}(z)$ and $\mathbf{c}(z)$ have conformal weight λ and $1-\lambda$.

b) Show that the central charge of this system is $c = -2(6\lambda^2 - 6\lambda + 1)$.

c) Which conformal field theory is identified with the generalized ghost system for $\lambda = \frac{1}{2}$.

d) For $\lambda = 2$ we obtain the reparametrization ghost system of the bosonic string. Quantum-mechanically a consistent string theory requires a worldsheet conformal field theory of vanishing central charge. The worldsheet of the bosonic string in D-dimensional Minkowski space $\mathbb{R}^{1,D-1}$ is described by D free bosons together with the reparametrization ghost system. Determine the critical dimension D of Minkowski space, which yields a consistent bosonic string theory on the quantum level.

H 6.3 The Schwarzian derivative *(3 points)*

Given to two successive local conformal transformations $z \to w(z)$ and $w \to u(w)$, demonstrate that the Schwarzian derivative fulfills the chain rule property

$$\{u; z\} = \{w; z\} + \left(\frac{dw}{dz}\right)^2 \{u; w\} \ .$$