Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 7
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Exercises on Conformal Field Theory

Dr. Hans Jockers, Andreas Gerhardus

http://www.th.physik.uni-bonn.de/klemm/cftss16/

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Due on June 10th, 2016

H 7.1 Correlation functions of descendant fields *(2 points)*

Give a general formula for the correlation function

$$\left\langle \phi_0^{\{\vec{k}\}\{\vec{\bar{k}}\}}(w,\bar{w})\,\phi_1(w_1,\bar{w}_1)\,\ldots\,\phi_N(w_N,\bar{w}_N)\right\rangle$$

of one arbitrary descendant field $\phi_0^{\{\vec{k}\}\{\vec{\bar{k}}\}}(w,\bar{w})$ with a product of primary fields $\phi_\ell(w_\ell,\bar{w}_\ell)$, $\ell = 1,\ldots,N$, in terms of the primary correlator

$$\left\langle \phi_0(w,\bar{w})\,\phi_1(w_1,\bar{w}_1)\,\ldots\,\phi_N(w_N,\bar{w}_N)\right\rangle ,$$

and suitable differential operators.

H 7.2 Energy momentum tensor as a descendant field *(3 points)*

We want to identity the energy momentum tensor a descendant field.

a) Express the holomorphic energy momentum tensor $T(z)$ as a descendant field of the identity operator $\mathbf{1}$.

b) Use the expression of the energy momentum tensor as descendant field to argue that $T(z)$ transforms as a quasi-primary field.

   Hint: Consider the Virasoro generators for global conformal transformations.

c) Use the expression for correlation functions of primaries with a descendant field to rederive the Conformal Ward Identity for primary fields.

H 7.3 Operator algebra of primary fields *(5 points)*

Let us consider the generic form of the operator algebra of two primary fields

$$\phi_k(z,\bar{z})\phi_\ell(0,0) \;=\; \sum_{\substack{s\\ \text{primaries}}}\sum_{\{\vec{k}\}}\sum_{\{\vec{\bar{k}}\}} C^s_{k\ell}\beta^{s\{\vec{k}\}}_{k\ell}\beta^{s\{\vec{\bar{k}}\}}_{k\ell}\, z^{h_s-h_k-h_\ell+|\vec{k}|}\,\bar{z}^{\bar{h}_s-\bar{h}_k-\bar{h}_\ell+|\vec{\bar{k}}|}\phi_s^{\{\vec{k}\}\{\vec{\bar{k}}\}}(0,0) .$$

For simplicity we assume in the following that the primaries ϕ_k and ϕ_ℓ have conformal weights $h_k = h_\ell = h$.

a) With the help of the Virasoro algebra evaluate the commutators $[L_1, L_{-1}^2]$, $[L_2, L_{-1}^2]$, $[L_1, L_{-2}]$, and $[L_2, L_{-2}]$.

b) In the lecture we determined $\beta_{k\ell}^{s\{1\}} = \frac{1}{2}$. Using the results of a), show that the constants $\beta_{k\ell}^{s\{1,1\}}$ and $\beta_{k\ell}^{s\{2\}}$ are given by

$$\beta_{k\ell}^{s\{1,1\}} = \frac{c - 12h - 4h_s + ch_s + 8h_s^2}{4(c - 10h_s + 2ch_s + 16h_s^2)}\,, \qquad \beta_{k\ell}^{s\{2\}} = \frac{2h - h_s + 4hh_s + h_s^2}{c - 10h_s + 2ch_s + 16h_s^2}\,,$$

in terms of the conformal weights h and h_s, and the central charge c.